Energy balance in a low pressure capacitive discharge driven by a double-saddle antenna

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A radio frequency (rf) plasma is created at low pressure (~1 mTorr) in the source tube of a “helicon” excited diffusion system in the absence of a dc magnetic field. The coupling is capacitive for the low source power of 160 W at 13.56 MHz considered here. Temperature measurements of the glass source tube yield a plasma power deposition of ~35 W. The plasma parameters (density, potential, electron temperature) were measured using a retarding field energy analyzer. An analytical model based on the measured plasma parameters and on additional external parameters measured in the matching box (rf voltages and phase, rf current) is developed. The model takes into account the geometry of the double saddle rf antenna. It is found that the inside of the glass wall adjacent to the antenna wire charges negatively. Ion acceleration into the glass along the antenna and fast electrons escaping the plasma account for most of the power deposition to the walls (~16.8 W). Secondary electrons liberated by ions impinging onto the glass along the antenna contribute a power of ~4.6 W. Adding the power of 3.7 W deposited to the part of the tube not affected by the antenna, the total power deposition responsible for the temperature rise of the tube is found to be about 25 W. The model shows that the power deposition is strongly nonuniform along the tube as a result of the antenna geometry. An estimate of the power deposited into the electrons by stochastic heating yields ~1.4 W, compared to an estimate of 5.8 W for the measured power loss from electrons. © 2003 American Institute of Physics. [DOI: 10.1063/1.1555058]

I. INTRODUCTION

In a conventional radio frequency (rf) discharge of interest to plasma processing, rf power is applied via a capacitor to one electrode of a parallel plate system. At low pressures the main mechanism for giving power to the electrons is their interaction with the movement of the plasma sheath surrounding the powered electrode. Plasmas are also generated by rf power applied to electrodes, or antennae, which are placed outside the plasma and are separated from it by an insulator which is generally part of the vacuum chamber. Here the insulator plays the role of a capacitor and is in contact with the plasma.

In this paper we describe an experiment where a “helicon” antenna (double-saddle-type) is used to excite a plasma in a 6-cm-diam 20-cm-long Pyrex glass tube connected to a diffusion chamber at one end. Generally there is a constant axial magnetic field associated with the helicon excitation, but here we are interested in the capacitive coupling of the antenna to the plasma and how the rf power is absorbed, so the solenoids were not activated.

It is well known in plasmas excited externally with some form of coil that at low powers the coupling is capacitive; that is, power is coupled to the electrons from the voltage difference between the ends of the coil, which produces an electric field. As the power is increased, a sudden jump occurs and the coupling becomes inductive, where the induced solenoidal electric field, created by the current flowing through the antenna, accelerates the electrons. This type of transition is also seen in “helicon” discharges and is characterized, in the capacitive mode, by a density increase with power which is less than linear and is commonly a square root dependence. In the inductive mode the dependence is typically linear and the plasma is generated by the double-saddle antenna in much the same way as in traditional inductive coupled systems. Although much has been written about helicon coupling in these systems little attention has been paid to the capacitive coupling mechanism.

In this paper we develop a model for the capacitive coupling from a double-saddle antenna and give experimental results of the plasma parameters for a low pressure discharge.

II. EXPERIMENTAL SETUP

A glass tube 6 cm in diameter and 20 cm long was sealed at one end with a metallic cap through which the argon working gas was delivered. The other end of the tube was connected to a 30 cm in diameter, 20 cm long aluminum chamber which was pumped by a 400 l s⁻¹ turbomolecular pump/rotary pump system as shown in Fig. 1. The base pressure of the system was ~10⁻⁶ Torr and for the experiments here, argon was injected at the top of the source tube to maintain a pressure of about 1 mTorr. No external magnetic field was applied.

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A "helicon" antenna of the double-saddle type was positioned around the source tube and connected to a matching network and rf generator. The shape is shown in Fig. 2(a) and to help visualize this rather complicated antenna, Fig. 2(b) shows the three-dimensional antenna "stretched out" into two dimensions. The antenna is made of a 3-mm-diam copper wire and the shape consists of four 12-cm-long sections, two 5-cm-long quarter turns and one 10-cm-long half-turn, so that the total length in contact with the tube (between points A and B on Fig. 2) is 68 cm. The two feeds to the capacitors give an additional 12 cm.

