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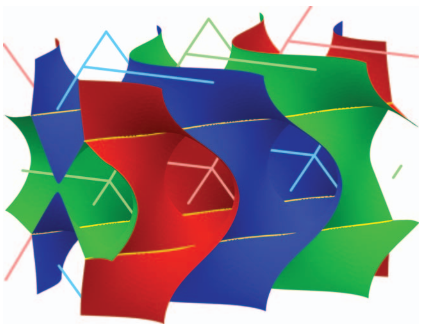
CONTEMPORARY GEOMETRY FOR THE BUILT DESIGN?

I explore the terrain that lies between architecture and geometry, from the perspective of a structural scientist with no professional architectural expertise. The divide between these disciplines perhaps stems from an ancient dichotomy between the art versus engineering schools of architecture, fertilised by the current dogma that art and science can never meet. Architects stand to gain much from study of the spectacular advances in geometry in recent decades, such as the growing understanding of cellular patterns in space, tiles, nets and curved surfaces. Some examples of those advances are discussed in detail. I conclude that both architecture and geometry would benefit from a renewed mutual interest.



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Introduction

I am neither a theoretician nor a practitioner of architecture. I am an amateur whose only qualification is an interest in architecture, that perhaps began as a young child, looking in awe at the scaffolded half-built shells of the Sydney Opera House rising over the water from the deck of a Californian passenger liner as it sailed into Sydney Harbour. As an adolescent in the 1960s, I became an aficionado of the built design, visiting vast tracts of bare subdivisions in beachside Perth with my family, impatiently waiting for our own house to be built. Out of that dazzling summer sand sprang streets, shopping centres and all manner of residential constructions. I took great pride in judging the quality of each design, from the A.V. Jennings box, to the outlandish announcements of WA Nickel Boom wealth, mentally comparing all of them with the clean, modern lines of our own emerging house.

Slowly, I became aware of the technical subtleties of architecture. A building must stand up as well as excite the eye. It should offer both shade and light. As a teenager, my favourite buildings were now-forgotten Modernist Perth suburban houses. As a young university science student, I came across the engineering masterpieces of Robert Maillart's bridges, and my inner meter of architectural purity swung towards the calculated beauty of Maillart.

The nineteenth century French engineer, Rondelet—who among other accomplishments saved Soufflot's Panthéon dome from collapse—expressed succinctly one aspect of this dichotomy:

> Architecture is not an art like painting or sculpture … It is a science whose essential aim is to construct solid buildings which deploy the finest of forms and the aptest of dimensions to unite all the parts necessary for their purpose.[1]

The complementary view is equally prevalent. For example, Richard Meier is unequivocal in his view of architecture:

> A work of architecture is … a work of art.[2]

Surely an architect must be both artist and engineer; those two faces of architecture have pushed and pulled architecture regularly over the past centuries. Every minute I spend in Jean Nouvel's dynamically-shuttered Institut du Monde Arabe in Paris thrills me, no more or less than my view through the open window as I write now, across the faded rooftops of the Albaicín district in Granada, Andalucía, to the hillside opposite, on which sits the jewel-like Alhambra Palace.

Nowadays I inevitably view architecture and engineering from the perspective of a practising scientist. I am a physicist by training, and spend most of my professional time exploring the links between pure geometry and the shapes adopted by atoms and molecules in crystals and liquid crystals. My own work is driven by the wealth of new discoveries and creations of modern geometers, crystallographers, physicists, chemists and materials scientists. We now routinely speak of "molecular architecture" and "materials design" to describe our own efforts to build structures and patterns of ever-increasing complexity in the microscopic world. At the same time, much of our work is driven by the acute awareness that the architecture of nature remains the pinnacle of materials design. The extraordinary structural complexity and economy of construction of materials in living systems, such as the silica skeleton of marine diatoms (Fig. 1(a)), or a wing scale in a butterfly (Fig. 1(b)), remains an ideal for modern materials design that scientists have yet to approach.

My view is that a third approach—Science—can be added to the Architectural and Engineering schools of design. This is surely shared by many architects and represents a point of contact between the sciences and architecture and engineering. It seems that one of the great scientists in my own area, D'Arcy Wentworth Thompson (who pioneered the understanding of natural materials and mathematical form in his magisterial book, *On Growth and Form*[3]), is also a hero to many architects. Thompson pioneered the idea of natural structure, such as bone, as a "diagram of forces" (see Fig. 2), a concept that Maillart would have felt comfortable with.

The Architect and Engineer as Artist

While some architects apparently embrace the findings of scientists, I sense a reluctance among other architects—even engineers—to delve into the more mathematical aspects of science. Modern science is specialised and comes with its own jargon. Those untrained in science are often unable to comment on the developments of pure scientific research, since its creations are often written in languages that can only be read after some learning. That situation is in stark contrast to architecture which perhaps suffers the opposite fate. Unlike science, even the uninitiated—from Prince Charles[4] to this scientist—assume the right to comment on the relative skills of the architect.

