EXPLORING SPECIAL RELATIVITY

Objectives
The aim of this experiment is to observe how things would appear if you could travel at speeds near that of light, $c = 3 \times 10^8$ m/s. From your observations you can deduce, or verify, aspects of relativistic physics. Both qualitative and quantitative understanding is sought.

Important Information
The "Real Time Relativity" (RTR) simulator you will use in this lab was created at ANU by Lachlan McCalman, Antony Searle, and Craig Savage. It is under development and has many limitations. In particular you may not be able to make precise measurements. When you are asked for a quantitative result just do the best you can: 10% accuracy is fine. If you have trouble using RTR, ask a demonstrator for help. Take a note of things that make it difficult to use, and include them on the post-lab evaluation. Thanks for your patience!

Evaluation
This is a new lab, and we are evaluating its effectiveness. To help us, please answer the questions on the evaluation handouts: complete one before the lab, and one after the lab. This will help us to assess the effectiveness of the lab, and to improve it.

Skills
In this experiment you will practice: making observations, critical evaluation, designing experiments.

Basic Principles
Computer simulations are used in science to conduct investigations that might otherwise be difficult or impossible.

The objects in the simulation are big, because the speed of light is fast. When you start RTR you can see the Earth and the Sun. These set the scale of the simulated world, and are respectively about 0.043 and 4.7 light-seconds in diameter. A light-second is the distance light travels in a second: 300,000 km. In the RTR world it is convenient to measure distance in light-seconds and time in seconds. Then the speed of light is $c = 1$ light-second per second.

The square plane on which most of the objects sit is 35 light-seconds on a side, the colored cubes are 2.5 light-seconds on a side, the distant bar is 25 light-seconds long, and the clocks are 10 light-seconds apart.

The first question to ask about a simulation is: Is it reliable? Although we believe RTR is quite accurate, it is under development, so you need to proceed with a healthy scepticism.

Three physical effects are particularly easily explored in RTR: the Doppler effect, aberration, and the headlight effect.

The Doppler effect is the difference between measurements of the frequency, or colour, of light by relatively moving observers. We consider the usual standard configuration observers S (world) and S′ (camera): see diagram [1]. Let the source of light, either an emitter or a reflector, be at rest in S. Let this frequency be f. Let the angle of the incoming light relative to the velocity vector be θ', see the diagram below. Then [1,4]
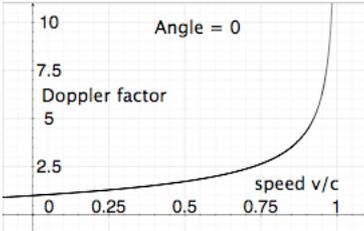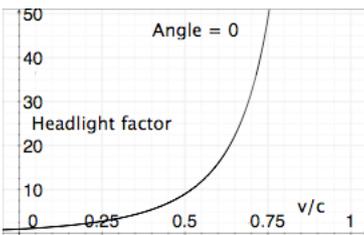
$$f' = f \frac{\sqrt{1 - v^2/c^2}}{1 - (v/c)\cos\theta'}.$$

(1)

Aberration is the difference in the angle of incoming light rays measured by relatively moving observers. An analogous everyday effect is the change in the angle of rain when you drive through it. Let the angles of the incoming ray be θ and θ' for the observers S and S′. Then the angles are related by [5]
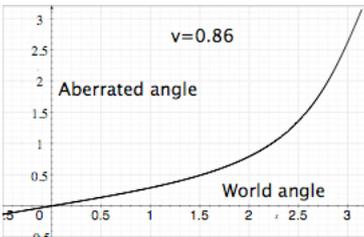
$$\tan\left(\frac{\theta'}{2}\right) = \sqrt{\frac{c-v}{c+v}} \tan\left(\frac{\theta}{2}\right).$$

(2)

The headlight effect is the change in light intensity measured by relatively moving observers. Aberration concentrates the incident light into a narrow cone centred on the direction of motion. In addition, time dilation decreases the measured photons per second in the camera frame. Overall there is a brightening in the direction of motion, and a darkening away from the direction of motion. The ratio of the intensities measured by observers S and S′ scales as the fourth power of the Doppler shift factor [6]

$$I' = I \left(\frac{\sqrt{1 - v^2/c^2}}{1 - (v/c)\cos\theta'} \right)^4.$$

(3)
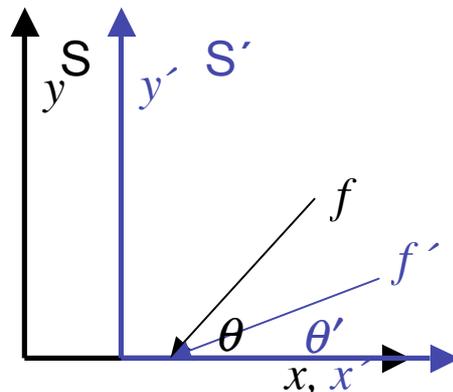
Standard configuration frames S and S′. S′ is moving along the positive x axis of S with speed v. The observed frequencies and incoming angles of the rays in each frame are indicated.

