Eigenmode Structure in Solar Wind Langmuir Waves

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Abstract

Standard theories of wave growth and evolution describe complex, nonlinear interactions that form the basis of many chorus events from space plasmas including auroral kilometric radiation, solar type II and type III radiation, radiation belts, and planetary radio emissions. Many of these theories, however, are built on the assumption that the background medium is uniform even though observations have shown quite the opposite. In many space regions, ambient density fluctuations are large enough to invalidate the “uniform background” assumption. Furthermore, observations show that waves interact in a highly localized fashion. For example, the narrow-band wave emissions of type II and III radio bursts are a set of weakly-coupled, strongly-localized pulsations. Auroral kilometric radiation displays highly modulated wave packets and discrete, distinct frequency lines. Motivated by observations, we investigate a hypothesis that the spatial and frequency-domain structures in waves observed in space plasmas are due to ambient density fluctuations. We compare waveforms extracted from STEREO, FAST, and sounding rockets to eigenmode solutions and find that, in most cases, ambient density perturbations are a plausible explanation for the packet shapes. Under this hypothesis, the ambient density perturbations that carry wave power are localized, and the wave eigenmode structures would be adequate to reproduce the observed behavior allowing for electromagnetic emissions to efficiently propagate. We also discuss the possibility that eigenmode growth can form the basis for the stochastic wave theory that describes the type III emissions.

Motivation: Rocket Observations

Sounding rocket observations [Allen and Lalonde, 1999] describe the structure of Langmuir waves in the auroral ionosphere. This structure was observed in the form of a series of evenly-spaced frequency bands with very close spacing (δn = 0). Allen and Lalonde [1999] showed that these bands could be explained as trapped eigenmodes of Langmuir torus. Their analysis showed that the trapping was perpendicular to the magnetic field and that the parallel wave number δk was a change so growth was continuous. The WSA analysis predicted evenly-spaced frequency bands separated by narrow frequency spacing, δn = δk, is consistent with observations.

Analytic Theory

We assume that the density perturbations pre-exist. The solar wind is a highly turbulent medium with observed magnetic and electric field perturbations that typically follow a Kolmogorov spectra [e.g. Bale et al., 2000], so assuming similar perturbations in density (e.g., REI) can be justified. For tractability, we assume a local density perturbation in the form of a parabolic well moving at the sound speed (v).

\[ \Delta \omega = \frac{\tilde{c}^2 - \tilde{c}_0^2}{2L^2} \]

where \( \tilde{c} \) is the electron density in the absence of the perturbation, B represents the depth of the density perturbation, and L represents the scale size of the well. We search for a local solution for Langmuir wave packets. Langmuir Eigenmodes

In a frame moving with the sound velocity c, the wave equation (1) simplifies to \( \Delta \omega = \frac{1}{2} \Delta n - \frac{\partial^2}{\partial t^2} \), where the prime symbol represents the moving frame. The high-frequency component of the one-dimensional Zakharov equations [IEEE] describes the interaction of a slowly-varying envelope of a Langmuir wave, \( E(x,t) = E_0 e^{i(\varepsilon x - \omega t)} \), containing the \( \alpha^2 \) and \( \beta^2 \) to a density perturbation, \( \Delta \omega \). For tractability, we assume a local density perturbation in the form of a parabolic well moving at the sound speed (v).

\[ \delta \omega(t) = \frac{c^2 - c_0^2}{2L^2} \]

where \( \Delta \omega \) is the difference between the Langmuir wave group velocity and the sound speed. \( \Delta n = \frac{\omega^2}{2} (c^2 - c_0^2) \), and \( \Delta o \), respectively, electron temperature and the electron plasma frequency. Since we assume the density perturbations pre-exist, the low-frequency part of Zakharov equation are not needed.

A solution can be formed by assuming a form \( E(x,t) = E_0 e^{i(\varepsilon x - \omega t)} \) where \( \Delta n \) is real and slowly varying in t. Ane is a constant frequency shift and \( \Delta o \) is a drift in wave number. Substitution of the above form into Equation (2) allows for separation into a real and imaginary part.

\[ \frac{\partial^2 E}{\partial x^2} - \frac{\partial^2 E}{\partial t^2} = \frac{1}{2} \Delta \omega \Delta n - \frac{\partial^2 E}{\partial t^2} \]

Conclusions

In case studies, we can describe the wave pulsar shape and spectral shape of observed solar wind Langmuir waves as trapped eigenmodes in one dimension:

• The trapped waves have a dominant eigenmode with inner contributions from only 1-2 other modes. The higher-order eigenmodes can dominate the observed result.
• The observed density perturbations are <1% with widths of a few km. The width of the density perturbations roughly the same as the wavelength.
• The observed spectral shape is a combination of frequency-quantized (n+1) and Doppler shifting of the n-th packet. The frequency quantization is often 100 kHz, so careful consideration of the Doppler shift is required.

Clarity, the imaginary part leads to \( \delta \omega = \frac{c^2 - c_0^2}{2L^2} \). With \( \Delta n \) defined, Equation (6) combines with Equation (1) to

\[ \frac{\partial^2 E}{\partial x^2} - \frac{\partial^2 E}{\partial t^2} = \frac{\Delta \omega \Delta n}{2} \]

subject to the quantization condition \( |\frac{\omega^2}{2} (c^2 - c_0^2)| < 2\pi r e^{-\frac{n^2}{2}} \). where \( r \) is a positive integer radius in the \( e^{th} \) harmonic. The parallel wave number \( \Delta k \) is the contribution (amplitude) of the \( m^th \) mode. The full set of solutions of Langmuir eigenmodes can be expressed as:

\[ E(x,t) = \sum_{e=1}^{n} \sum_{m=1}^{\Delta k} e^{-\frac{\omega^2}{2} (c^2 - c_0^2) r e^{-\frac{n^2}{2}}} \]

subject to the quantization rule:

\[ \Delta \omega = 2n + 1 \Rightarrow \Delta k = 2n + 1 \Rightarrow \Delta \omega = \frac{c^2 - c_0^2}{2L^2} \]

The Case 1 waveform is not a Gaussian shape with a slight asymmetry. The fit is dominated by the n=1 mode with a small contribution from higher-order modes. The observed waveform was in the high-speed solar wind (100 km/s) and the Case 2 waveform is the only peaked in its spectrum. This case appears to be dominated by the n=1 mode with a small contribution from the next higher-order modes. The solar wind speed was roughly 310 km/s.

The Case 3 waveform is very complex at first glance. However, both the wave forms and spectral shapes are described by a few harmonic polynomial combined with consistent n=6 mode. The solar wind speed was roughly 350 km/s.