

Cellular automata model in particle transport studies in magnetized plasma

H. Punzmann and M. G. Shats,

*Plasma Research Laboratory, Research School of Physical Sciences and Engineering,
Australian National University, Canberra ACT 0200, Australia*

Email: Horst.Punzmann@anu.edu.au

Email: Michael.Shats@anu.edu.au

Abstract

Transport in a magnetized plasma is determined by a large number of particle interactions which lead to a diffusion-like behavior. Due to the nonlinear interplay of equations describing transport mechanisms, the spatial structure and the temporal dynamics of plasma profiles have non-trivial solutions. In this paper, we model transport properties of a bi-stable diffusive sandpile, which is based on a one-dimensional cellular automata algorithm, and compare them with experimental results obtained from toroidally-confined plasma experiments. Despite the simplicity of the sandpile model, a striking similarity between spatial structures of transport parameters in both systems is found.

1. Introduction

The cellular automata (CA) model in its simplest form (one-dimensional (1D)), is a process of repetitive update of a local rule applied to discrete cells. Even the most basic families of CA systems have been shown to exhibit a variety of complex phenomena such as self-organization, metastability, turbulence, self-similarity, and so forth. As has been pointed out (Gravner and Griffeath, 1998), CAs can be viewed as discrete counterparts to nonlinear partial differential equations. As such, CA are able to emulate many aspects of real world physical systems and have been used in a wide range of applied science.

In this paper we use the paradigm of a one-dimensional cellular automata model of a bi-stable diffusive sandpile to model the spatial structure of particle transport properties. The computational results are qualitatively compared with experimental data obtained in toroidal plasma confinement experiments. The modelling results of the sandpile show a transition from a continuous and flat sandpile slope to a condition with a distinct discontinuity in the gradient as the deposition rate is slowly increased. Such a “kink” in the sandpile slope is known as a “pedestal” in the profile and is associated with a spatially-localized change of transport. The equivalent plasma experimental observation of the development of such a pedestal phenomena in a toroidally-confined, magnetized plasma is called the “H-mode” (Wagner et al., 1982). Plasma bifurcates from a so called low confinement mode (L-mode) to a mode of high or improved confinement (H-mode). The analysis of spatial profiles of particle transport parameters shows that the sandpile model and the plasma discharges have qualitatively similar characteristics in both the L-mode (subcritical state) and the H-mode (supercritical state).

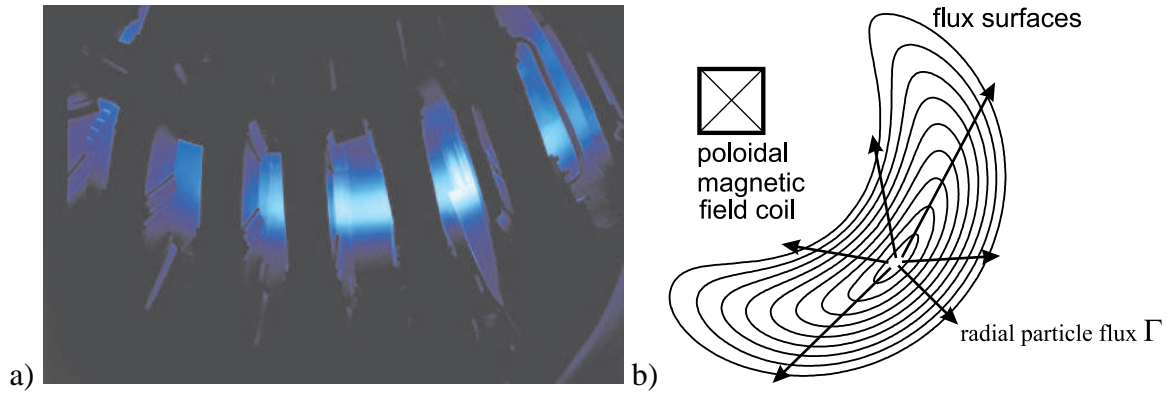


Figure 1. a) An image taken inside H-1 heliac showing an argon plasma discharge and b) the contour plot of isotropic magnetic flux surfaces in the plasma poloidal cross section.

