Model for relaxation oscillations in a helicon discharge

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Relaxation oscillations observed in the large-volume, helicon plasma experiment WOMBAT (Waves on Magnetized Beams and Turbulence) [R. W. Boswell and R. K. Porteous, Appl. Phys. Lett. 50, 1130 (1987)] are modeled. These oscillations have a period of several milliseconds and have been identified as transitions between a low-density, inductive discharge and a high-density, helicon-wave discharge. In the model, it is assumed that the mode transitions are triggered by variations in the neutral density in the source region. The neutral density decreases due to ionization augmented by ion pumping and increases due to refilling of the source chamber from the much larger diffusion chamber. The system is modeled using two, coupled, nonlinear, ordinary differential equations that describe the neutral and plasma densities in the source chamber. Ionization by inductively-coupled fields and ionization due to electrons accelerated by helicon waves with phase velocities near the threshold electron velocity for ionization are considered. The model is found to reproduce experimentally measured variations of the plasma density and helicon wave phase velocity with rf power, neutral pressure and magnetic field. The negative impedance needed for the existence of a relaxation oscillation is provided by the helicon-wave coupling mechanism. © 1999 American Institute of Physics. [S1070-664X(99)03405-9]

I. INTRODUCTION

Millisecond-period relaxation oscillations have recently been observed in the large-volume, helicon-wave plasma WOMBAT (Waves on Magnetized Beams and Turbulence). In these oscillations, the discharge switched periodically between two states; a high-density helicon-wave discharge (W-mode) and a lower-density inductively-coupled discharge (H-mode). The W-mode discharge was characterized by a high central plasma density ($\sim 5 \times 10^{18} \text{m}^{-3}$), bright blue Ar II emission along the axis of the source chamber (hence the name ‘‘blue-mode’’) and a helicon wave phase velocity corresponding to electron energies close to the ionization threshold for Ar. The H-mode discharge had a broader density profile and a very high helicon phase velocity. Consequently, resonant wave–electron interactions (i.e., surfing) probably do not contribute significantly to ionization in the H-mode discharge.

In this paper, we investigate the physics responsible for the relaxation oscillation. The lengthy period suggests that neutral density variations in the source chamber determine the timescale. This hypothesis is tested by constructing a global dynamical model that includes the depletion of neutral atoms in the source chamber by ionization augmented by momentum transfer from ions leaving the source chamber to the neutrals (i.e., ion pumping) and refilling of the source chamber by neutral atoms from the diffusion chamber. (In WOMBAT, the diffusion chamber has a volume $\approx 80$ times that of the source chamber, and so can act as a gas plenum.) Ionization by electrons accelerated by the helicon wave and ionization from ohmic heating due to inductive coupling are included. The model predicts that neutral depletion decreases the ionization rate in the source chamber to the point where the high density helicon plasma is unsustainable, whereupon a transition to a low-density H-mode discharge occurs. The much lower ionization rate in the inductive discharge allows the neutral density in the source chamber to recover. As the neutral density increases, the ionization rate, and hence the plasma density, increase to the point where a transition to helicon coupling occurs and the ionization rate and plasma density increase dramatically to reignite the W-mode.

II. GLOBAL DYNAMICAL MODEL

The unstable nature of the high density helicon mode in WOMBAT on time scales of a few milliseconds strongly suggests that variations in the neutral density are responsible, since neutrals are the only species moving slowly enough to cause such a long timescale effect within the source chamber. Variations in the neutral density will clearly influence the plasma density by affecting the ionization rate, and if sufficiently large may trigger transitions between discharge modes. In particular, we consider the possibility that neutral depletion in the source when the plasma density is high drives a transition from helicon to inductive coupling, causing a rapid drop in the plasma density. Neutrals are then able to flow back into the source chamber, increasing the ionization rate and plasma density to the point where a transition back to the helicon mode occurs. This cycle then repeats, leading to a relaxation oscillation.