I suspect that many designers feel a fundamental distinction between design and the mathematical sciences. The painter and friend of Frank Gehry, Ed Ruscha, expresses this sense all too clearly while describing Gehry's work in Sydney Pollack's documentary on Gehry:

> He mixes the free-wheelingness of art with something that is really concrete and unforgiving, which is the laws of physics.[5]

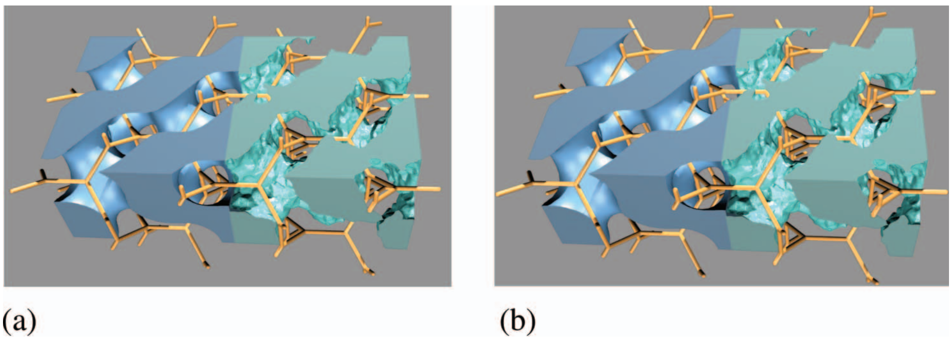

Figure 1. Natural form in living systems. A diatom skeleton, reproduced with kind permission of Hallegraeff (a); and reconstruction from electron microscopic tomography of the chitin wing scale structure in the *Callophrys rubi* (b). The net within the voids of the structure indicate edges of the ideal net that describes the complex 3D arrangement of channels in this sponge-like structure. The (leftmost) smoothed structure represents a mathematical idealisation of the structural data (rougher, rightmost): an ideal mathematical form known as a triply periodic minimal surface. *Source*: Gustaaf Hallegraeff, *Plankton. A Critical Creation*, Sandy Bay, Tas., University of Tasmania, 2006, p. 49.

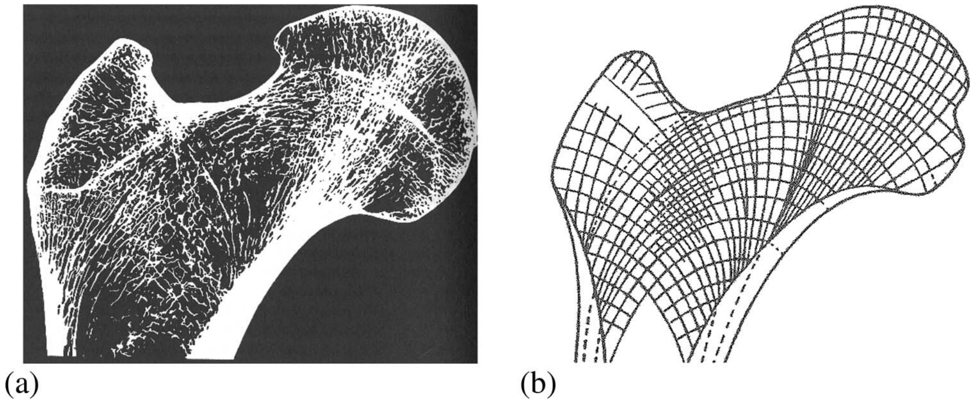

Figure 2. An image of a sectioned human femur (a) and D'Arcy Wentworth Thompson's diagram of forces acting on the bone (b).

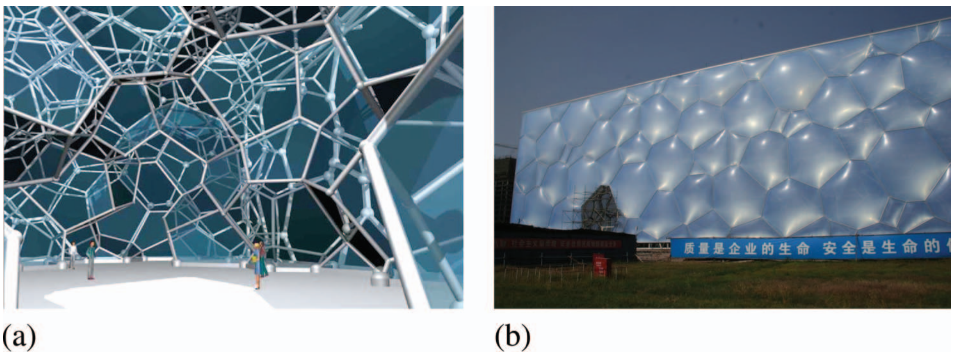

Figure 3. The Water Cube, engineered by Tristram Carfrae, constructed for the 2008 Beijing Olympics. Maquette of the space frame (a). External view of the construction (b). (All images by Arup + PTW + CCDI and are reproduced with the kind permission of Tristram Carfrae).

