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# Electron scattering as a tool to study zero-point kinetic energies of atoms in molecules



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# ABSTRACT

High resolution electron compton scattering (ECS) is being used to study the atomic momentum distributions and hence the zero-point kinetic energies (ZPKE) of the scattering atoms. Such studies have shown that the scattering is from a single atom of the scattering sample. For an electron beam with a well defined incident energy, the scattered electron energy at any angle from each atomic species is Doppler broadened. The broadening reflects the atomic momentum distribution contributed by both the internal and external motions of the molecular system. By measuring the Doppler broadening of the scattered electron lines it was possible to determine the kinetic energies in gases such as H<sub>2</sub>, D<sub>2</sub>, HD, CH<sub>4</sub> and in H<sub>2</sub>O, D<sub>2</sub>O and NH<sub>3</sub> were measured and compared with those calculated semi-empirically using the measured optical infra red (IR) and Raman frequencies of the internal vibrations of the molecules. In general, good agreement between the measured and calculated values was found. Electron scattering was also used to study the ratio of e-scattering intensities from the H- and O-atoms in water (H<sub>2</sub>O), where some anomalies were reported to exist.

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# 1. Introduction

In recent years many scattering studies were carried out using electron beams with incident energies between 1 and 40 keV. The targets were in various forms and the electrons were found to scatter independently from each atomic mass and not from the molecule as a whole. This behavior is contained in the assumption of the impulse approximation (IA) found to hold in many cases studied using the electron scattering method; it is due to the high incident electron energies compared to the vibrational energy of the atoms in the samples.

Few methods were reported in the literature in which a direct measurement of the atomic kinetic energies was carried out: (1) Nuclear resonance photon scattering (NRPS) [1], (2) neutron compton scattering (NCS) [2,3], and (3) electron compton scattering (ECS) [4–6]. Here we use the abbreviation ECS, to emphasize the similarities with the well-established NCS technique. When the same technique is used for determination of the composition of surfaces, it is often referred to as electron Rutherford

Backscattering (ERBS). The same spectrometer can study electronic excitations, in which case the technique is called reflection energy loss spectroscopy (RELS). The NRPS method used either gamma rays (emitted from thermal neutron capture reactions on various elements) or bremsstrahlung photons created by 4–10 MeV electrons striking a high-Z target. In the first case a nuclear reactor was employed to provide the high flux of neutrons for producing the intense photon beam; it was used extensively for several applications [1,7–9]. The other case employed an electron Linac to produce the high intensity bremsstrahlung beam [10].

The second method using NCS was found to have several properties in common with ECS in the sense that the neutron scatters from an atom in a molecule as if it was free, thus the kinematics in the two cases are quite similar. In fact, the atomic kinetic energies measured by the two methods were found in many cases to be close to each other. Moreover, as mentioned below, it was possible to check some anomalies reported by the NCS method [11–14]; the result was that those anomalies did not persist using the ECS technique [15].

The atomic kinetic energies in molecules and in solids were calculated using a semi-empirical (SE) method [1,16] in which the vibrational frequencies are taken from experiment and it is assumed that there is no coupling between the different modes

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of motion. This method was found to yield in several cases the best and the closest agreement to experiment.

Because of the small electron mass, the recoil energy transferred to the scattering atom is very small [17–19]. Thus, the scattering process operates as a mass analyzer causing the energy spectrum of the deflected electrons to split according to the masses of the scattering atoms of the molecule. In order for the energy separations between the scattering peaks to be resolved, a high resolution spectrometer had to be used [4]. The Doppler broadening  $\Delta = (4K_eE_{rec})^{1/2}$  of each electron line is related to the mean kinetic energy  $K_e$  of the scattering atom and to the recoil energy  $E_{rec}$ ; it provides information on  $K_e$  in the same fashion as that of NCS [19].

The measurement of the  $K_e$  of a scattering atom in a molecule is important as it includes the part contributed by the zero-point vibrational motion and may be obtained by measuring the Doppler broadening of the scattered electron lines. In any H-containing molecule the internal kinetic energy is carried mostly by the light partner, i.e. the H-atom and can be measured with good accuracy as in NH<sub>3</sub>, H<sub>2</sub>O, CH<sub>4</sub> (see below).