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Prelab Study
Read through this entire document. Then study the following tutorials on the "Through Einstein's Eyes" web site: relativistic aberration, Doppler effect, headlight effect. These are available on CD from Craig, or online at:
http://www.anu.edu.au/Physics/Savage/TEE/site/tee/learn.html

Prelab Problems
0. Complete the pre-lab evaluation handout.

1. Does the Doppler effect increase or decrease the frequency when the ray is coming from:
(a) in front, $\theta' < 90$ degrees?
(b) behind, $\theta' > 90$ degrees?

2. Show that a ray incoming at $\theta' = 90$ degrees has the Doppler shift
$f' = f\sqrt{1 - v^2/c^2}$.
This is called "time dilation". The source of light in S can be thought of as a clock producing f oscillations per second. In the S´ frame a smaller number f' oscillations per second is observed. Hence each oscillation takes longer, and the S time is said to be "dilated".

3. Consider two incoming rays with slightly different angles. Does the angular difference increase or decrease for rays coming from:
(a) in front, $\theta' = 0$ degrees.
(b) behind, $\theta' = 180$ degrees.

4. Does the headlight effect increase or decrease the intensity when the light is coming from:
(a) in front, $\theta' = 0$ degrees?
(b) behind, $\theta' = 180$ degrees?

5. (a) Express in metres the length of the 25 light-seconds long bar. (b) Express a light-second per second in ms^{-1}. (c) Express a light-second per second per second in ms^{-2}, and in units of "gees", the acceleration due to gravity, $g = 10$ ms^{-2}.

 Prelab problems

should be done before you come to lab!

Instructions for using Real Time Relativity, version 0.6
See the RTR Manual provided in the lab, and on the website for more detailed instructions:
http://www.anu.edu.au/Physics/Savage/RTR/instructions.html

To start RTR double click the RTR icon on the desktop. To exit it press **Escape**.

The **mouse** controls where you are looking, but not the direction your rocket is moving. Try looking around a bit with the mouse. You'll notice that the red cross always points in the same direction-this tells you which way your ship is facing. Placing the red cross in the centre (**F key**) means you are looking in the direction of travel. It is useful in the lab to be able to look 90 degrees to the direction of motion: **L & R keys**.

The **keyboard** controls the rocket. The **A and D keys** control yaw (rotation side to side), and the **up and down arrow keys** control pitch (tilting the nose up and down). Try moving around a little with the keys. You'll notice that the red cross remains fixed in the centre of the screen.

To accelerate and decelerate, use the W and S keys. To stop, use the **1 key**. To instantly accelerate to 0.866c, use the **2 key**. To instantly accelerate to -0.866c, use the **3 key**. These speeds correspond to a length contraction, and time dilation, factor of 2. The **4 & 5 keys** accelerate to a contraction factor of 4.

If you get lost you can reset to the starting configuration by using the O (Oh) key. To go to a suitable position for the length contraction experiment use the **I key**. To go to a suitable position for the time dilation experiment use the **C key**.

By default, the **Doppler effect** and **headlight effect** are turned off in the simulation, but can be toggled on and off with the **K & H keys** respectively. Indicators at the top left of the screen tell you whether they are on or off.

To Pause RTR press the **F2 key**. To **capture** the RTR window press the two-key combination **Alt-PrintScreen**. This puts the image on the clipboard. It can then be pasted (**control-v**) into an application such as Paint, annotated if you like, and **printed**.

Paint may be opened while RTR is paused. It is found under "Accessories" in the "All Programs" menu. If possible, reduce images in size (Stretch/Skew under the Image menu of Paint: 40%-50% for both horizontal and vertical), and paste several on the same page. Before printing, use Page Setup to ensure that the paper is set to "Landscape" and to "Fit to 1 by 1 pages". Use Print Preview to check the printout before actually printing it.

The experiment
Explore the simulated world for 15 minutes or so. It takes some time to get used to the strange way things appear as you accelerate and decelerate. You also need to get used to steering the "rocket".

Write down one or two questions that you would like to try and answer during this lab, if you have time.

Print out one or two images that show interesting effects and secure them in your logbook (printing instructions above) with a description of what they show. As you work through the lab include illustrative images in your logbook.

After your initial exploration, describe what you see as you continuously accelerate from rest from the starting configuration in front of the columns ("O" key). Holding the "A" key produces <u>constant</u> proper acceleration of 0.2 light-seconds per second^2, or $5.6 \times 10^6\, g$. This is the acceleration you would feel if you were actually in the rocket (Would you survive?). However, since you cannot exceed the speed of light relative to the world frame, your world frame acceleration decreases as you speed up. When the acceleration is along the direction of motion, proper acceleration a_p and world frame acceleration a_w are related by [3]

$$a_p = a_w \left(1 - v^2 / c^2\right)^{-3/2}.$$

Seeing behind. Fly along parallel to the line of cubes. Note that if you go fast enough you can see behind the cubes, i.e. you can see the face that is normally obscured when looking forward. How fast do you have to go for this to happen? Explain how it is possible [7]. Make a comparison to the everyday effect in which you can see a plane in front of you, but hear it behind you.