The model developed here uses a volume-averaged approach, and is simplified to two representative regions, the source and diffusion chambers, as shown in Fig. 1. This as-
assumption allows the dynamics to be described using ordinary differential equations, rather than more accurate (and more complicated) partial differential equations. The characteristic ion velocity (the Bohm velocity \( v_B \)) and the neutral atom velocity \( v_n \) are assumed constant. The dynamical variables are the volume-averaged ion density \( N_i \) and neutral density \( N_g \) in the source chamber. (We use \( N_i \) to refer to the ion, electron or plasma density, as appropriate.)

Our model is represented by two, coupled nonlinear, ordinary differential equations. The ion density increases due to ionization in the source chamber, and decreases due to the flux of ions out of the source chamber and into the diffusion chamber, so that

\[
\frac{dN_i}{dt} = N_g N_i K(N_i) - \frac{A}{V} \Gamma_i.
\]

The neutral density decreases due to ionization and ion pumping (momentum transfer from outward flowing ions to neutrals) and increases due to neutrals flowing into the source chamber from the diffusion chamber, giving

\[
\frac{dN_g}{dt} = -N_g N_i K(N_i) - \frac{A}{V} \Gamma_g.
\]

Here \( \Gamma_i \) and \( \Gamma_g \) are the net ion and neutral fluxes across the surface bounding the source region, \( K \) is the ionization rate coefficient, \( V \) is the source region volume and \( A \) is the surface area bounding the source region (hence the length scale of the source region is \( L = V/A \)). In this model, the ionization rate depends upon a time dependent neutral gas density, while more typically the neutral density is fixed and the second equation is a power balance equation that determines the electron temperature through \( K \). The dependence of \( K \) on \( N_i \) is determined by the power coupling mechanism as discussed in Sec. II A. The ion and neutral flux terms are considered in Sec. II B.

A. Ionization mechanisms

The ionization rate coefficient \( K \) depends on the mechanism coupling energy from the rf fields to the plasma electrons, and therefore on the plasma density. Two coupling mechanisms will be considered in deriving the variation of \( K \) with \( N_i \)—inductive coupling and helicon coupling.

1. Ohmic heating by inductive coupling

Inductive coupling to a magnetized plasma delivers power to the electrons via evanescent antenna fields penetrating the plasma across the magnetic field.\(^2\) The perpendicular rf electric field component decays spatially according to

\[
E(x) = E_0 \exp \left( i \omega t - \frac{x}{\delta_x} \right),
\]

where \( E_0 \) is the electric field amplitude at the plasma edge, \( x \) is the distance from the antenna, and \( \delta_x \) is the perpendicular skin depth. Here \( \delta_x \) is associated with the collisionless (for the low pressures in the experiment) conductivity of the plasma across the magnetic field, and can be estimated by

\[
\delta_x = \frac{c \omega_{pe}}{\omega_{ce}} \frac{B}{\omega} \left( \mu_0 m_i n_i \right)^{-1/2},
\]

where \( \omega_{pe} \) and \( \omega_{ce} \) are the electron plasma and cyclotron frequencies, and \( c \) is the speed of light in vacuum.

The rf fields cause the electron distribution function to oscillate in velocity space with an amplitude \( v_D \) proportional to the local field strength, given by

\[
v_D = \frac{eE(x)}{m \omega_{ce}},
\]

which has the time-averaged effect over a rf cycle of broadening the distribution function and increasing the ionization rate accordingly. For this model, the average drift is calculated by integrating Eq. (5) across the source region radius \( R \) to give

\[
v_{Dav} = \frac{eE_0}{\sqrt{2m \omega_{ce}}} \frac{\delta_x}{R} \frac{1 - \exp \left( - \frac{R}{\delta_x} \right)}{\left( \frac{R}{\delta_x} \right)^2}.
\]