## 2. Experimental details

In the measurements, an electron beam with a well defined incident energy between 1 to 40 keV and low thermal spread was used. The energy analysis of the scattered electrons requires spectrometers of very high resolution to resolve the small difference in recoil energy of the atoms present as well as the total energy spread of the electrons scattered from a specific isotope. The energy resolution of our spectrometer is 0.3 eV; it includes the thermal spread of the electrons emitted from a barium oxide cathode. Besides the experimental resolution the energy of the scattered electron is affected by the Doppler broadening caused by the internal and external motions of the atoms of the sample. This is the quantity we want to measure. Fig. 1 depicts the Doppler broadened shape of electrons scattered at 135° from the H-atom (of a solid ammonia (NH<sub>3</sub>) sample) at 80 K using a 2.5 keV incident e-beam. It shows the energy separation between the peak intensities of the electrons scattered from N- and H-atoms in NH<sub>3</sub>, illustrating the type of resolution required in such measurements [4]. As the N peak is much narrower than the H peak, it is immediately clear that the width of the latter is dominated by Doppler broadening. Apart from  $NH_3$ , the samples discussed in this work are  $H_2$ , HD, D<sub>2</sub>, CH<sub>4</sub> and H<sub>2</sub>O. The electron incident energies employed in the studies discussed here were between 1 and 6 keV.



**Fig. 1.** Spectrum of 2.5 keV electrons scattered over  $135^{\circ}$  from a solid ammonia (NH<sub>3</sub>) sample at 80 K. It illustrates the energy separation between the N- and Hatoms and the Doppler broadened shapes of the peaks. It also shows the fitting procedure and assumed background due to detected electrons that have also created electronic excitations in the target.

Gas phase measurements are near room temperature as the pressure differential over the nozzle is small and adiabatic cooling should thus be minor. The low temperature measurements are made from frozen samples using either a cold finger (H<sub>2</sub>O case at 118 K) or a LN<sub>2</sub> cryostat (NH<sub>3</sub> case at ~80 K). From the spectra we determined the intrinsic width (Doppler broadening  $\Delta$ ) of the peaks (by subtracting the experimental resolution from the observed width in quadrature) and for isotropic systems the intrinsic width is related to the mean kinetic energy  $K_e$  according to [6]:  $\Delta = \sqrt{\frac{4}{3}K_eE_{rec}}$ . With  $E_{rec}$  the recoil energy for scattering from a stationary atom.

In the following we explain how the atomic kinetic energies were calculated.

# 3. Calculations and results

## 3.1. Diatomic gases

As an example we show how to calculate the H-kinetic energy,  $K_e(H)$ , in a diatomic molecule such as HD in gaseous phase. Note that  $K_e(H)$  is contributed by three components: Translation, *free* rotation and internal vibration of the molecule. By assuming a decoupling of the three motions, and using the harmonic approximation, we may write [1,16]:

$$K_{e}(\mathbf{H}) = S_{t} \frac{3}{2}kT + S_{r}kT + S_{v} \frac{hv_{0}}{2} \left(\frac{1}{e^{hv_{0}/kT} - 1} + \frac{1}{2}\right)$$
(1)

where  $S_t$ ,  $S_r$  and  $S_v$  are the kinetic energy fractions shared by the H-atom in the translational, rotational and vibrational motions of the H-atom in the HD molecule respectively. Here, the vibrational motion of HD is represented by a harmonic quantum oscillator with frequency  $v_0$  (taken from experiment) [20]. Eq. (1) may be viewed as representing the kinetic energy of any isotope in H<sub>2</sub>, HD and  $D_2$ . The energy fractions of Eq. (1) for those isotopic gases are listed in Table 1 from which the atomic kinetic energies of H and D were calculated (Table 1). It was assumed that the D mass is twice the H-mass and that the spring constant of the oscillator is the same for the three gases. As expected, the calculated  $K_e(H)$ in H<sub>2</sub> is smaller than that in HD but higher than  $K_e(D)$  in D<sub>2</sub> as well as in HD. This may be understood by noting that the oscillator kinetic energy in HD is divided between the H- and D-masses in a 2:1 ratio. Table 1 also lists the calculated  $K_e(H)$  values at 145 K and 0 K. Note that the zero-point value was calculated by accounting for the contributions from both the external-lattice and the internal motions, while in the gaseous phase where the molecule freely rotates, the major contribution to  $K_e$  is from the internal vibration. In HD at 145 K for example, the major part, 84% of  $K_{e}(H) = 89.1$  meV, is contributed by the zero-point motion of the

#### Table 1

Calculated energy fractions  $S_t$ ,  $S_r$ ,  $S_v$  together with the kinetic energies,  $K_e(H)$  and  $K_e(D)$ , at 0 K and 145 K of H<sub>2</sub>, HD and D<sub>2</sub>. Values of the experimental frequency  $v_0$  are also listed together with the *measured*  $K_e$  values using the ECS method. Indicated in the last column are the calculated total zero point energies (bold) of the H and D atoms, which include contributions from both external lattice and internal modes of motion, where the lattice modes were calculated assuming a pure Debye solid [22]. Parenthesized italics values are the calculated zero point energies of the internal vibrations obtained using Eq. (1).