2. Plasma Experiment

The plasma experiments have been performed in the H-1 heliac (Hamberger et al., 1990), a plasma research facility located at the Australian National University. The H-1 heliac is an experimental device which magnetically confines charged particles (electrons and positive ions) in a toroidal magnetic field. The complicated, three dimensional magnetic field is designed such that charged particles, which follow magnetic field lines around the torus, stay confined on concentric surfaces of isotropic magnetic flux as shown in Fig. 1(b). Figure 1(a) shows an image of an argon plasma discharge in H-1 inside a section of some of the toroidal magnetic field coils. Due to the magnetic configuration, particle transport across the magnetic flux surfaces (i.e., radially) is reduced by several orders of magnitude compared to the transport along the magnetic field lines. Time-averaged plasma parameters such as temperature and density are constant on these surfaces. The reduction of the radial transport to improve plasma confinement is a main topic in high-temperature plasma research. Radial transport is driven by particle collisions and turbulence-driven losses originating from a wide spectrum of plasma instabilities. In the plasma discharges investigated in this paper, particle collisions are a dominant transport mechanism in the low-temperature plasma operating regime of the H-1 heliac.

Transport processes in a magnetized plasma, equivalent to transport phenomena in statistical mechanics, can be described *via* the continuity equation

$$\frac{\partial n}{\partial t} = -\nabla \cdot \Gamma + S. \quad (1)$$

Here $\partial n / \partial t$ represents the rate of change of the particle density n which is balanced by the particle source S and reduced by the particle loss term $\nabla \cdot \Gamma$, where Γ is the particle flux. In the cylindrical plasma approximation the particle flux Γ can be derived in plasma equilibrium conditions ($\partial n / \partial t = 0$) based on Eq. 1 to be $\Gamma(r) = \frac{1}{r} \int_0^r r S dr$. The measurement details of how to derive the particle source term S , the description of the spectroscopic tools used and details on the diagnostic techniques are described in (Punzmann et al., 2003). The spectroscopy diagnostic installed on H-1 can reconstruct spatially-resolved radial profiles of the plasma electron density n_e and the net electron particle flux Γ_e . To compare transport characteristics in different plasma confinement modes we calculate the effective diffusion coefficient $D_{eff} = \Gamma_e / \nabla n_e$ which relates the electron particle flux Γ_e to the density gradient dn_e / dr in the direction perpendicular to the flux surfaces.

The experiments were performed in the H-1 toroidal heliac, a 3-field-period helical axis stellarator, which has a major radius of $R = 1$ m and a mean minor plasma radius of $\langle a \rangle \approx 0.2$ m.

Plasma is produced by ~ 60 kW of radio-frequency waves at 7 MHz at a magnetic field strength in the range of $B = 0.05 - 0.15$ T. Plasma parameters are: $n_e \approx 1 \cdot 10^{18} \text{m}^{-3}$, $T_e \approx 10$ eV, $T_i \approx 40$ eV in argon at filling pressure of $\sim 3 \cdot 10^{-5}$ Torr. Radial profiles of plasma parameters in such conditions will be compared with modelling results later in the paper.

3. Sandpile Model

Cellular automata have been used as a metaphor for numerous phenomena in physics in the last decade. In plasma physics, one and two-dimensional models of the “running” sandpile, are used as a paradigm for transport properties and many other hypothetical characteristics of magnetized plasma. The sandpile has been shown to incorporate features such as self-organized criticality (Bak et al., 1987), the appearance of spatial self-similarity, $1/f$ noise dynamics and intermittent transport events of sandpile avalanches (Bak et al., 1988; Kadanoff et al., 1989). In magnetized plasma the sandpile concept has been applied to model various complex phenomena such as particle and energy transport processes (Carreras et al., 1996, 1999; Sanchez et al., 2001; Garcia et al., 2002; March et al., 2004), turbulent transport dynamics (Newman et al., 1996a,b; Sanchez et al., 2003), the stiffness of profiles (Carreras et al., 1998a), the self-similarity of fluctuations (Carreras et al., 1998b), the transition dynamics (Newman et al., 2002; Gruzinov et al., 2003) and the formation of pedestals in plasma profiles (Gruzinov et al., 2002; Chapman et al., 2003). Attempts have even been made to directly map physical quantities in the plasma to model parameters of the sandpile (Newman et al., 1996b).

