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# Extreme concentration fluctuations due to local reversibility of mixing in turbulent flows

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Mixing of a passive scalar in a fluid (e.g. a radioactive spill in the ocean) is the irreversible process towards homogeneous distribution of a substance. In a moving fluid, due to the chaotic advection [H. Aref, *J. Fluid Mech.* **143** (1984) 1; J. M. Ottino, *The Kinematics of Mixing*: *Stretching,Chaos and Transport* (Cambridge University Press, Cambridge, 1989)] mixing is much faster than if driven by molecular diffusion only. Turbulence is known as the most efficient mixing flow [B. I. Shraiman and E. D. Siggia, *Nature* **405** (2000) 639]. We show that in contrast to spatially periodic flows, two-dimensional turbulence exhibits local reversibility in mixing, which leads to the generation of unpredictable strong fluctuations in the scalar concentration. These fluctuations can also be detected from the analysis of the fluid particle trajectories of the underlying flow.

*Keywords*: Mixing; FTLE; turbulence.

## 1. Introduction

Mixing of a passive scalar is a transient process in which initially segregated pollutant becomes homogeneously distributed in a fluid. Stirring a fluid greatly reduces the mixing time. To understand mixing in a moving fluid it is essential to use the Lagrangian description, i.e. a point of view of a moving fluid particle. This trajectory-based representation of a flow, which captures both its kinematics and dynamics, is suitable for studying stretching and folding, the two cornerstone processes, which determine the distribution of a passive scalar concentration in the process of mixing.[2] These two deformation mechanisms are at play in a variety of flows from simple stirring protocols to turbulent flows in a pipe.[4–7] Despite an impressive progress in understanding mixing in recent decades,[8–13] the impact of

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turbulence on mixing remains a challenging question and several related issues remain outstanding. Among them is the question about the similarity and difference between the mixing process in simple flows showing Lagrangian chaos, and in turbulence. Another crucial question is what key properties of the underlying flow best describe mixing dynamics in a turbulent flow? To address these questions, we measure local concentration levels and analyze the dynamics of Lagrangian trajectories to characterize turbulent mixing as interplay between extreme stretching and compression.

## 2. Experiments and Results

We study mixing in electromagnetically driven planar flows, which exhibit a gradual transition from spatially periodic flows to two-dimensional (2D) turbulence, as the flow kinetic energy is increased.[14] A fluorescent dye solution is placed on the surface of the fluid in the shape of 4 circular blobs (each about 40 mm in diameter) before the flow is energized. In these experiments, we study the mixing of the dye during the slow stage in which the probability density functions (PDF) of the dye concentration becomes self-similar. In these experiments, this stage is reached about 40 s after driving a flow.

We compare two types of flows. A spatially periodic quasi-stationary flow is produced at low electric current flowing through the top fluid layer (with a forcing scale Reynolds number of Re = 30), while fully developed 2D turbulence is generated at a higher current.[14] (Re = 112). In the first case, the flow is dominated by the forcing scale vortices. In turbulence, vortices merge forming a broad range of turbulent eddies represented by the Kolmogorov–Kraichnan $k^{-5/3}$ spectrum in the inverse energy cascade range.[14]

A dramatic difference between these two flows is observed when one measures locally in space the dynamics of the dye concentration. The dye concentration in two areas (shown in Figs. 1(a) and 1(b)) in these two flows are measured locally. At low Reynolds numbers the concentration fluctuates modestly (Fig. 1(c)), while

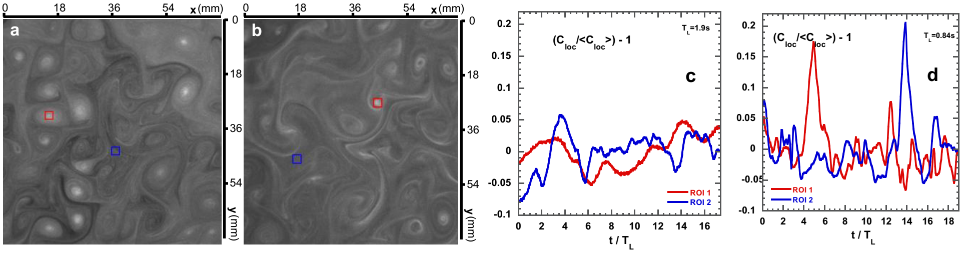


Fig. 1. (Color online) Patterns of the fluorescent dye in the process of mixing in (a) spatially periodic and (b) developed turbulent flow. Temporal evolution of the local dye concentration $C_{\rm loc}$ normalized by the mean concentration $\langle C_{\rm loc}\rangle$ in (c) periodic and (d) turbulent flows. Blue and red curves correspond to the regions of interest (2.9 × 2.9 mm) in Figs. 1(a) and 1(b) shown as the blue and red boxes respectively.