A retarding field energy analyzer (RFEA) previously described was inserted through various ports near the top of the diffusion chamber (Fig. 1) and oriented to measure ions coming from the source (i.e., upward). The RFEA itself is a 1-cm-long 1-cm-diam cylinder inserted in a 3-cm-long 1-cm-diam cylindrical metal casing attached to a supporting tube. Two supporting tubes with "dog legs" were used to obtain radial measurements at the exit of the source tube and axial measurements in the bottom half of the source. A thermocouple connected to the tube at $z = -0.04$ m, i.e., close to the half-turn of the antenna (Fig. 2), was used to measure the temperature rise during operation.

Measurements of the rf potentials at both ends of the antenna (points A and B on Fig. 2) as well as the phase difference between the two ends were obtained using two rf probes and the rf current through the antenna was measured using a rf current probe.

### III. EXPERIMENTAL RESULTS

The power meter on the rf generator was set to 300 W but as this was considered to be a possible area of unreliability, the generator was connected to a large 50 ohm load (previously tested) and the voltage at the load was measured with two independent rf probes (previously calibrated). This measurement yielded a power of 160 W.

The voltage measured along the antenna showed a linear variation from point A to point B and the phase measured between both points was $160^\circ$. The peak voltage along the helicon antenna is shown schematically in Fig. 3 (a phase of $180^\circ$ is chosen for the modeling). Due to the $\pi$ type of matching network, there is a zero in voltage near one end of the antenna with the rf voltage changing phase at this point. The measured peak rf current was 16 A.
The current measured by the RFEA was calibrated to allow density measurements to be made at the same time as the ion energy was measured. By moving the RFEA radially across the exit of the source, the radial profile of the plasma density was measured and is shown in Fig. 4 (open squares). The two triangles correspond to the density at the plasma sheath edge predicted by the free fall model described in the next section. The solid line is a polynomial fit of the data used for the modeling.

The measured axial profile of the plasma density is shown in Fig. 5 (open squares). The two points at the gas feed end of the source (shown by triangles) were not measured in this experiment but are the densities measured in other helicon sources similar to the one described here, and are normalized to fit the present data. They also respect the free fall model at the top end of the source. The solid line is the result a polynomial fit of the data used for the modeling.

With these data it is now possible to plot the density along the length of the antenna. As seen from Fig. 6, it is far from being constant. This will be used in deriving the power losses in the plasma.

Starting from room temperature (~20 °C), the rf was switched on with the cooling fan off and the temperature increase was measured as a function of time for 110 W and 1.5 mTorr (Fig. 7). The saturation temperature obtained after 15 min of operation was measured to be ~120 °C at 160 W and 1 mTorr.

IV. MODELLING THE DISCHARGE

A. Plasma density profiles

The gas pressure in the discharge is low (~1 mTorr) and the mean free paths for all collisional phenomena are greater than the source diameter. Hence the radial density profile is described by the free fall model as distinct from a collisional model where diffusion dominates. For a low pressure argon plasma coupled in a cylinder of radius R, the ratio between the sheath edge density \( n_s \) and the maximum density \( n_0 \) along the radial axis is:

\[
\frac{n_s}{n_0} = \frac{1}{1 + \frac{R}{2a}}
\]

where \( a \) is the electron mean free path.
\[ \frac{n_s}{n_0} = \frac{0.8}{(4 + R/\lambda_i)^{1/2}}, \]  

(1)

where \( \lambda_i \) is the mean total free path for ion-neutral collisions: \( \lambda_i (\text{cm}) \sim (330p)^{-1} \) with \( p \) in Torr. Since the radial measurements of the density with the RFEA positioned at the source exit (\( z = 0.1 \text{ m} \)) do not cover the entire radius range, the free fall model described by Eq. (1) is used to calculate the sheath edge density \( n_s \) close to the source tube (triangles in Fig. 4). The two calculated values \( (n_s \sim 0.3n_0) \) fall on the interpolated fit of the experimental data giving us confidence in the free fall model applied radially. We have arbitrarily positioned the triangles at \( R = -3 \text{ cm} \) and \( R = +3 \text{ cm} \), neglecting the sheath width on first approximation. As discussed below, this approximation is valid on most of the tube area but not along the rf antenna.