The cross section for ionization is assumed to be zero for electron velocities less than \( v_{iz} = 2.35 \times 10^6 \text{ m/s} \), and \( \sigma_{iz} = 1.0 \times 10^{-20} \text{ m}^2 \) for electron velocities greater than \( v_{iz} \), where this is a characteristic value of \( \sigma_{iz} \) in Ar for electron energies of 20–30 eV. The ionization rate coefficient for a Maxwellian distribution function with the above average drift velocity and given thermal velocity \( v_{th} \) is then

\[
K_{th} = \sigma_{iz} \left[ v_{Dav} \left( 1 + \text{erf} \left( \frac{v_{Dav} - v_{iz}}{v_{th}} \right) \right) + \frac{v_{th}}{\sqrt{\pi}} \exp \left( - \frac{(v_{Dav} - v_{iz})^2}{v_{th}^2} \right) \right].
\]

2. Electron trapping by helicon waves

Helicon waves contribute to the ionization rate by perturbing the electron distribution function within the trapping width \( v_w \) of the wave phase velocity, \( v_w = \omega/k_z \), where the trapping width\(^4\) is defined by

\[
v_w = 2 \left( \frac{eE_z}{m_i k_z} \right)^{1/2}.
\]

This effect is evident in many experiments carried out by many authors\(^5-8\) and has been investigated theoretically\(^9,10\). The relevance to plasma production of such perturbations to the
Effects on trapping are taken into account by multiplying the phase velocity for helicon waves is modeled using the simple plane wave dispersion relation (with $k_\perp = 0$),

$$v_\phi = \left( \frac{\omega B}{\mu_0 n_e e} \right)^{1/2},$$

that was previously found to agree with the experiment. The bump is modeled by a Gaussian function with a width defined by $v_{\phi u}$, and a height defined by the total number of trapped electrons $n_{tr}$, that is, by the integral from $(v_{\phi} - v_{ru})$ to $(v_{\phi} + v_{ru})$ of the drifting distribution function obtained from inductive coupling, to give

$$n_{tr} = \frac{n_0}{2} \left[ \text{erf}\left( \frac{v_{\phi} + v_{ru} - v_{Dan}}{v_{th}} \right) - \text{erf}\left( \frac{v_{\phi} - v_{ru} - v_{Dan}}{v_{th}} \right) \right].$$

Collisions with the neutrals tend to scatter trapped electrons out of the wave frame, decreasing the value of $n_{tr}$ as the neutral density increases. This is taken into account in the model by comparing the “bounce” frequency $\omega_n$ of the trapped electrons (i.e., the frequency with which they complete one orbit in the wave frame) with the collision frequency $\nu_\phi$ of electrons traveling at the phase velocity to give

$$\gamma = \frac{\omega_n}{\omega_{\nu} + \nu_\phi},$$

where $\omega_n$ and $\nu_\phi$ are defined by

$$\omega_n = \left( \frac{eE_z \omega}{m_\nu v_\phi} \right)^{1/2}$$

and

$$\nu_\phi = N_\phi \sigma v_\phi.$$  

In the high $N_\phi$ limit where $\nu_\phi \gg \omega_n$, $\gamma$ is small and the collision frequency is large enough to prevent a bump on the tail from forming, whereas in the opposite case ($\nu_\phi \ll \omega_n$) $\gamma$ approaches unity and the density of trapped electrons approaches the value of $n_{tr}$ obtained above. Hence, collisional effects on trapping are taken into account by multiplying $n_{tr}$ by $\gamma$ when calculating the ionization rate coefficient. The contribution of the bump on the tail and hence the helicon wave to the ionization rate coefficient $K_W$ is calculated using the same assumptions used for $K_H$, to obtain

$$K_W = \frac{\sigma_{iz}}{2} \left[ v_{\phi} \left[ 1 + \text{erf}\left( \frac{v_{\phi} - v_{iz}}{v_{tr}} \right) \right] + \frac{v_{ru}}{\sqrt{\pi}} \exp\left[ -\left( \frac{v_{\phi} - v_{iz}}{v_{ru}} \right)^2 \right] \right].$$

A typical example showing the variation of $K$ with plasma density and the individual contributions of $K_H$ and $K_W$ is shown in Fig. 2. For low densities, the skin depth is larger than the system radius so that the evanescent fields from the antenna are large throughout the source, hence the average drift velocity $v_{Dan}$ and the resulting value of $K_H$ are large. As the density increases, the skin depth becomes less than the system radius and the plasma becomes more conductive, and the value of $v_{Dan}$ decreases. Equations (4) and (6) indicate that when $\delta_i \ll R$, $v_{Dan}$ is proportional to $n_e^{-1/2}$. The ionization rate from inductive coupling $N_i K_H$ thus increases linearly for low values of $n_i (R \ll \delta_i)$ and decreases as $n_i$ becomes large ($R \gg \delta_i$).