While that view is perhaps understandable coming from a visual artist, it appears to permeate through to even the most quantitative extremes of design, including civil engineers. Tristram Carfrae, the engineer responsible for the Water Cube in Beijing (Fig. 3), has spoken of a transformation of the discipline of engineering from a form of applied mathematics, to one where the engineer is "happily wallowing in complexity" claiming that with the increased sophistication of today's tools, understanding of the details of their innards is no longer possible. Nowadays, engineers have to be comfortable with "using stuff you don't understand" on structures and processes that "you can't draw; a comfort with informational processes upon which they deploy critical judgements", forcing the practitioner to become "more judgemental, less absolute".[6]

It is certain that, as in science, computers and the web have transformed engineering and architecture in dramatic fashion. The Water Cube arose after Carfrae "found everything I needed on the net in a week".[7] Frank Gehry's spectacular curvilinear forms are refined on the computer and his modelling team are a vital part of the practice. In the Pollack documentary, Gehry describes how his frustration with the design of the external staircase of the Vitra Design Museum led to computational design:

> I started playing with the spiral stair; I loved the way that curve read against the rectilinear. I tried to draw it with descriptive geometry, but when the guy built it there is a kink in it. … That was when I got frustrated and asked the guys in the office: "isn't there a better way to describe these things, cos I'd like to play with curved shapes. If I could just describe them." That's what led us to the computer.[8]

Despite Gehry's heavy reliance on the computer to quantify designs, he remains reliant on physical models: according to Pollack's film, his design work is grounded in model building, often realised by gluing and bending sheets of cardboard, folded, cut and glued to yield a form that pleases. Gehry clearly views the computer as a black-box, from which the computer specialist, the magician, can extract the rabbit of quantitative design. Carfrae is more in tune with the scientific view that a computer is a powerful design medium in its own right, to be harnessed first-hand to realise novel three-dimensional (3D) designs.

A Role for Geometrical Science?

Despite the sheer beauty of their realised designs, I feel some disappointment with both Gehry's and Carfrae's attitudes to modern geometry. Given that both Gehry's helicoidal stair at Vitra and Carfrae's tetrahedral space frame in Beijing are structures long known to certain scientists, many aspects of their design work could have been done with just a modest dose of geometrical training, rather than relying on computational black-boxes.

Sadly, this "tyranny of discipline" can be found among scientists too. A disturbing divide between practitioners of pure mathematics and materials science is also evident.[9]

In Rondelet's day, the Ecole des Beaux Arts in Paris had four professors: one each for history, theory, construction and mathematics.[10] At the time of Rondelet's pronouncement on architecture quoted above, he was the incumbent of the construction post. The explicit inclusion of mathematics as an integral part of architecture education in nineteenth century France was not surprising given the legacy of Napoleonic France, concerned above all with building an empire to rival that of the Romans, characterised by bridges, roads and other monuments to the boundless potential of rational thought. Is the built environment of the twenty-first century a very different place?

Bridging distinct cultures, whether mathematics and physics or chemistry, or architecture and engineering and mathematics, is a big ask. Yet no matter how fast and powerful our computers become, they cannot produce forms from thin air, only refine forms according to human instructions. Given the explosion of new forms in the past few decades in geometry and materials science, architects and engineers should perhaps revisit the philosophy of Rondelet; a wealth of novel forms and concepts are to be found in the sciences. What can modern science offer the aspiring architect? We are now living in a golden age of geometry, with a number of advances that signal the most significant evolution in geometric thinking since the ancient Greeks, whose codification of geometry set the stage for the past two millennia. I have only the room here to mention a few of these advances, necessarily selected from my own areas of interest.

The quantum leap in geometrical thinking that we are currently digesting springs from a truly radical source: non-Euclidean geometry. Though now almost 200 years old, its applications to scientific thinking were until recently confined largely to the rarified echelons of theoretical physics. Nowadays, it is permeating less ethereal areas of science. For example, in my own work an explicitly non-Euclidean approach is routinely used to generate, classify and relate various spatial patterns of all sorts, from complex 3D weavings and knots in space, to novel cellular decompositions (Fig. 4). We are interested in these structures, that arise from the abstract universe of hyperbolic two-dimensional (2D) geometry, in order to understand how atoms and molecules arrange themselves into crystals and liquid crystals at the microscopic scale, visible only at very high magnifications.

Non-Euclidean geometry offers an increasingly interesting aspect of design to our own 3D Euclidean world: curvature. (Non-Euclidean geometry has a lot more to say than that, but this essential core is a theme that allows us to view structure itself in novel ways.) Freed from the Euclidean constraint of flatness, the vocabulary of form now available far exceeds that available to the classical Greeks and their