		-					
Gas	St	Sr	Sv	$v_0  [cm^{-1}]$	$K_e(H,D)$ [meV]		
					<i>T</i> = 145 K		0 K
					Calc	Exp	Calc
H <sub>2</sub> (H)D H(D) D <sub>2</sub>	1/2 1/3 2/3 1/2	1/2 2/3 1/3 1/2	1/2 2/3 1/3 1/2	4155 3599 3599 2939	80.1 89.1 48.4 61.3	74 95 46 70	<b>69.3</b> (64.5) <b>78.9</b> (74.5) <b>42.3</b> (37.3) <b>50.2</b> (45.6)

internal vibration being 74.5 meV. In fact the zero-point value at 0 K is always higher than that of internal vibration because of the effect of vibration and libration of the  $H_2$ , HD and  $D_2$  molecules in a *solid lattice* formed on approaching 0 K. Moreover, the Debye temperature of e.g. solid  $H_2$  and  $D_2$  is quite high being 118 and 114 K respectively [21]. Accordingly, the zero point values at 0 K in Table 1 were calculated by assuming a pure Debye solid [22] where an average Debye temperature of 116 K was assumed for HD.

Table 1 shows also the experimental values of  $K_e(H)$  deduced by analyzing the results of ECS measurements [23], revealing large deviations of ~10% from the calculated values. To improve statistics we may consider the *sum* of the four measured  $K_e(H)$  values of Table 1 which differ by only 2% from that of the calculated sum. The deviations may be due to the assumptions and uncertainties involved in analyzing the data of [23]: (1) The temperature T of the reference gas (He) was estimated from the measured Doppler width of He to be 145 K indicating a huge drop in the gas temperature after leaving the needle near the electron beam. (2) The temperature T of H<sub>2</sub>, HD and D<sub>2</sub> *was* assumed to be the same namely 145 K during the measurement. (3) The instrumental resolution was taken to be 0.2 eV and assumed to be the same for H<sub>2</sub>, HD and D<sub>2</sub> at all incident e-energies between 1 and 6 keV.

## 3.2. The NH<sub>3</sub> molecule

The H-kinetic energy  $K_e(H)$  in ammonia is obtained by considering the geometry of the NH<sub>3</sub> molecule which has a pyramidal shape where the H-atoms form an equilateral triangle, with the N-atom at the top [24]. Here we also assume that the H-atom in NH<sub>3</sub> has three motions: translation, rotation and internal vibration which in the SE approach are assumed to be uncoupled. Hence  $K_e(H)$  for the vapor phase may be written as:

$$K_{e}(\mathrm{H}) = S_{t} \frac{3}{2}kT + S_{r} \frac{3}{2}kT + \sum_{j=1}^{6} S_{j} \frac{1}{2} \left( \frac{h\nu_{j}}{e^{h\nu_{j}/kT} - 1} + \frac{h\nu_{j}}{2} \right)$$
(2)

The first two terms on the right of Eq. (2) represent the classical expression of translation and rotation of the entire molecule (each being 3kT/2) with S<sub>t</sub> and S<sub>r</sub> the fractions shared by H-atom in those modes. The third term is the contribution of the six internal vibrational modes of the NH<sub>3</sub> molecule with frequencies  $v_i$ . Note that  $S_i$ is the fraction of the kinetic energy shared by the H-atom in the *j*th vibrational mode; the factor 1/2 in the third term arises from the fact that in a harmonic oscillator, the kinetic energy is half its total energy. Table 2 lists the six measured internal vibrations frequencies [24,25] of <sup>14</sup>NH<sub>3</sub> in its three phases. To calculate  $K_e(H)$  using Eq. (2), the main problem is to deduce the fractions  $S_i$ , obtained using computational methods of molecular spectroscopy described in [1,24]. Supplementary parameters resulting from such a calculation are the values of the spring constants which are also depicted in Table 2. In addition, the free rotations and translations of the vapor turn in the liquid and solid phases into librations and vibrations with frequency distributions  $g_L(v)$  and  $g_T(v)$  respectively. Thus  $K_e(H)$  in a condensed phase may be written as:

#### Table 2

Normal frequencies  $v_j$  (j = 1,...,4) in units of cm<sup>-1</sup> of <sup>14</sup>NH<sub>3</sub> in the solid, liquid and *vapor* forms (taken from Refs. [27,28]) showing the frequency differences between the three phases. The corresponding spring constants k (in units of 10<sup>5</sup> dyn/cm) and the calculated fractions  $S_j$  of the kinetic energies of the H-atom are listed only for the *vapor* phase.