As the density increases from ionization through inductive coupling, the helicon wave phase velocity $v_{\phi}$ in the model decreases according to the dispersion relation. The value of $n_{tr}$ given by Eq. (10) increases exponentially. However, the value of $K_W$ does not increase significantly in this model until $(v_{\phi} - v_u) < v_{iz} < v_{\phi}$, that is, until the phase velocity is situated such that trapped electrons are accelerated from below $v_{iz}$ (where their probability of ionizing is zero) to some value above it. When $v_{iz}$ falls within this range, the value of $K_W$ increases rapidly, causing a dramatic increase in the density, which is identified as the transition to helicon coupling. The value of $K_W$ peaks when $v_{\phi} = v_{iz}$ and decreases as the density is further increased to become negligible when $v_{iz} > (v_{\phi} + v_u)$, as none of the trapped electrons are accelerated across the ionization threshold.

### B. Ion and neutral fluxes

The effects considered in modeling $\Gamma_i$ and $\Gamma_g$ are momentum transfer from ions directed out of the source chamber to the slower neutrals (which also represents a frictional force slowing the ions) and the refilling of neutrals from the diffusion chamber, which is here considered to be a reservoir of neutrals with constant density $N_{e0}$. That is, neutrals can be added to (or removed from) the diffusion chamber with-
out changing the density there because of its large volume. Consequently, the total number of particles in the source chamber, \( N(N_i + N_g) \), is not conserved. In addition, we have implicitly assumed that axial transport of neutrals between the source and diffusion chambers is much more important than radial transport within the source chamber, since ions and neutrals transported radially to the walls of the source chamber should be quickly recycled back into discharge. The net neutral flux \( \Gamma_g \) is then estimated as

\[
\Gamma_g = \frac{1}{2} (N_g - N_{g0}) v_g + L N_g \sigma_{px} \Gamma_i,
\]

where \( \sigma_{px} \) is the cross section for momentum transfer between ions and neutrals. The first term represents the flux associated with a density gradient between the source and diffusion chambers and the second term represents the additional contribution to the neutral flux of momentum transferred from ions moving out of the source. The ion flux \( \Gamma_i \) is approximated by

\[
\Gamma_i = \left[ \frac{\sqrt{2}}{2} \left( 3 + \frac{L}{\lambda_{mp}} \right)^{-1/2} \right] N_i v_B = \alpha N_i v_B,
\]

where \( \lambda_{mp} = 1/(N_g \sigma_{px}) \) is the mean free path for ion-neutral collisions. The term in brackets (here called \( \alpha \)) models the decrease in flux with increasing collisionality assuming discharge equilibrium, and is the result of a balance between the presheath electric field that continually accelerates ions and the collisional slowing of ions.

**III. RESULTS AND DISCUSSION**

**A. Equilibrium and stability**

Our model is described by two coupled, nonlinear, ordinary differential equations for the plasma density \( N_i \) and the neutral density \( N_g \). As such, the state of the system at any time is completely specified by a point \( (N_i, N_g) \) in two-dimensional phase space. In equilibrium, the time derivatives of the ion and neutral densities [Eqs. (1) and (2)] are zero. In phase space, each equilibrium condition is represented by a curve. Let the ion equilibrium curve, where \( dN_i/dt = 0 \), be given by \( N_i = F(N_g) \), and the neutral equilibrium curve, where \( dN_g/dt = 0 \), be given by \( N_g = G(N_g) \). The intersections of \( F \) and \( G \) then give equilibrium values of the neutral and ion densities \( (N_i^e, N_g^e) \), and the phase space trajectory \( [N_i(t), N_g(t)] \) represents the temporal evolution of the system. From the definitions of \( F \) and \( G \), the system’s trajectory must cross \( F \) horizontally and \( G \) vertically. These considerations largely characterize the dynamics in terms of \( F \) and \( G \) (given that the direction of time along the trajectory is known), where the relative slopes of \( F \) and \( G \) at an equilibrium point determine its stability.