In this paper we use the CA model formulated by Bak et al. (1987) and extend it by introducing a second stability threshold in the model as proposed by Gruzinov et al. (2002). In this model of the bi-stable “running” sandpile, the first stability threshold reflects a gradient-dependent local diffusivity in the plasma. As the gradient increases the plasma bifurcates into a state of reduced transport which is equivalent to the upper stability range in the sandpile model. As the gradient increases further, a second critical-gradient threshold is reached which corresponds to a plasma stability limit where the onset of plasma instabilities relaxes the gradients to subcritical values. Additionally, the model is complemented with a global background diffusive term, characterized by the diffusion coefficient (Gruzinov et al., 2003). Thus the model incorporates the main features of a magnetically confined plasma, namely diffusion, the bi-stability of transport coefficients and the local gradient limits due to the onset of instabilities.

In the model, illustrated in Fig. 2(a), a fixed number of sand grains n_f is randomly deposited with a uniform distribution across the sandpile of cell height h_x and total width L . The local gradient Z_x of neighboring cells h_x and h_{x+1} is relaxed iteratively by toppling sand grains down the slope according to the following automata rules.

description	model rules
relaxation rule	$h_{x,t+1} = h_{x,t} - \Delta g + \Gamma_d$
local gradient	$Z_x = h_x - h_{x+1}$
diffusive flux	$\Gamma_d = D_0 \cdot (Z_{x-1} - Z_x)$
automata flux	$\Delta g = \begin{cases} 0 & Z_x < Z_{c1} \text{ or } Z_{c2} < Z_x < Z_{c3} \\ g_c & Z_{c1} \leq Z_x \leq Z_{c2} \\ 1 + (Z_x - Z_{c1})/2 & Z_x \geq Z_{c3} \end{cases}$
parameters	$Z_{c1} = 8, Z_{c2} = 20, Z_{c3} = 30, g_c = 3, D_0 = 0.2, n_f = 8, 14$

Table 1. Cellular automata rules for the bi-stable, diffusive sandpile model.

The time step t in table 1 corresponds to the repetitive relaxation of the sandpile slope

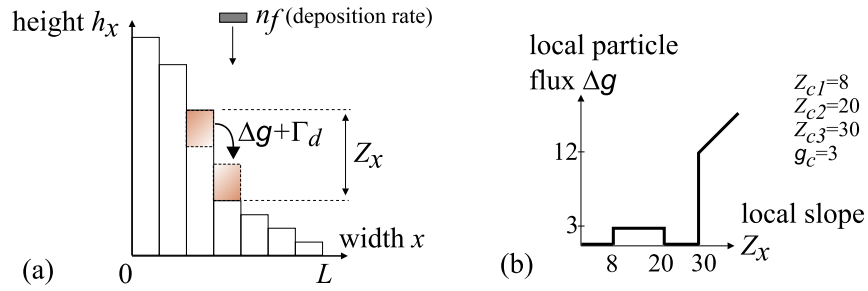


Figure 2. (a) The one-dimensional sandpile model of width L and cell height h_x and (b) the local flux versus gradient corresponding to the automata rules.

according to the automata rules. In our simulations we chose three relaxation iterations within each time step t . The model includes a generic background diffusive flux Γ_d , which is a function of the diffusion coefficient D_0 . A fixed number of sand grains $g_c = 3$ slide off the sandpile slope if neighboring cells exceed the critical gradient Z_{c1} . Below the first threshold Z_{c1} , and between Z_{c2} and Z_{c3} , the sandpile is stable. Once the critical threshold Z_{c3} is reached, the sandpile is relaxed below the subcritical threshold Z_{c1} . The diagram of Figure 2(b) visualizes the applied model rules for Δg by showing the local flux of sand Δg versus local gradient Z_x . The boundary conditions of the model are at $x = 0$ the sandpile is closed ($h_{(0,t)} = h_{(1,t)}$) and at $x = L$ the sandpile is open ($h_{(L+1,t)} = 0$). The sandpile builds up by randomly dropping grains onto the sandpile until the sand slides off, reaching an equilibrium slope after a certain number of iterations.

4. Comparison of Experimental Plasma and Sandpile Model Results

Toroidally confined plasma can bifurcate from a turbulence dominated state with poor transport properties (L-mode) into a state of improved confinement (H-mode). The discovery of the so called “L-H transition” in 1982 (Wagner et al., 1982) is now considered to be a universal phenomena confirmed in several different types of magnetically-confined toroidal plasma experiments. The improved confinement properties of the H-mode are crucially important to develop an efficient and economical viable fusion reactor. In the field of high-temperature plasma physics, understanding the physics mechanism of H-mode is of fundamental importance.