Following the free fall model in a cylinder of length \( L \), the ratio between the sheath edge density \( n_s \) and the maximum density \( n_0 \) along the main \( z \)-axis would be\(^1\)

\[ \frac{n_s}{n_0} = \frac{0.86}{(3 + L/2\lambda_i)^{1/2}}, \]  

(2)
i.e., a density ratio of about 0.3 for a cylinder length \( L = 0.2 \text{ m} \). The measurements of the axial density in the bottom half of the source tube (\( 0 \leq z \leq 0.1 \text{ m} \)) show an exit/center density ratio of 0.1, suggesting that the model does not hold axially in the bottom half of the source when assuming a cylinder length \( L \) equal to that of the tube. This could be a result of the plasma expansion into the processing chamber or of the asymmetric position of the rf antenna along the source (between \( z = -0.07 \text{ m} \) and \( z = 0.05 \text{ m} \)) resulting in a localized ionization. Still, recent axial density measurements in a similar system with a larger diameter tube and longer antenna\(^1\) suggest that the free fall approximation can be used to describe the top part of the source by using an effective length \( L \) smaller than that of the tube length. We used the polynomial fit shown in Fig. 5. Additionally, there is a small gas pressure gradient measured in the 6-cm-diam source tube and \( \lambda_i \) varies on axis between 2 cm near the gas inlet and 5 cm near the exit, suggesting that axially we are in an intermediate regime between free fall and ambipolar diffusion (\( \lambda_i \ll L \)). This variation along \( z \) will be neglected and identical radial profiles will be assumed for all values of \( z \) with an average \( n_s/n_0 \) density ratio of 0.3.

**B. Initial power balance**

Neglecting the effect of the antenna for the moment and carrying out a simple power loss estimate using the measured densities next to the source walls and assuming the plasma escapes out from the end of the source into the diffusion chamber at the Bohm speed, we find

\[ P_{\text{plasma}} = Aen_vB(V_p + E_{\text{ion}} + E_{\text{exc}} + 2T_e), \]  

(3)

where \( A \) is the area of the tube contacting the plasma (including the top and bottom ends \( A \sim 0.0433 \text{ m}^2 \)), \( n_s \) is the plasma density next to the wall (sheath edge density), \( v_e \) is the electronic charge, \( v_p = (T_e/M_p)^{1/2} \) is the Bohm (sound) speed, \( V_p \sim 5.2 \text{ T.e} (\sim 30 \text{ V}) \) is the difference between the plasma potential on axis and the floating potential at the wall, \( E_{\text{ion}} \sim 15 \text{ V} \) is the ionization energy, \( E_{\text{exc}} \sim 13 \text{ V} \) is the excitation energy, and we assume that each escaping electron carries \( 2T_e \) of kinetic energy to the wall (the measured electron temperature is \( \sim 6 \text{ eV} \) and \( v_R \sim 3 \times 10^3 \text{ m} \text{ s}^{-1} \)). Assuming an average density of \( 8.4 \times 10^{15} \text{ m}^{-3} \) obtained from the axial fit (Fig. 5) and applying the free fall model radially, the plasma density next to the wall is about \( 2.5 \times 10^{15} \text{ m}^{-3} \) and the power calculated using Eq. (3) is 4.6 W. This value is considerably smaller than the input power.

Although some authors have found power transfer efficiency as low as 0.1% in capacitive coupling modes,\(^8,9\) showing that major loss can occur in the matchbox, eddy and skin currents, and other resistive losses, it is of interest to initially estimate the power loss using our tube temperature measurements. As shown in Fig. 7, the increase is linear during the first minute and the power deposition \( P_{\text{temp}} \) into the tube can be estimated using the tube temperature rise during the initial linear increase and neglecting the heat lost by the tube

\[ P_{\text{temp}} = \frac{C_v m_0 \Delta T}{\Delta t}, \]  

(4)

where \( C_v \) is the heat capacity for Pyrex glass (\( C_v = 753 \text{ J K}^{-1} \text{ kg}^{-1} \)), \( m_0 \) is the mass of the tube (0.210 kg) and \( \Delta T \) is the temperature increase during the time interval \( \Delta t \). The temporal measurements made for 110 W and 1.5 mTorr give an estimated power of 29 W. As shown in Sec. IV D 3 below, we would expect a power deposition of 36 W for 160 W and 1 mTorr rf power and pressure conditions using a correction factor of 1.22 determined from the power deposition model and the plasma parameters measured for both power cases.