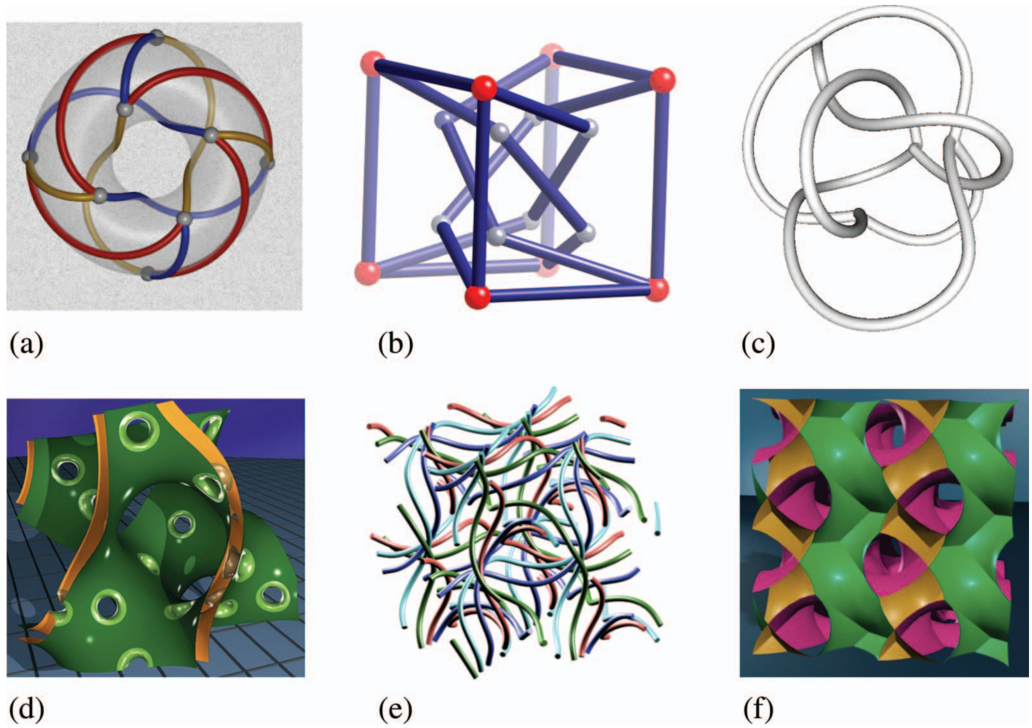

Figure 4. Some patterns that we have explored and designed using ideas and approaches from non-Euclidean geometry.

followers. And that vocabulary allows us to think about the shapes that preoccupy designers, from Gehry to Carfrae, in a practical, quantitative fashion. The mathematical details are to be found elsewhere; here I illustrate that approach with some concrete examples of forms, taken largely from our own interests: tilings, nets and space partitions.

Tiling theory looks at the shapes that can be replicated to fill a space without any overlaps or gaps between those shapes. One of the most fundamental rules of tiling is surely known to all designers: if we are to cover a flat wall with equal and regular polygons: only those with three, four or six sides will tile (Fig. 5).

Artisans have long known that we can go a lot further once we allow the edges bounding the tile to curve. Supremely beautiful examples are to be seen in the Alhambra, such as illustrated in Figure 6.

There are many simple tiling questions that until very recently remained not only unanswered, but fundamentally unable to be answered. That situation has changed radically in recent years: we can now systematically explore tilings at will. A group of mathematicians in Bielefeld, Germany, have in the past two decades constructed a complete constructive theory of tilings that can and is being used to build tilings. Delaney–Dress tiling theory, developed by Andreas Dress[11] with

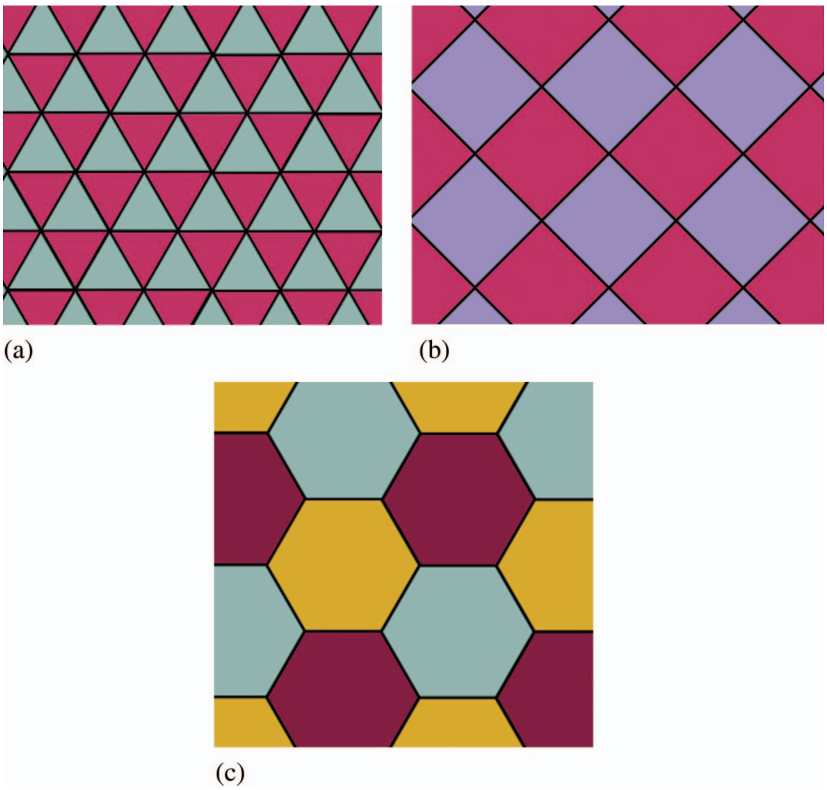

Figure 5. Planar tilings with Platonic polygons: (a) triangles {3,6}, (b) squares {4,4} and (c) hexagons {6,3}.