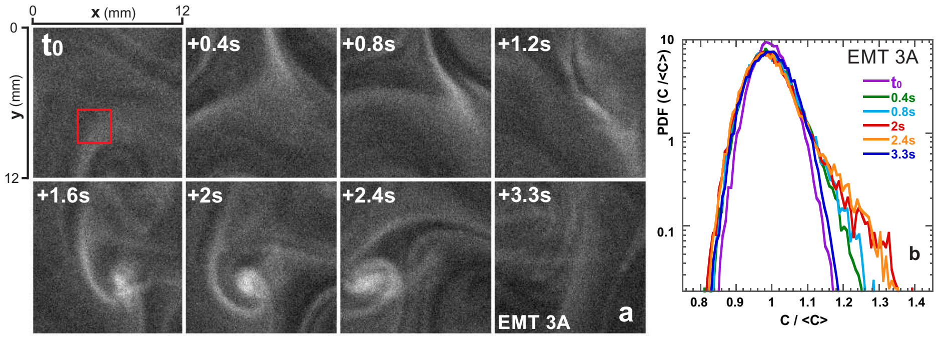


Fig. 2. (Color online) (a) Image sequence of the fluorescent dye in turbulent flows. (b) PDF of the normalized local concentration measured in the region marked as the red box (a). Red box indicates the same region of interest as in Figs. 1(a) and 1(b).

turbulence shows strong intermittent concentration events (Fig. 1(d)). Note that the time axes are normalized by the respective Lagrangian integral times $T_L$.[15]

Strong bursts in the local dye concentration in turbulence are due to complex topological reconnections of the stretched scalar filaments. Strongly elongated filaments recoil in the vicinity of focal points, or spirals, as seen in Fig. 2(a). The recurrence of bright concentration blobs, appearing randomly in time and in space, represent local “unmixing” events, Fig. 2(a). The generation of bright spots leads to the formation of strong tails in the local concentration PDFs (Fig. 2(b)). The statistically averaged PDFs (in space and ensemble averaged) converge and have exponential tails. In contrast, at low Reynolds number bright spots evolve in time very slowly; they remain mostly stationary. Concentrations in this flow do not fluctuate as much as they do in turbulence.

What statistics of the underlying flow can explain such sharp differences in the mixing process? Extreme fluctuations appear as a result of intense stretching (generation of long thin filaments) and topologically complex trajectory folding. Stretching dominates at shorter time scales, while folding becomes more important after fluid elements become more elongated.[7] We study the stretching by analyzing fluid particle trajectories. First, we check if the statistics of stretching events can reveal differences in the concentration dynamics in these two flows. If the underlying velocity field is known, stretching fields based on the finite-time Lyapunov exponents (FTLE) can be computed to characterize mixing.[12–16] FTLE[17,18] is the logarithm of the stretching divided by the finite integration time $\tau$, and they are often used to characterize stirring on the ocean surface.[19] FTLE is determined at location $\mathbf{x}_0$ and time $t_0$ as $\Lambda(\mathbf{x}(\mathbf{x}_0, t_0), \tau) = (1/\tau)\log(|\delta\mathbf{x}(\tau)|/|\delta\mathbf{x}(0)|)$, where $\delta\mathbf{x}(\tau)$ is the separation at time $t_0 + \tau$ between two points which were close together and centered at location $\mathbf{x}_0$ at time $t_0$.

We analyze FTLE for the same flows as those in Fig. 1. The integration time $\tau$ is chosen shorter than the Lagrangian integral time $T_L$ to capture the full dynamic range of $\Lambda$. Values of FTLE in turbulence are substantially higher than those in the

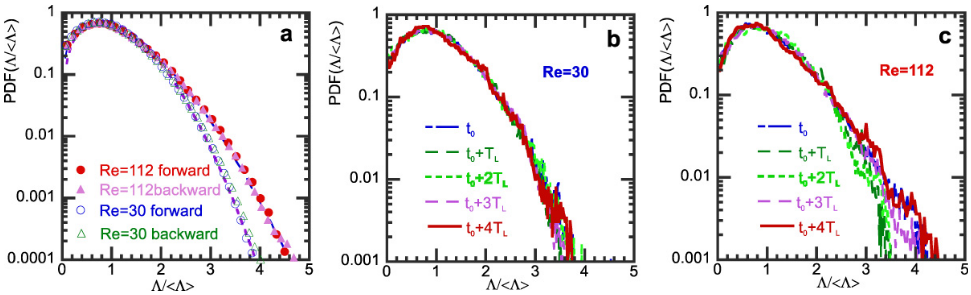


Fig. 3. (Color online) (a) PDFs of the FTLE computed in the flows at low and high Reynolds numbers plotted versus $\Lambda/\langle\Lambda\rangle$. In both flows PDFs are well-fitted by the Weibull distributions (dashed lines). Instantaneous PDFs of the FTLE (integration time $\tau = 0.6T_L$) at different moments of time in (b) spatially periodic flow and (c) in turbulence.

periodic flow. The two PDFs of $\Lambda$ normalized by their mean values $\langle\Lambda\rangle$ are illustrated in Fig. 3(a). The figure shows spatially and ensemble averaged PDFs of the forward and backward FTLEs. Forward FTLEs characterize the divergence of the adjacent fluid particle trajectories (stretching), while the backward FTLEs characterize the convergence of trajectories computed by reversing time (compressive motions). Statistically averaged PDFs of forward and backward FTLE are effectively the same. This confirms that the studied flows are incompressible 2D flows since the sum of positive and negative Lyapunov exponents vanishes, as it should.[20]

Probabilities of large $\Lambda$ are notably higher in turbulence, as seen from the PDF tails. If we plot these PDFs without the ensemble averaging, as in Figs. 3(b) and 3(c), substantial differences in the PDF tails are observed in turbulence and far less in the periodic flow. This behavior is similar to the dynamics in the tails of the concentration PDFs of Fig. 2(b). The PDFs of the FTLE reveal differences at short times ($t < T_L$) in the probabilities of extreme stretching (and compression) events in the two flows.

## 3. Summary

The above results highlight the difference in the dynamics of mixing in regular and in turbulent flows, namely the high concentration fluctuation level in turbulence and the generation of spontaneous extreme concentration spots. The observation of the high concentration fluctuation is correlated with the fluctuation in the stretching/compression field through the measurement of the FTLE field.

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