We can confirm the above estimate by using Stephan’s law for radiation

\[ J = \sigma (T^4 - T_{\text{amb}}^4), \]  

(5)

where \( \sigma \) is Stephan’s constant (\( 5.7 \times 10^{-8} \text{ W m}^{-2} \text{ °C}^{-4} \)), \( T_{\text{amb}} \) is the initial ambient temperature (\( \sim 20 \text{ °C} \)) and \( T \) is the saturation temperature of \( 120 \text{ °C} \) measured for 160 W and 1 mTorr. The radiant heat loss from the tube is \( J \times A_{\text{cy}l} \sim 35 \text{ W} \), where \( A_{\text{cy}l} \) is the tube area excluding the top and bottom ends (\( A_{\text{cy}l} \sim 0.0377 \text{ m}^2 \)). However, although similar values are obtained from both methods, they only correspond to a local measurement at the top of the source tube close to the antenna half-turn. The tube is not a good conductor and the heating may not be uniform. Hence, the power may be overestimated.

The power of 4.6 W obtained from Eq. (3) cannot fully account for the measured tube temperature increase. The first place to look for power loss is in Joule heating of the antenna,

\[ P_{\text{heat}} = \frac{R_{\text{HF}} I_{\text{rf}}^2}{2}, \]  

(6)

where \( I_{\text{rf}} \sim 16 \text{ A} \) is the peak rf current measured with a current probe and \( R_{\text{HF}} \) is the high frequency skin depth resis-
tance, estimated to be 0.08 ohm for our copper wire at \(\sim 25^\circ C\) and a frequency of 13.56 MHz. This yields a power of 10 W.

It has been shown that this calculation for \(R_{RF}\) is often not valid when estimating power loss.\(^8,^{10}\) Power loss from the radiation fields of the antenna increases the effective resistance of the antenna in the matching circuit, and the vacuum resistance needs to be measured. Suzuki et al.\(^8\) used a loop antenna made of \(1/4\) in. copper tube with \(R_{RF}\) \(\sim 0.028\) ohm and measured an antenna resistance \(R_a\) of 0.19 ohm, i.e., about 6.8 times higher than \(R_{RF}\). Degeling et al.\(^10\) used an antenna made of 3 mm-thick 10 mm-wide copper strip with \(R_{RF}\) \(\sim 0.024\) ohm and measured an antenna resistance \(R_a\) of 0.1 ohm to 0.7 ohm when varying the input power and the magnetic field, i.e., 4–30 times higher than \(R_{RF}\). These measurements suggest that the rf power loss in the antenna could be many tens of watts. Consequently, it was decided to investigate all the possible routes for power deposition.

\section*{C. rf biasing of the glass adjacent to the antenna}

The rf voltage on the antenna penetrates the plasma and is separated from it by the 4-mm-thick Pyrex glass tube. As the dielectric constant of glass is about 4.5, most of the rf is seen at the edge of the plasma and consequently, the glass surface will charge negatively like the bias capacitor in an asymmetric capacitive discharge.\(^11\) Ions in the plasma will be accelerated to the biased section of the glass wall and dissipate power in the wall. Hence we need an estimate of the bias voltage and the area which has charged negatively. In addition, a certain proportion of secondary electrons will be liberated by an ion colliding with the surface. Some of these electrons will be accelerated by the sheath (along the rf antenna) back into the plasma bulk and will produce secondary electrons when escaping the plasma and hitting the tube. Although this second order effect by the electron-induced secondaries is minimal and will be neglected, the effect of the ion-induced secondaries will be taken into account.

The situation here is rather complicated as the antenna is a 3-mm-diam wire contacting the outside of the glass tube, within which is the plasma, a truly 3D problem. In order to construct a tractable model of the plasma sheath in such a configuration we will investigate two simplifications corresponding to a planar and a cylindrical geometry and schematized in Figs. 8(a) and 8(b), respectively. Four sheath parameters need to be determined in order to model the power deposition: the sheath thickness, the sheath “shape,” the sheath area, and the sheath capacitance.