Doppler and aberration. Next check for approximate quantitative agreement with the formulae (1) and (2) for the Doppler shift and aberration. For each effect choose some particular case to make a comparison between what you observe and what is predicted by the formula. These are basic checks that the simulation is behaving sensibly.

When studying the Doppler effect you should turn the headlight effect off ("H" key). Otherwise it swamps the Doppler effect. Think of it as "computer correction" of your camera image.

Verify the time dilation formula discussed in prelab problem 2: $f' = f\sqrt{1 - v^2 / c^2}$, where f' is the frequency (colour) observed when moving with speed v relative to an object whose rest frame frequency is f. The verification should be quantitative, but approximate, and be for a range of speeds. The "L" and "R" keys should be useful for this. They point the camera 90 degrees to the left or right of your direction of motion. The "F" key points the camera forwards, and "B" backwards.

Extra Information:

Red light frequency: 4.3×10^{14} s^{-1}.

Green light frequency: 5.7×10^{14} s^{-1}.

Blue light frequency: 6.8×10^{14} s^{-1}.

Length contraction. Aberration makes it tricky to observe the relativistic "**length contraction**" effect [1]. However it occurs for distant objects viewed at 90 degrees to the direction of relative motion. The apparent size is then less by the length contraction factor $\sqrt{1 - v^2/c^2}$ than when viewed from the same perpendicular distance at rest relative to the object [2].

When studying length contraction you should turn the Doppler and headlight effects off ("K" & "H" keys).

Verify the length contraction formula: $L = L_0 \sqrt{1 - v^2/c^2}$, where L_0 is the length of the object at rest, and L is the length measured by an observer moving with speed v. The verification should be quantitative, but approximate, and be for two different speeds: e.g. 0.866c and 0.9682c, obtained with the "2" & "4" keys.

Use the fact that for distant objects angular size is proportional to length. This is because distances perpendicular to the motion are the same for all observers [1]. Here's a suggestion for how to proceed: Use the "I" key to get to a position on the edge of the world plane, and look 90 degrees to the right ("R" key). Use the grid ("G" key) to measure the approximate size of the long rod. Now accelerate and measure the length. Note that as you accelerate aberration moves the rod forwards in your visual field. This is a good place to print comparative images.

Time dilation. There are two clocks in the world that read world frame time in seconds. What you see is the clock time at the time the light left the clock; e.g. for a clock 10 light-seconds away you see the clock as it was reading 10 seconds ago.

Fly towards and away from them and describe what you observe (look back at the clocks by pointing the camera, e.g. with the mouse or the "B" key to look backwards). Explain what you observe.

Design and perform an experiment that minimises the light delay effect so that you can observe pure time dilation. The "C" key puts you in a suitable position for this, equidistant from the two clocks. Note that the onscreen timer registers your proper time, which is also your actual ("wrist-watch") time.

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Extension work (optional).

Questions. Look at the questions you wrote down at the start of this lab. Have you answered them? If not, and you have time, try and answer them now using RTR.

Spheres. Describe how spheres look at relativistic speeds. View the Sun and the large Earth (not to scale) under the plane. Explain what you see.

Relativity of simultaneity. Design and perform an experiment that allows you to observe the relativity of simultaneity.
Hint: Observe the two clocks while you are equidistant from each according to your own observations from the moving rocket. If you are equidistant (according to you!) then there is no differential light delay, and any time difference between the clocks is due to the relativity of simultaneity.

Does this give you insight into how the relativity of simultaneity arises?

Evaluation. Please answer the post-lab evaluation questions on the sheet provided, a day or so after doing the lab, and hand it in.

Achievements
You can design and carry out experiments, even better than before.
You can critically evaluate the correctness of simulation software.
You know how things would look in a relativistic world!

References
[1] Tipler & Mosca 39.3. Relativity lecture notes 1, section 4.
[2] H. Muirhead, "The special theory of relativity", Macmillan 1973. Section 3.6.
[3] W. Rindler, "Relativity: special, general and cosmological". Section 5.5.
[4] Reference [3]. Section 4.3.
[5] Reference [3]. Section 4.4.
[6] J. McKinley, "Relativistic transformations of light power", American Journal of Physics 47, pg. 602, 1979.
[7] Through Einstein's Eyes web site, section on "Relativistic Aberration". See animation at the bottom of the page. Bookmarked in Explorer on the lab computers.
http://www.anu.edu.au/Physics/Savage/TEE/site/tee/learning/aberration/aberration.html

(i) extra information