Figure 6. Some decorations in the Alhambra, Granada. The richness of these ornaments is due, in part, to the presence of curvature. A {3,6} tiling (cf. Fig. 5), transformed by colouring and curvature (a). Ignore the six-pointed stars and the hexagons and a variation of the {6,3} tiling (cf. Fig. 5) is seen (b).

Daniel Huson[12] and Olaf Delgado Friedrichs,[13] is spectacularly useful. It allows us to construct tilings up to an arbitrary degree of complexity in spaces of any dimension or curvature! Olaf Delgado Friedrichs' tiling software is freely available from the web;[14] it can be used to build an endless variety of 2D or 3D tilings with just a modest dose of training.

Look for example, at 3D tiles that can tesselate flat volumes (i.e. Euclidean 3D space), one dimension richer than tessellations of flat walls. Some examples spring to mind immediately. For example, space can be tiled by cubes (Fig. 7). Many other tilings of space by polyhedral cells with straight edges and flat faces—consistent with the Greek notion of polyhedra—are possible. For the first time we are able to construct catalogues of tilings, opening the door to systematic searches of all possible structures up to a given level of complexity.

If we allow the faces of these 3D tiles to be curved rather than just flat—just as for the edges of the Alhambra 2D tiles—the variety of 3D tilings explodes, and a number of new tiles emerge. Some of these tiles were in fact already known in the 1970s: the US architect Peter Pearce wrote a seminal book[15]—perhaps better known to scientists than architects! Indeed, Pearce lamented in 2006:

> I have received much more acknowledgement and citations from the scientific community than from the design/architecture community. … This is particularly perplexing, even troubling, to the extent the fundamental content of the book was driven by design intentions, not scientific discoveries and insights.[16]

Pearce makes some pointed comments on the conservatism of architects in looking towards novel designs for sustainable buildings. In particular, he writes:

> Although there were "moments of glory" along the way, and certainly an amazingly useful "learning curve", in the end I was not able to get beyond the fundamental conservatism that dominates protocol, methodologies, and the limited design visions that constrain the design of buildings in our culture.

One of Pearce's examples is a splendid four-faced saddle polyhedron, formed from a

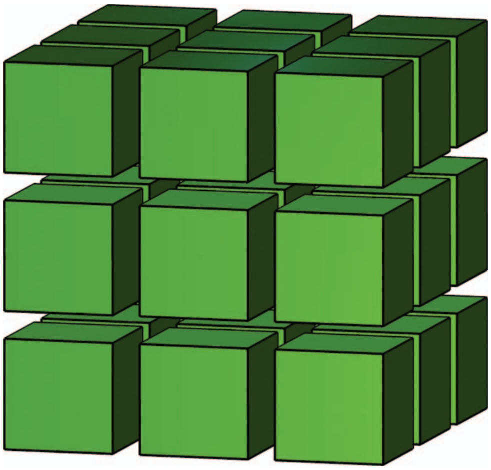

Figure 7. Tiling of (Euclidian) 3D space by cubes.

regular tetrahedron by kinking all the edges at their midpoints. The faces of his 3D tiles are curved, bounded by skew hexagons. All angles between edges in this tile are exactly tetrahedral, as shown in Fig. 8.

Delgado Friedrichs and Michael O'Keeffe have developed this tiling concept in a direction that is of immediate use to our understanding of atomic and molecular crystals. These crystals are usefully described as 3D crystalline networks, or, for short, "nets", whose edges are the chemical bonds within the crystal. Such nets, composed only of edges and nodes, are known to architects as space frames, such as the skeleton of the Water Cube (Fig. 3). For example, a tiling of space with Pearce's saddle polyhedra results in a net of edges well known to all solid state scientists as the structure of diamond. In the diamond crystal, carbon atoms are located at each vertex, each bonded to four neighbouring atoms along tetrahedral directions, illustrated in Fig. 9. Thus the diamond net is a tetrahedral space frame.

This technique of generating nets has led to some important fundamental structural ideas. For the first time, a rigorous classification of the simplest nets has now been done, in some senses on a par with the classical Greek enumeration of Platonic (regular) and Archimedean (semi-regular) polyhedra.[17] There are five and only five regular nets according to this classification, just as there are five Platonic polyhedra (Fig. 10).

Indeed, the regular nets share properties of the Platonic polyhedra: the latter can be viewed as regular nets on the sphere. A student of design will be familiar with these regular nets, since they deserve to be as well known as the regular (Platonic) polyhedra! An excellent website is now online (hosted by the Australian National University) that describes these structures in detail.[18]

We have developed an alternative path to enumerating nets. Our approach is conceptually simple. First we recognise the seminal result of non-Euclidean geometry: there are three generic 2D homogeneous geometries, defined as spaces of constant curvature. These are elliptic, Euclidean and hyperbolic 2D space. These three spaces have respectively positive curvature (as in the surface of a sphere), zero curvature (for example the Euclidean flat plane), or negative curvature (such as a saddle). Fragments of each space are illustrated in Fig. 11. Another way of describing these spaces is to think of them as 2D surfaces, with the important proviso that one does not think of those surfaces as inhabiting some higher dimensional space. Rather we must think of these surfaces as spaces in themselves, much as an ant would conceive the universe were it confined to wander these surfaces without any ability to look anywhere but along the surface.