\subsection*{1. Planar geometry}

The planar sheath model [Fig. 8(a)] consists of the antenna wire of radius \(r_w\) in contact with the glass tube of thickness \(S_{TU}\), itself in contact with the plasma sheath of thickness \(S_{CL}\). To determine \(S_{CL}\), we use the Child law for a collisionless sheath in an argon plasma capacitively coupled between two planar electrodes.\(^1\) In planar geometry, the Child law current density is given by

\begin{equation}
 j_C = K_i e_0 \frac{2 e V_{dc}^{3/2}}{M_i S_{CL}^{1/2}},
\end{equation}

where \(e_0\) is the free space permittivity, \(e\) and \(M_i\) are the argon ion charge and mass, \(V_{dc}\) is the dc component of the voltage across the sheath, and \(K_i\) is a constant (\(\sim 0.823\) for rf Child law—presently used—and 4/9 for dc Child law). We use the relation \(V_{dc} = V_{rf}\) for a sinusoidal voltage driven sheath where \(V_{rf}\) is the measured peak rf voltage shown in Fig. 3.

The Bohm current density at the sheath edge is given by

\begin{equation}
 j_B = e n_B v_B.
\end{equation}

Equating Eqs. (7) and (8), we obtain the sheath thickness,

\begin{equation}
 S_{CL} = \left( \frac{K_i e_0}{e v_B} \right)^{1/2} \frac{V_{dc}^{3/4}}{n_B^{1/2}}.
\end{equation}

The sheath thickness along the rf antenna is shown in Fig. 9 (dotted line). For a good part of the antenna, it is greater that \(r_w + (S_{TU}/4.5) \approx 2.4\) mm so that the rf voltage is mostly dropped across the sheath and not across the tube of dielectric constant 4.5, as verified below.

\subsection*{2. Cylindrical geometry}

The cylindrical sheath model consists of two concentric cylinders, an outer cylinder of radius \(r_0\) acting as the emitter (the plasma sheath edge) and an inner cylinder of radius \(r\) acting as the collector (the glass tube wall). The radius \(r\)
Since $r$ as a function of known parameters:

$\frac{\text{sheath potential}}{\text{rf potential}} = \frac{V_{\text{dc}}}{V_{\text{rf}}}$

We used the following analytical fit to their data in our parameter range:

$\beta^2 = \frac{2eV_{\text{dc}}^{3/2}}{M_i\beta^2r_0r}$

where $\beta^2$ is a geometric factor which depends on $r_0/r$ and $K$, is the same constant as in Eq. (7). $\beta^2$ has been given numerically by many authors for an extended range of $r_0/r$ ratios.\(^{2,15}\) We used the following analytical fit to their data in our parameter range:

$\frac{V_{\text{dc}}}{V_{\text{rf}}} = \left(\frac{r_0}{r}\right)^{2.4} \left(\frac{r_0}{r} - 1\right)^{0.4}$

Since $(r_0 - r)^2$ in Eq. (11) is equivalent to $S_{\text{CL}}^2$ in Eq. (7), going from planar to cylindrical geometry is equivalent to using the correction factor $(r_0/r - 1)^{0.4}$. Equating Eqs. (8) and (10), and inserting Eq. (11) we can express the ratio $r_0/r$ as a function of known parameters:

$\frac{r_0}{r} = \left(\frac{K_i\epsilon_0}{e}\right)^{0.42} \left[\sqrt{2eV_{\text{dc}}^{3/2}}/M_i\eta_s\right]^{0.42} + 1.$

In our system, $r_0/r$ varies between 1 and 8 along the antenna. The sheath thickness $r_0 - r$ is shown in Fig. 9 (solid line) and is slightly less than the planar result $S_{\text{CL}}$, as observed by many authors.\(^{2,15}\)

### 3. Sheath area

The shape of the sheath cross section along the antenna needs to be determined in order to calculate the sheath collection area for the power deposition along the antenna. The simplest approximation is to maintain the cylindrical geometry and to assume a circle of radius $r_0 = r_w + S_{\text{CU}} + S_{\text{CL}}$, as shown by Figs. 8(a) and 8(b). Taking into account the presence of the glass tube, the sheath area $A_{\text{sh}}$ per unit length of antenna in cylindrical geometry is approximated by

$A_{\text{sh}} = \pi r_0 - 2r$

with $r = r_w + S_{\text{CU}}$. Eliminating $r_0$ and $r$ gives

$A_{\text{sh}} = \pi(r_w + S_{\text{CU}} + S_{\text{CL}}) - 2(r_w + S_{\text{CU}})$.