For example, given that \( N_g \) initially decreases with time as \( N_i \) increases (hence the trajectory moves anticlockwise on a graph of \( N_i \) vs \( N_g \)), and that for all cases \( dG/dN_g < 0 \), if \( dF/dN_g < 0 \), then \( (N_i^e, N_g^e) \) is a stable equilibrium point, and unstable if \( dF/dN_g < 0 \). This can be easily verified by drawing intersecting curves for \( F \) and \( G \) of different slopes and sketching the resulting trajectory given the rules mentioned above. The trajectory must converge on the equilibrium point in the former case (i.e., stable) and diverge in the latter case (i.e., unstable) when the trajectory moves in an anticlockwise sense.

To calculate the curves \( F \) and \( G \), the model was solved numerically for conditions relevant to the experiment WOMBAT; \( R = 0.1 \text{ m} \) and \( L = 0.5 \text{ m} \) (source chamber dimensions), \( N_{g0} = 10^{20} \text{ m}^{-3} \) (a neutral pressure of 3 mTorr Ar in the diffusion chamber), \( B = 0.010 \text{ T} \), \( v_th = 10^5 \text{ m/s} \) (electron temperature \( kT_e = 3 \text{ eV} \)), \( v_i = v_B \), \( v_g = 300 \text{ m/s} \), \( E_z = 0.015 E_0 \), and \( \sigma_{px} = 24 \times 10^{-20} \text{ m}^2 \). [Here \( \sigma_{px} \) is the approximate cross section for charge-exchange calculated using Eq. (3.4.32) of Ref. 2. This value somewhat underestimates the total cross section for momentum transfer.] Figure 3 shows some examples of the equilibrium curves as the antenna electric field \( E_0 \) (and hence the input power) is increased, as well as vectors showing the relative magnitude and direction of the rate of change of the ion and neutral densities. Phase space trajectories are also shown. In each case, the initial values are \( N_i = 10^{23} \text{ m}^{-3} \) and \( N_g = 10^{12} \text{ m}^{-3} \).

Figures 3(a), 3(b), and 3(d) show that when the ion equilibrium curve \( F \) intersects the neutral equilibrium curve \( G \) with a positive slope a stable equilibrium results, with an
inductive mode discharge in (a) and (b), and a helicon wave discharge in (d). In Fig. 3(c), the curve $F$ intersects the neutral equilibrium curve $G$ with a negative slope, making the equilibrium point unstable. That is, the equilibrium point falls in a negative impedance region, leading to oscillations. The phase space trajectory is driven away from this point, however it is also bounded by the requirements that $0 < N_g < N_{g0}$ and $N_i > 0$ and is limited from above by the decreasing returns of the ionization rate with density (and an effective power limit set by the finite value of $E_0$). The result is a limit cycle encircling the negative impedance region, i.e., a relaxation oscillation. Each of the behaviors shown in Fig. 3 have been observed in WOMBAT.

B. Physical interpretation of curves $F$ and $G$

As stated earlier, the dynamics of the system are largely determined by the equilibrium curves $F$ and $G$, which represent a balance between the ionization rate and the ion loss rate, and a balance between the removal of neutrals from the source by ionization and ion pumping against refilling from the diffusion chamber. Hence the conditions that give rise to these curves in the model are likely to be consistent with the experiment if the model and the experiment show similar dynamics. Therefore the physical properties determining the curves $F$ and $G$ in the model will be outlined.