Two-dimensional tilings, explored so elegantly (and completely) in the patterns on the flat

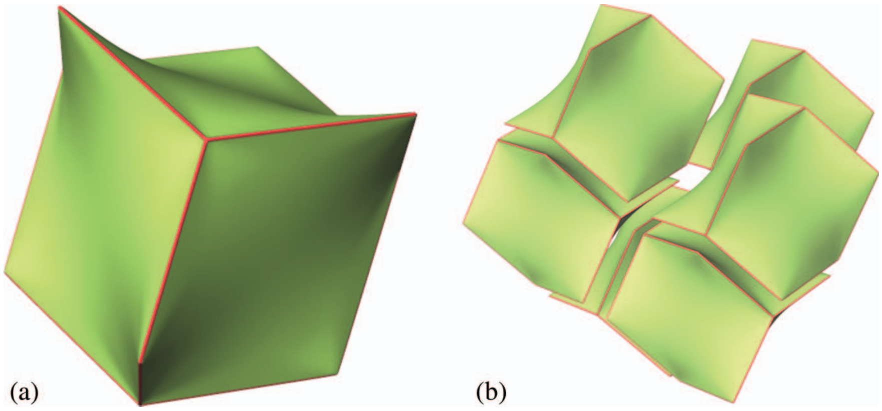

Figure 8. A "saddle-polyhedron" with four identical saddle-shaped faces (a). Tiling of Euclidean 3D space by these saddle polyhedra (b). The net of edges formed by the tiling describes a tetrahedral framework. This tiling was first described by the architect Paul Pearce in the 1970s.[15]

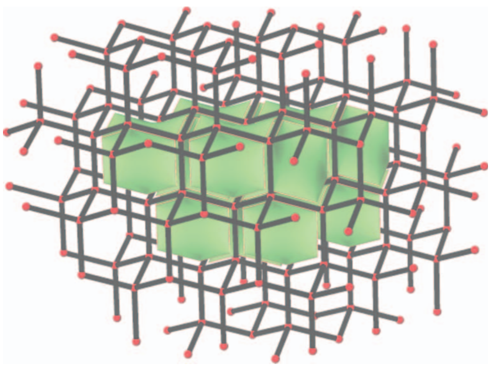

Figure 9. The diamond net, formed by edges of Pearce's saddle polyhedral tiling.

walls of the Alhambra, can be extended to these curved spaces or surfaces, giving a far richer variety than is found in Euclidean geometry, and allowing tilings of curved surfaces. For example, the constructive tiling theory of Dress *et al.* can be applied to 2D curved spaces. Examples of similar motifs "mutated" to form tilings of the simplest 2D spaces, known to mathematicians as the *elliptic*, *Euclidean* and *hyperbolic planes* are shown in Fig. 12.

Hyperbolic 2D tilings have been explored systematically only recently. Two-dimensional hyperbolic space is so superficial, so area-rich, that only very small fragments of it fit into our 3D space, unless we allow the 2D hyperbolic plane to wrap onto itself, over and over, much like the flat plane can be wrapped onto a cylinder with endless windings. If we try to place the hyperbolic plane in 3D Euclidean space it displays a massive degree of crenellation, much like seaweed (Fig. 13) and soon crowds in on itself, so that only small fragments can be "embedded" in our 3D space.

That is why the hyperbolic tilings in Fig. 12 appear to contain smaller and smaller tiles, In actuality, all (like-coloured) tiles are identical. A beautiful movie by Stuart Ramsden[19] shows how this space can be neatly folded within 3D flat space to minimise those distortions. The folding is done by wrapping the hyperbolic plane onto sponge-like surfaces, known as three-periodic minimal surfaces. If we choose simple tilings of hyperbolic space and then embed the edges of the tiling into 3D Euclidean space via these three-periodic

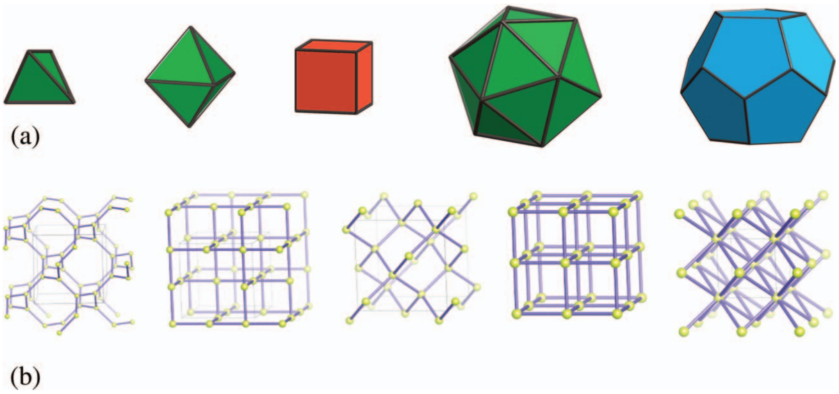

Figure 10. Regular or Platonic polyhedra, described for millennia (top). Regular nets, according to the recent definition of O'Keeffe et al. (bottom).[18]