An estimate of the sheath collection area averaged over the antenna length gives 0.017 m\(^2\) in cylindrical geometry, i.e., 45% of the glass tube area $A_{\text{cyl}}$ (excluding both ends).

### 4. Sheath capacitance

Using the expressions for the sheath thickness and the sheath area, the validity of the assumption on the sheath potential ($V_{\text{dc}} \sim V_{\text{rf}}$) can be verified. The peak voltage $V_{\text{rf}}$ measured at the metal antenna is applied to the plasma through the glass tube and a sheath and is divided across the tube of capacitance $C_{\text{CU}}$ and the sheath of capacitance $C_{\text{sh}}$. Hence the sheath potential is given by

$V_{\text{dc}} = \frac{C_{\text{CU}}}{C_{\text{CU}} + C_{\text{sh}}} V_{\text{rf}}.$

With a dielectric constant of 4.5 for the glass and unity for the sheath, the capacitance ratio for planar geometry is expressed as

$\frac{C_{\text{CU}}}{C_{\text{sh}}} = \frac{4A_{\text{TU}}S_{\text{CL}}}{A_{\text{sh}}S_{\text{CU}}}$

where the tube area $A_{\text{TU}}$ per unit length of antenna is approximated by

$A_{\text{TU}} = 2(S_{\text{TU}} + S_{\text{CL}})$.

We use an iterative procedure to determine $V_{\text{dc}}/V_{\text{rf}}$. Starting initially with the assumption $V_{\text{dc}}/V_{\text{rf}} = 1$ and solving the previous three equations leads to a new value of $V_{\text{dc}}/V_{\text{rf}}$ along the antenna. This ratio can then be used to recalculate the sheath thickness and area. The iteration converges rapidly. The results obtained for $V_{\text{dc}}/V_{\text{rf}}$ along the antenna initially (dotted line) and after the fifth iteration (dashed line) are plotted in Fig. 10. The result for the sheath thickness after the fifth iteration is plotted as the dashed line in Fig. 9.
for the planar case. The results show that the assumption
\(V_{dc} \sim V_{rf}\) is not valid only for a small region 0.05 m \(\leq z \leq 0.15\) m. Excluding that part, the \(C_{TU}/C_{sh}\) ratio varies along the
antenna similarly to the sheath thickness and is in the range
5–15, i.e., almost all of the rf voltage is dropped across the
sheath.

D. Final power balance

The total power input \(P_{\text{tot}}\) of 160 W injected into the
system will be dissipated by two main mechanisms, the
dissipation losses \(P_{\text{ext}}\) in the matching box (eddy and skin cur-
cents and vacuum antenna radiation), and the plasma
losses \(P_{\text{plasma}}\) due to ionization, excitation, and the kinetic energy
carried by electrons and ions to the walls,

\[
P_{\text{tot}} = P_{\text{ext}} + P_{\text{plasma}}.
\]

(18)

\(P_{\text{plasma}}\) corresponds to sum of the power \(P_{\text{ant}}\) deposited along the
glass adjacent to the rf antenna and the power \(P_{\text{rf}}\) de-
posited on the rest of the tube walls. As discussed below
there is an additional power \(P_{\text{sec}}\) corresponding to the
ion-induced electron secondaries

\[
P_{\text{plasma}} = P_{\text{ant}} + P_{\text{rf}} + P_{\text{sec}}.
\]

(19)

1. Along the antenna

Now it is possible to calculate the power loss from the
plasma to the walls via the rf sheath for the planar and cy-
lindrical cases using the density along the antenna shown in
Fig. 6. Using the sheath model previously described, \(P_{\text{ant}}\) per unit length is

\[
P'_{\text{ant}} = A_{sh} n_s v\theta (V_{dc} + V_{rf} + E_{\text{ion}} + E_{\text{exc}} + 2T_e),
\]

(20)

where \(V_{dc} = V_{rf}\) is the peak rf voltage measured on the
antenna (Fig. 3). The power \(P'_{\text{ant}}\) dissipated by ions accelerated
by the rf produced bias along the antenna and by fast elec-
trons escaping the plasma is shown in Fig. 11 (cylindrical
geometry only for better clarity). Integrating over the an-
tenna length gives 18.1 W and 16.8 W for \(P_{\text{ant}}\) in the planar
and cylindrical cases, respectively. These values are not very
different so we can have some confidence in the approxima-
tions made in the model and subsequent calculations will be
made in cylindrical geometry only.