When plasma bifurcates from L- to H-mode, plasma profiles develop a steep gradient at the edge of the plasma. This is one of the characteristic features of the H-mode phenomena. The increase in plasma edge gradients in the electron density n_e as measured in discharges of the H-1 heliac is shown in Figure 3(a). The graph shows two radial electron density profiles of both L- and H-mode as a function of the normalized plasma radius $\rho = r/\langle a \rangle$. The radial coordinate $\rho = 0$ corresponds to the plasma center and $\rho = 1$ to the edge of the plasma. Due to the improved confinement properties of H-mode, the electron density doubles across the L-H transition. The significant increase in the plasma edge density of H-mode leads to a distinct discontinuity in the profile of the gradient. This is called the H-mode pedestal. The dotted line at $\rho = \rho_{ped}$ in Figure 3(a) marks the location of the H-mode pedestal top which is identified by the “kink” in the density gradient. The change in the density gradients on both sides of ρ_{ped} is associated with a change in transport properties. The radial location of the H-mode pedestal top marks the location of a transport barrier.

Using the CA model as described in the previous section, we compare the plasma elec-

tron density profiles in the L- and H-mode to profiles of the diffusive sandpile. As has been shown (Gruzinov et al., 2002), the minimalistic bi-stable diffusive sandpile model reproduces the phenomena of the pedestal formation. Profiles of the sandpile simulation in equilibrium, after approximately 10^5 iteration time steps, are shown in Fig. 3(d). By slightly increasing the control parameter of the deposition rate from $n_f = 8$ to $n_f = 14$ in the simulation, the sandpile develops the distinctive pedestal in the profile. The dotted line marks the location of the largest change in the gradient for the case of increased deposition rate.

As has been pointed out (Newman et al., 1996b), the analogue to the sandpile deposition rate n_f in plasma is the heating power or the plasma fluctuation level which is proportional to the anomalous energy loss term. In the L-H confinement bifurcations of H-1 (Shats et al., 1996) the fluctuation level is reduced and fluctuations and the corresponding anomalous particle transport vanishes. The increased deposition rate in the sandpile model simulation is therefore qualitatively equivalent to the suppression of turbulence in plasma during the transition to H-mode. The measured plasma profiles in Fig.3(a) in L- and H-mode and the sandpile profiles for different deposition rates in Fig.3(d) are qualitatively very similar. The sandpile model captures the essential feature of the pedestal formation in the profile. Thereafter we will adopt the plasma physics terms of L- and H-mode for the sandpile model where low deposition ($n_f = 8$) refers to L-mode and high deposition profile ($n_f = 14$) refers to H-mode.

First, we compute the particle flux which is required for calculation of the effective diffusion coefficient as described above. The net electron particle flux for both L- and H-mode is shown in Figure 3(b). The particle flux, being an integral across the radius, shows the expected monotonic increase towards the plasma edge. In H-mode the particle flux is increased across the whole plasma radius. Correspondingly, the computational results of the sandpile model in Fig. 3(e) show the same trend of increased particle flux in H-mode compared to L-mode.

To compare the spatial structure of transport properties we calculate the effective diffusion coefficient $D_{eff} = \Gamma / \nabla n$. As a measure of confinement, radial profiles of D_{eff} are used to compare relative changes in L- and H-mode confinement for both the plasma and the sandpile. The profiles of D_{eff} for the plasma are shown in Fig. 3(c) and for the sandpile in Fig. 3(d). In both cases L- and H-mode profiles of the diffusion coefficient intersect around the radial position of the pedestal location in H-mode.

Due to the flat density gradients inside the pedestal top ($\rho < \rho_{ped}$), confinement properties deteriorate towards the plasma (or sandpile) center in H-mode compared to L-mode. Outside the transport barrier ($\rho > \rho_{ped}$), in the pedestal region, confinement is significantly improved in both cases. The improvement of confinement in the pedestal region in both the plasma and the sandpile leads to a significant increase in the H-mode density gradient. The drop in the effective diffusion coefficient D_{eff} at the density pedestal top ρ_{ped} marks the location of a transport barrier which is associated to the H-mode pedestal formation.