Figure 11. The three 2D geometries: elliptic, Euclidean and hyperbolic (left to right).

minimal surfaces, we generate nets. So, paradoxically, we gain a privileged entree to 3D flat space via 2D curved space! That concept has allowed us to generate numerous interesting patterns and 3D weavings that are otherwise difficult to build geometrically. We have spent almost a decade now mining that approach and the results are slowly accumulating on the "EPINET" structural database, freely accessible on the web.[20]

En route, we have also found some interesting novel ways to partition space. Look first at the three-periodic minimal surfaces. These surfaces sub-divide space into two interwoven volumes. Each volume forms an open, infinite, extended 3D tile that weaves through space. We therefore have a tiling of space, where each tile contains just a single curved face. And only two are needed to fill space! An example is shown in the left-hand image of Fig. 14. These three-periodic minimal surfaces have been known for many years. Even more complex partitions are now emerging. For example, we have found a number of ways to tile space with just three open interwoven tiles, each containing a single face and an infinite number of straight edges, leading to inter-growth of three discrete volumes (inside, outside and something else …). An example of this structural class can be seen in the right-hand image of Fig. 14.

Closing

We hope that these examples give some idea of the richness of structure that emerges once

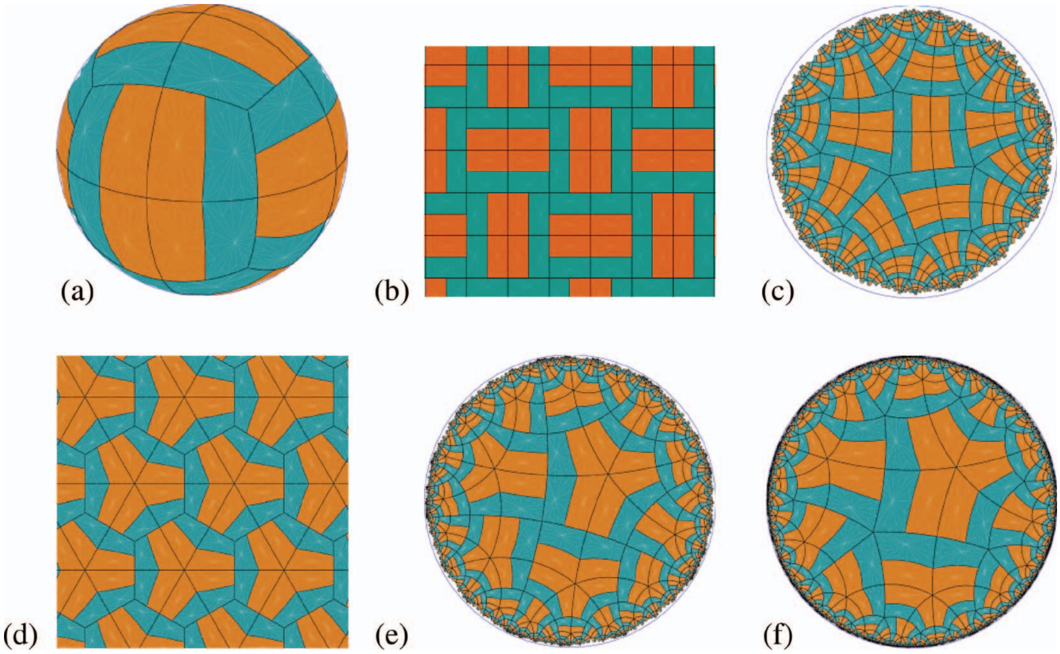

Figure 12. The top row shows related tilings of (a) elliptic, (b) Euclidean and (c) hyperbolic 2D space. Each examples contains two tile shapes. Both tile types are four-sided polygons in each case; the tiling morphs from positive to zero and negatively curved space by changing the number of tiles sharing the star-shaped corners (3, 4 and 5 respectively). The bottom row shows Euclidean (d) and hyperbolic tilings (e, f) formed from hexagonal and quadrilateral tiles. For convenience of representation only, the hyperbolic examples are projected onto a flat circular disc leading to extreme distortions; in actuality these patterns afford tilings of saddle-shaped surfaces with equal tiles, just as the elliptic example tiles the sphere surface.

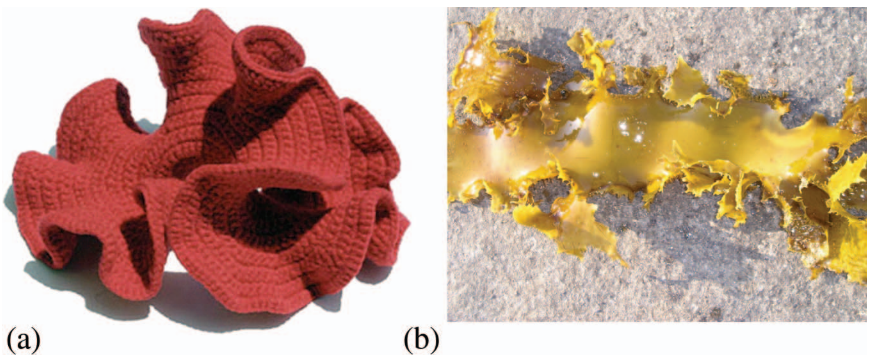

Figure 13. Some fragments of 2D hyperbolic geometry in our own space: a crocheted portion of the hyperbolic plane by D. Taimina (reproduced with permission) and seaweed.