2. Secondary emission

Extensive work on ion implantation using plasmas has
shown the importance of ion-induced secondary
electrons.\(^{18–21}\) Although data for glass are not available for the
presently required energy range (\(V_{rf} \leq 1.5\) kV), secondary
electron emission from the ion bombardment along the an-
tenna cannot be neglected. We assume a coefficient \(K_{\text{sec}}\) of
0.3 (Auger) which is probably an underestimation,

\[
P_{\text{sec}} = A_{sh} n_s v_B K_{\text{sec}} (V_{dc} + V_{rf}).
\]

(21)

\(P_{\text{sec}}\) is shown along the antenna in Fig. 11 (cylindrical geo-
metry) and integrating over the length gives \(P_{\text{sec}}\) = 4.6 W.

3. Along the tube

The power \(P_{\text{tube}}\) dissipated along the undriven part of the
tube corresponds to the sum of the power deposited on the
top and bottom ends \(P_{\text{top}}\) and \(P_{\text{exit}}\), respectively and the
power \(P_{\text{glass}}\) deposited on the cylindrical glass tube, from
which is subtracted the power \(P_{\text{sheath}}\) corresponding to the
glass tube affected by the rf antenna,

\[
P_{\text{tube}} = P_{\text{top}} + P_{\text{exit}} + P_{\text{glass}} - P_{\text{sheath}}.
\]

(22)

where

\[
P_{\text{top}} = e \left(2 \pi \int_{0}^{R} n_s(r,-0.1) r\,dr\right)
\]

\[
\times v\theta (V_{rf} + E_{\text{ion}} + E_{\text{exc}} + 2T_e) = 1.53 \text{ W}
\]

(23)

and \(P_{\text{exit}}\) calculated using \(n_s(r,0.1)\) gives 0.23 W. \(P_{\text{glass}}\) is
obtained similarly:

\[
P_{\text{glass}} = e \left(2 \pi R \int_{-0.1}^{0.1} n_s(r,z) z\,dz\right)
\]

\[
\times v\theta (V_{rf} + E_{\text{ion}} + E_{\text{exc}} + 2T_e) = 4.04 \text{ W}.
\]

(24)

The power \(P'_{\text{sheath}}\) per unit length can be approximated in
cylindrical geometry as follows:

\[
P'_{\text{sheath}} = 2r_o n_s v_B (V_{rf} + E_{\text{ion}} + E_{\text{exc}} + 2T_e).
\]

(25)

Integrating over the antenna length gives \(P_{\text{sheath}} = 2.1\) W. Consequently, Eq. (22) leads to a value of 3.7 W for \(P_{\text{tube}}\),
which is about four times smaller than \(P_{\text{ant}}\).

Returning to Eq. (19), we find \(P_{\text{plasma}}\) to be 16.8 +3.7
+4.6 = 25 W which is five times more than the power of 4.6
W initially estimated from Eq. (3) and closer to the power
deposition \(P_{\text{temp}}\) or \(J \times A_{cyt}\) of ~35 W estimated from the
temperature measurements.

The model can now be used to estimate the power depo-
sition along \(z\) and the power calibration factor used for the
calculation of \(P_{\text{temp}}\) [Eq. (4)]. Averaging over the tube cross
section, and assuming \(P_{\text{sheath}} \approx P_{\text{glass}}\) in Eq. (22), the power
deposition \(P_{\text{plasma}}\) along the \(z\)-axis, which leads to heating
of the source tube is shown in Fig. 12. The two largest values
are the last two, which correspond to the antenna half-turn at the top at \(z = -0.07\) m and to the two antenna quarter turns at the bottom of the
source at $z=0.05$ m. These are not reflected in the axial density profile (Fig. 5), probably due to the low pressure used, which leads to a large mean free path for ionizing electrons. Most importantly, Fig. 12 shows that the averaged power deposition in the top half of the tube (where the tube temperature measurements were performed at $z=-0.04$ m) is about twice that of the bottom half. This is possibly one reason for the larger power of 35 W obtained from the tube temperature rise measurements, compared to 25 W from the modeling results based on the electrical and plasma measurements. Other reasons might be an underestimation of the secondary emission coefficient $\delta_{\text{sec}}$ or some heating from the antenna by direct contact with the tube.