5. Discussion and Conclusion

In this paper we show measurement results of transport properties in toroidally-confined plasma of H-1 heliac and compared them with modelling results of the cellular automata of a bi-stable diffusive sandpile algorithm. The radial profiles of electron density versus sandpile gradient, particle fluxes and the diffusion coefficients obtained in two different confinement modes are qualitatively in good agreement. Both systems show a significant drop in the effective diffusion coefficient at the location of the largest profile gradient in H-mode. The sandpile model captures the characteristic features of the H-mode in magnetized plasma, namely the formation of the density pedestal and the confinement improvement within the pedestal region. The sand-

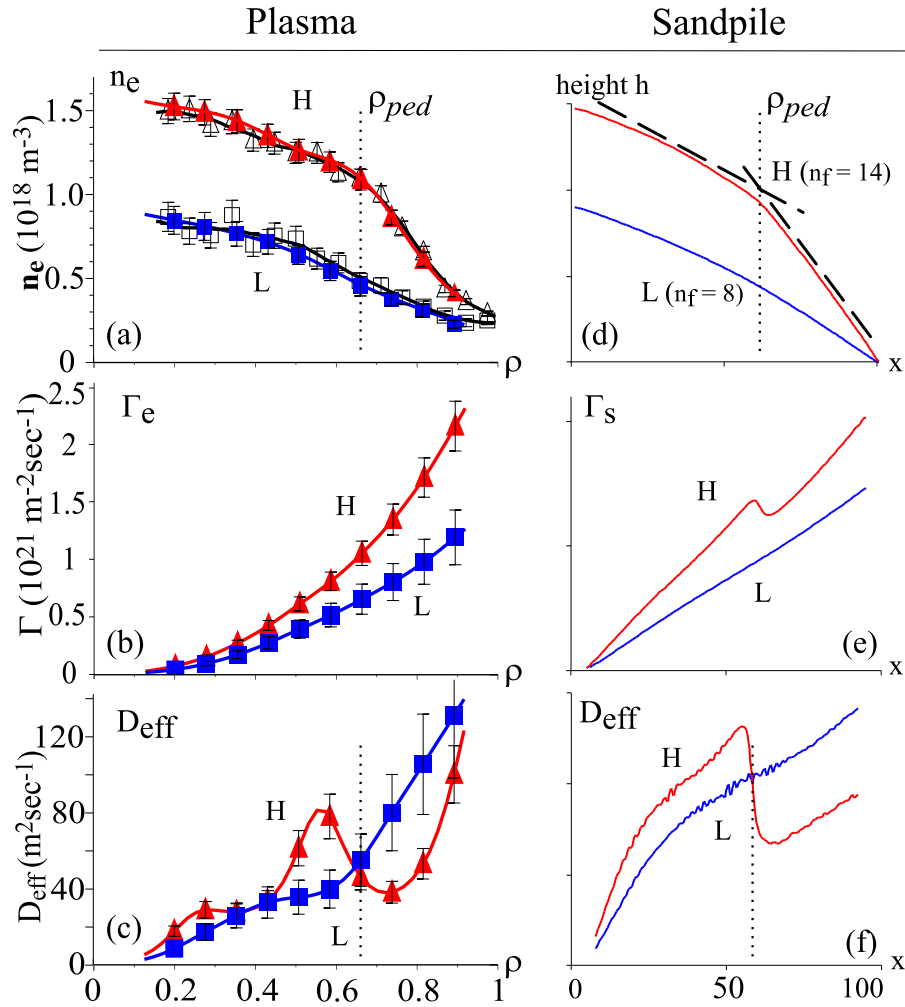


Figure 3. Measured radial plasma profiles of (a) electron density, (b) net electron particle flux and (c) the effective diffusion coefficient in L (squares) and H (circles) confinement modes. Sandpile modelling results show (d) sandpile height, (e) net sand flux and (f) the effective diffusion coefficient with subcritical (L-mode) and supercritical (H-mode) grain deposition. Dotted lines at ρ_{ped} mark the location of the density pedestal top in H-mode.

pile model is seen to be a useful tool to reflect the understanding of complex plasma transport processes. Even though the bi-stable diffusive sandpile is a minimal model, it is capable of capturing fundamental aspects of transport in magnetized plasma such as the transport barrier formation.

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