Setting the derivative in Eq. (1) to zero and making the approximation that $L \gg \lambda_{mfp}$ it can be shown that for ion equilibrium (curve $F$), $N_g$ is specified by

$$N_g = \frac{1}{L} \left( \frac{3v_B^2}{4K^2\sigma_{px}} \right)^{1/3}. \tag{18}$$

That is, whenever $K$ increases, the neutral density required to balance the ionization rate against the ion loss rate must decrease. Equation (18) is actually the inverse function $N_g = F^{-1}(N_i)$ since $K$ is a function of $N_i$. In an inductively-coupled plasma, it can be shown$^2$ generally that $K \propto N_i^{-1/2}$ for high plasma densities. Equation (18) then implies $N_i = F(N_g) \propto N_g^2$ for ion equilibrium. The curves for $F$ in Fig. 3 broadly exhibit this relationship, with the exception of the s-shaped part of the curves occurring for ion densities around $10^{18}$ m$^{-3}$. This particular shape is a result of the helicon wave ionization mechanism, and is maximised when $v_\phi = v_{ci}$, occurring when $N_i = 10^{18}$ m$^{-3}$ for the present conditions.

The limit cycle behavior exhibited in Fig. 3(c) is clearly a result of curve $G$ intersecting the unstable portion (i.e., the portion with a negative slope or “negative impedance”) of the s-shaped part of curve $F$. Physically this region of phase space corresponds to a positive feedback situation where an increase in the ionization rate leads to an increase in the ion density, a lowering of the phase velocity towards the optimum value for ionization, and hence a further increase in the ionization rate. In all cases affected by helicon wave coupling, this positive feedback results in a transient overshoot of the ion density above the optimum value for ionization, as seen in Figs. 3(b), 3(c), and 3(d).

By setting the derivative in Eq. (2) to zero, we find that the curve $G$ is given by

$$N_i = \frac{v_x}{4L} \left( \frac{N_{g0}/N_g - 1}{K + \alpha \sigma_{px} v_B} \right). \tag{19}$$

This expression shows that generally the neutral equilibrium curve has an $N_i^{-1}$ dependence and that $N_i$ approaches zero as $N_g$ approaches $N_{g0}$, as expected since any ionization must decrease the neutral density. The rapid divergence of $N_i$ required to maintain neutral equilibrium as $N_g$ decreases is a result of the fact that the neutral refilling rate from the diffusion chamber depends on the difference in neutral density between the source and diffusion chambers and this difference becomes increasingly large as $N_i$ decreases. Therefore the rate of removal of neutrals by ionization and ion pumping must increase as $N_g$ decreases and this is further exacerbated by the dependence of the ionization rate on the neutral density. Hence, the ion density required to maintain equilibrium increases rapidly as the neutral density decreases.

Taking values from the model, it can be shown that $K < \alpha \sigma_{px} v_B$ along the neutral equilibrium curve, hence the equilibrium condition [Eq. (19)] is determined by the rate at which neutrals are removed by ion pumping rather than ionization. This is not to say that neutral losses from ionization are not important in determining the trajectory of the plasma state before the curve $G$ is approached. In particular, the transition from inductive to helicon-wave coupling involves a rapid increase in the ionization rate from an initially low plasma density, hence the depletion of neutrals must be dominated by ionization in this case.

C. Comparison with experiment

Figure 4 shows the time variation of the plasma and neutral densities and Fig. 5 shows the helicon wave phase
velocity calculated from the model for the cases in Fig. 3. The parameters used are consistent with the experimental conditions$^1$ for WOMBAT, and closely resemble the experimental results, which are shown in Fig. 6. In particular, the initial, transient, high-density overshoot, followed by a gradual decay over a few milliseconds and then a rapid decrease in density by a large factor is well reproduced. Also, the initial ion density peak for the model is greater in magnitude than subsequent peaks, in qualitative agreement with the experiment. Figure 4(c) is a textbook$^{15}$ example of a relaxation oscillation, where the slow variation in the neutral density, which determines the period, triggers rapid changes in the plasma density.