The temperature measurements with time (Fig. 7) were performed for slightly different operating conditions (110 W and 1.5 mTorr) for which the model was run by inserting the maximum measured peak voltage and plasma density (the measured electron temperature was not changed) and assuming similar profiles for the peak voltage along the antenna (Fig. 3), the radial density (Fig. 4) and the axial density (Fig. 5). Since the power term $P_{\text{plasma}}$ [Eq. (19)] is responsible for most of the source tube heating, the corresponding correction factor of 1.22 obtained by running the model for both operating conditions was applied to $P_{\text{temp}}$. An estimated power deposition of 36 W for 160 W and 1 mTorr power and pressure conditions was obtained, in good agreement with the power of 35 W calculated using Stephan’s law [Eq. (5)].

4. **Stochastic heating**

In addition to the total power balance, we can examine the power balance of the electron component only. The power $P_{\text{stoc}}$ per unit length transferred to the electrons by the rf sheath along the antenna can be estimated as

$$P'_{\text{stoc}} = 0.45A_{\text{sh}} \sqrt{\frac{m}{e}} \varepsilon_0 \omega^2 T_e^{3/2} V_{\text{rf}},$$

where $m$ is the electron mass, $\omega = 85.2 \times 10^6$ rads$^{-1}$ is the radiant frequency and $V_{\text{rf}}$ if the peak rf voltage measured

along the antenna. $P'_{\text{stoc}}$ along the antenna is shown in Fig. 11 for the cylindrical case and integration over the antenna length gives $P_{\text{stoc}} = 1.4$ W.

We can estimate the power lost by the electrons by using

$$P_{\text{el}} = P_{\text{top}} + P_{\text{exit}} + P_{\text{glass}}.$$  

We find that $P_{\text{el}} = 5.8$ W, about four times higher than the theoretical estimate from Eq. (26). Ohmic heating of electrons by capacitive and/or inductive rf currents flowing in the plasma bulk might be one source of additional power deposition.

5. **Power efficiency**

The total power input $P_{\text{tot}}$ into our system is 160 W. Using the tube temperature measurements we estimate a power efficiency $P_{\text{temp}}/P_{\text{tot}}$ of 0.22 and using the model based on the measured plasma and external parameters we find a power efficiency $P_{\text{plasma}}/P_{\text{tot}}$ of 0.16. The operating conditions correspond to a capacitive coupling with a density range of $10^9 \leq n \leq 10^{10}$ cm$^{-3}$. Our results are in excellent agreement with those obtained by Suzuki et al.$^8$ who calculated a power transfer efficiency of 0.1–0.3 for the same density range by measuring the radiative resistance of the external rf single-loop antenna ($R_a \sim 7 R_{\text{HF}}$).

Using an average power deposition of $\sim 30$ W, we find that the remaining 130 W are lost in our matching box which gives an antenna radiative resistance of $R_a \sim 0.1$ ohm, i.e., 13 times bigger than $R_{\text{HF}}$. This result agrees very well with other experimental$^{10}$ and numerical$^{22}$ studies; Kamenski$^{22}$ modeled the antenna radiation resistance in a cylindrical helicon wave driven plasma source for four commonly used antennas. Radiation resistances of $\sim 0.05$–0.5 ohm were found in the $10^9 \leq n \leq 10^{10}$ cm$^{-3}$ density range for a double-saddle antenna.

V. **CONCLUSION**

The main plasma parameters (density, potential, and electron temperature) have been measured axially and radially in a helicon source equipped with a double-saddle antenna and operating in a low pressure low density capacitive mode in the absence of a dc magnetic field. Additional measurements of the plasma temperature and of the antenna rf voltages and rf current have been carried out. A model of the energy balance based on the experimental data has been developed by initially modeling the plasma sheath along the double-saddle antenna of rather complex geometry. For the present capacitive coupling, the rf antenna voltage is mostly dropped across the sheath and the glass surface along the antenna charges negatively like the bias capacitor in an asymmetric discharge. The fraction of the glass tube area affected by the biasing is large ($\sim 45\%$). Consequently, the main power deposition terms correspond to ion acceleration into the glass along the antenna and to the subsequent ion-induced secondary electrons, both contributing to the low power transfer efficiency of this capacitively coupled discharge.
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12 J. D. Cobine, Gaseous Conductors (McGraw–Hill, New York, 1941), Chap. 6.