we move beyond Flatland, as well as the sense that these studies remain very active areas of research for many scientists. I suspect that the work required for a designer to digest the tools and results of modern geometry will be amply repaid, and lead to a deeper exploration

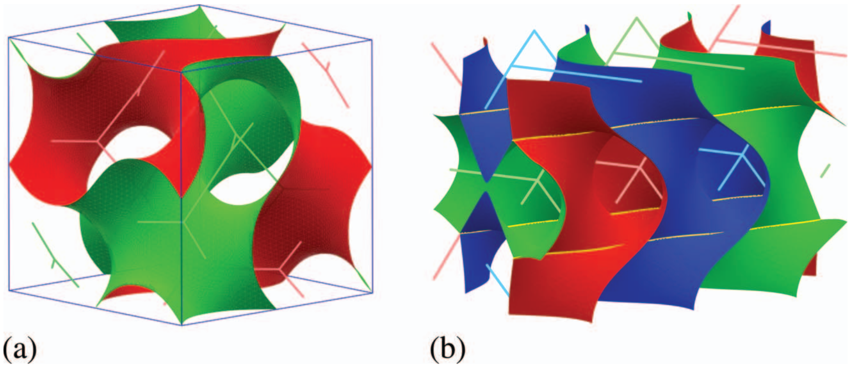

Figure 14. A bicontinuous structure, with just two interwoven 3D tiles (that are each infinite), separated by the three-periodic minimal surface known as the 'gyroid'. The coloured nets indicate the convoluted channels of each tile (a). A new tricontinuous pattern, with three infinite and interwoven 3D tiles, each bounded by minimal surfaces. Each tile is equivalent, and can be interpreted as inflated diamond nets (b) (see also Fig. 9).

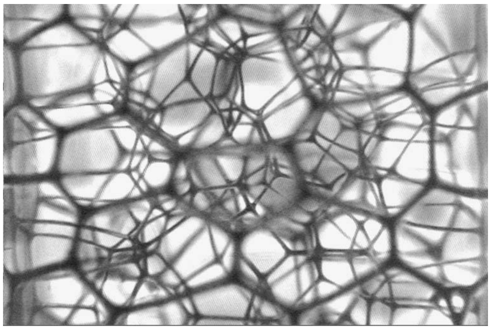

Figure 15. Image of a soap froth or foam. The stable form of any froth follows Plateau's Rules: three faces meet along each edge at 120° and four edges meet at each node, forming tetrahedral junctions. Compare this image with the space frame in Fig. 3.

of form and function of the built environment. Armed with this new perspective, I conclude the paper with another look at the designs of Gehry and Carfrae discussed above.

The Water Cube is surely an impressive example of the potential for cross-fertilisation between science and design. Carfrae states that the Weaire–Phelan pattern he uses to build this structure is structurally interesting in that it is a network with tetrahedral edge angles and is fundamentally novel. That is not the case. There are many, many other examples of tetrahedral nets (or space frames), such as the diamond net, shown in Fig. 9. Indeed, the edges of all closed-cell foams formed by soap films also realise tetrahedral angles at their junctions, as discovered by the blind Belgian physicist, Plateau, in the nineteenth century and proven by Jean Taylor in the 1980s (Fig. 15). So the variety is infinite! To go further, the designer needs to delve into the now mature area of net geometry, and associated fields, such as foams research[21] and the science of cellular materials.[22]

Gehry is an architect of curved surfaces par excellence. Yet I cannot avoid the hunch that he could go much further in his explorations of form were he schooled in aspects of non-Euclidean and curved surface (differential) geometry. For example, a beautiful theorem of differential geometry by Gauss implies that the only shapes that can be constructed from bent planes or sheets of cardboard are developable surfaces (intrinsically flat, with zero curvature). To generate intrinsically curved structures, the planes must be warped, by insertion or deletion of patches (such as "darts" in dressmaking, used to make for example, *jupes à godets*.[23]) Conversely, (doubly-) curved forms cannot be realised with single curved panels. Indeed, that particular design challenge has produced novel and interesting geometry and architecture,[24] resulting in the Austrian "Architectural Geometers" school of Helmut Pottmann and colleagues.[25]

Lastly, I see no reason why architects and engineers cannot contribute to science as much as science can contribute to design in general. For example, the American architect Peter Pearce in the 1970s investigated periodic minimal surfaces and related structures for their design possibilities. Perhaps Pearce was ahead of his time; witness the fascinating design of the Australian Wildlife Health Centre (Healesville, Victoria) by Minifie Nixon from 2006, based on another (non-periodic) minimal surface known to mathematicians as the Costa minimal surface. Periodic minimal surfaces are also reappearing in an architectural context via the work of Pottmann and colleagues. This work exemplifies a promising way ahead for both architects and mathematical scientists, whereby architecture and geometry inform each other. The hoary divide between architecture and engineering—like that between art and science—is a false dichotomy.

Acknowledgements

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