The qualitative agreement between the model and experiment suggests that the initial transient high density peak in the experiment is due to a rapid transition to the helicon coupling mechanism when the rf power is turned on, where trapped electrons near the wave phase velocity are the primary contributors to the ionization rate and act to increase the density in a positive feedback between the ion density and the phase velocity (as determined by the dispersion relation). The neutral density in the source then decreases due to the rapid attrition of neutrals during this period of high ionization, which process is amplified by ion pumping. As the neutral density decays, the ionization rate, and therefore the plasma density, gradually decrease until the plasma density is no longer sufficient to sustain efficient ionization by the helicon wave coupling mechanism, and a transition to inductive coupling then occurs. The loss rate of neutrals from the source is now less than the refilling rate from the diffusion chamber and the neutral density in the source increases over a period of a few milliseconds until the plasma density is sufficient to cause a transition back to the W-mode discharge. The neutral density at which this transition occurs is less than the initial neutral density, so that the maximum ionization rate, and hence the plasma density, is lower than the initial peak.

Parameter scans have been taken to compare the model and experiment for a range of conditions. The easiest parameter to vary in both cases is the input power (represented by the electric field strength $E_0$ in the model). Figure 7(b) shows the variation of the ion density with time as a function of $E_0$, with the gray scales representing ion density, for a magnetic field setting of 0.010 T and $N_{e0} = 10^{20}$ m$^{-3}$. This plot clearly shows three regimes as the electric field is increased, which correspond to cases (b), (c), and (d) in Fig. 3; a stable H mode (after a transient W mode), a relaxation oscillation between H and W modes, and a stable W mode.

The period of the oscillation shows an interesting variation as $E_0$ is increased; the time spent in the H mode decreases, while the time spent in the W mode increases. The first of these two effects is consistent with the variation found in the experiment as shown in Fig. 7(a), and is probably due to the fact that as $E_0$ is increased, the unstable equilibrium position around which the limit cycle is centred moves to a lower value of $N_e$. The rate at which neutrals refill the source depends on $N_{e} - N_{e0}$, which on average increases as the equilibrium value of $N_e$ decreases. Hence the refilling rate of neutrals into the source chamber increases with $E_0$, and less time elapses in the H mode before the neutral density has increased sufficiently to trigger a transition to the W mode. The increase in time spent in the helicon wave mode with $E_0$ arises because in the model the ion flux out of the source depends on the ion speed $v_i$, which is assumed constant. As $E_0$ is increased the peak density attained in the W mode increases, however the rate of (quasi-exponential) decay after reaching this value depends on $v_i$, so a longer time is taken for the density to decrease to the critical density below which the W mode is unsustain-

![Figure 5](image_url)

**FIG. 5.** Predicted time dependence of the helicon wave phase velocity corresponding to the cases in Fig. 3.

![Figure 6](image_url)

**FIG. 6.** Experimental measurements of (a) the central plasma density and (b) the helicon wave phase velocity 16 cm from the end of the source chamber.
able. That this behavior is not seen in the experimental suggests that the ion speed is not constant as assumed in the model which is a crude assumption in any case. However, there are other assumptions in the model which could give rise to this effect.

Temporal variations of the plasma density with neutral Ar pressure as measured experimentally and as predicted by the model are shown in Fig. 8. Experimentally, as the pressure increases, we find that the oscillation period decreases, while the amplitude of the initial peak and its width increase. The decreasing period is consistent with the idea that the time spent in the low density mode is determined by the refilling rate from the diffusion chamber, which decreases with $N_0$, while the width of the initial peak increases since more neutrals need to be pumped out of the source region for higher initial neutral densities. These trends are qualitatively reproduced by the model, though over a smaller pressure range.

A discrepancy between model and experiment, evident in Figs. 7 and 8, is that the oscillation period predicted by the model is about twice the experimental value. In the model, this time scale depends largely on the ion and neutral velocities and the length of the source region, which are fixed according to the relevant parameters in the experiment. It is considered likely that the extension of the model to include spatial variations of the ion and neutral densities would reduce this discrepancy. However, much of the simplicity in the model gained by taking a volume-averaged approach would be lost.

Another important qualitative result found in both the model and the experiment is that it becomes more difficult to tune to the oscillation as the source magnetic field is decreased below $0.010$ T, while a stable helicon mode occurs more easily at lower input powers. This effect is due to the $B$-field dependence of dispersion relation for helicon waves [i.e., $v_\phi \approx (\omega B/n_e)^{1/2}$], and presumably could be reproduced by variations in the rf frequency. As the magnetic field is decreased, the optimum phase velocity, and therefore the peak in the ionization rate, moves closer to the peak in the ionization rate due to inductive coupling, so it becomes increasingly easy to transit to a stable W-mode discharge. In the experiment, it becomes increasingly difficult as $B$ is decreased to cause the instability by varying the tuning and the input power—the plasma either exists in a stable inductive or stable helicon mode. This behavior enabled the cw measurements reported previously to be easily obtained at magnetic fields around $0.0050$ T, and became a source of frustration as the magnetic field was increased and the discharge became less stable.

This effect is also predicted by the model, as shown in Fig. 9(a), which shows the peak-to-peak variation of $N_i$ as a function of magnetic field for various $E_0$. It is seen that the maximum density achieved and the amplitude of the density oscillation increase with magnetic field. The experimentally measured maximum and minimum excursion of the oscillat-

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**FIG. 7.** Gray scale plots of plasma density vs time and input rf power (or $E_0$ for the model) from (a) experimental measurements and (b) the model. In both cases $B=0.010$ T.

**FIG. 8.** Gray scale plots of plasma density vs time and neutral pressure (a) as measured in WOMBAT and (b) as predicted by the model for $B=0.010$ T and $E_0=3.2$ kV/m.
ing plasma density in the diffusion chamber of WOMBAT is shown as a function of the source magnetic field in Fig. 9(b), and clearly reproduces the model result.

The model predicts that the input power needed to produce the oscillation (represented by $E_{\text{in}}$) increases with magnetic field. This is also evident in measurements of the net input power (i.e., forward power minus reflected power) taken simultaneously with the density measurements, as shown in Fig. 9(c). However, the extent of the variation in power level is not as large as predicted by the model. This is most probably due to the limitation in the model that the effect of the magnetic field on the ion loss rate is not considered. In the experiment the effective loss rate for the plasma decreases as the magnetic field increases, so the increase in the power input required to cause the instability is reduced. Figure 9(a) shows that the stable high density mode in the model does not vary significantly with magnetic field when the power input is kept constant. This is because the wave trapping mechanism can only affect the ionization rate when the threshold for ionization is within the range $(v_\phi - v_u) < u_{iz} < (v_\phi + v_u)$. If the plasma density is sufficiently high, $v_\phi + v_u < u_{iz}$ and inductive coupling dominates. That inductive coupling is able to support the plasma at such high densities is considered unrealistic, and points to a deficiency in the estimation of the skin depth [Eq. (4)]. However, it may be that the electromagnetic helicon wave fields (which are of the same order as the inductive field) contribute to the ionization rate in the same way as the inductive coupling model, with the electric field simply causing the electron distribution function to oscillate in velocity space. Hence the model may inadvertently have approximately the correct behavior for the case where the helicon wave amplitude is independent of wavelength (i.e., the $k$-spectrum of the launching antenna is large).

IV. CONCLUSIONS

The cause of instability in a high density helicon plasma over millisecond time scales and the relaxation-oscillator behavior observed in the experiment WOMBAT (Ref. 1) have been modeled. A global model predicting the plasma and neutral density variations with time, given the hypothesis that changes in the neutral density trigger transitions between inductive and helicon wave discharges, was constructed. As the model requires the power coupling, or ionization mechanism, to be specified, it provides a test of whether ionization by electron trapping in helicon waves is consistent with the observed dynamical behavior. That a model based on such rough approximations demonstrates qualitative and even semi-quantitative agreement with experiment over a large parameter range (with only a few adjustable parameters) strongly suggests that both the ionization mechanisms and neutral depletion effect included in the model contribute significantly to the experimentally observed dynamics. In particular, the model shows that the negative impedance necessary to produce relaxation oscillations results from a peak in the ionization rate coefficient $K$ due to helicon wave power coupling. This peak occurs when the helicon wave phase velocity (i.e., the plasma density) is such that electrons with energies near the ionization threshold gain enough energy to ionize neutrals by “surfing” the helicon wave.

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