Solid	Liquid	Vapor	$k_{\rm j}$	Sj
3217	3300	3336.2	5.8658	0.3251
3371	3380	3443.3	0.3184	0.3104
1070	1060	950.2	0.2219	0.2824
1626	1610	1627.8	0.6314	0.3096
	Solid 3217 3371 1070 1626	Solid         Liquid           3217         3300           3371         3380           1070         1060           1626         1610	Solid         Liquid         Vapor           3217         3300         3336.2           3371         3380         3443.3           1070         1060         950.2           1626         1610         1627.8	Solid         Liquid         Vapor         kj           3217         3300         3336.2         5.8658           3371         3380         3443.3         0.3184           1070         1060         950.2         0.2219           1626         1610         1627.8         0.6314

$$K_{e}(H) = S_{T} \int_{\nu_{T0}}^{\nu_{T}} g_{T}(\nu) \alpha(\nu) d\nu + S_{L} \int_{\nu_{L0}}^{\nu_{L}} g_{L}(\nu) \alpha(\nu) d\nu + \sum_{j=1}^{3} S_{j} \frac{h\nu_{j}}{2} \left( \frac{1}{e^{h\nu_{j}/kT} - 1} + \frac{1}{2} \right)$$
(3)

with  $\alpha(\nu) = \frac{\hbar\nu}{2} \left( \frac{1}{e^{\hbar\nu/kT} - 1} + \frac{1}{2} \right)$  the kinetic energy of the corresponding harmonic oscillator where  $\nu_{T0}$ ,  $\nu_{T}$ ,  $\nu_{L0}$  and  $\nu_{L}$  are the limits of integration;  $g_{T}(\nu)$  and  $g_{L}(\nu)$  are normalized so that:

$$\int_{\nu_{T0}}^{\nu_{T}} g_{T}(\nu) d\nu = \int_{\nu_{L0}}^{\nu_{L}} g_{L}(\nu) d\nu = 1$$
(4)

Note that the  $g_{T}(v)$  and  $g_{L}(v)$  values for solid and liquid NH<sub>3</sub> are taken from inelastic neutron scattering (INS) data [26].

Fig. 2 displays the calculated values of  $K_e(H)$  for ammonia between 5 K and 300 K and applies to the gaseous, liquid and solid phases. Note that the calculated curve of the solid was smoothly joined continuously with that of the liquid and the vapor phases. The deviation from one curve to the other is less than 1%. The *classical* value of  $K_e(H) = 3kT/2$  is also shown illustrating the huge contribution of the zero-point value of  $K_e(H)$  being 148 meV which constitutes the major part, ~98%, of the kinetic energy in the vapor phase at 200 K. The same nearly continuous behavior of the  $K_e(H)$ curve joining the three phases was also found in the case of water (see below). A good agreement may be seen between the calculated and the measured  $K_e(H)$  value at 80 K.

Note that the validity of the SE calculations was in fact established in a previous work [22] where the kinetic energy of the *N*-atom in NH<sub>3</sub> was studied using a different technique, namely the NRPS and an excellent agreement was obtained between the measured and SE calculations.

## 3.3. The water molecule

The calculation of  $K_e(H)$  for a triatomic H<sub>2</sub>O molecule (in *vapor* form) proceeds in the same manner as in ammonia but has only 3 normal modes and was discussed in detail elsewhere [29]. In the liquid and solid case however, there is a major difference because the experimental vibrational modes are strongly influenced by the effect of the *hydrogen bonds* which weakens the OH stretch



**Fig. 2.** Calculated  $K_e(H)$  values versus *T* for ammonia (NH<sub>3</sub>) between 5 K and 320 K, covering the gaseous, liquid and solid phases. Note the slow increase of  $K_e(H)$  with *T* and the good agreement with the ECS measured preliminary value at 80 K (solid circle with error bar). The *classical* value  $K_e(H) = 3kT/2$  is also shown revealing a huge difference from the present calculation which is due to the contribution of the zero-point motion. The  $K_e(H)$  value at 80 K was obtained by averaging the results for incident electrons of 1.5, 2.0 and 2.5 keV.



**Fig. 3.** Calculated  $K_e(H)$  values versus *T* for water (H<sub>2</sub>O) between 5 K and 320 K, covering the liquid and solid phases. ECS measured value at 118 K is indicated (solid triangle) together with INS (solid circles) and DINS (solid and open stars) experimental values and theoretical predictions based on thermodynamic macroscopic data [29].

frequencies  $v_j$  from 3686, 3738 cm<sup>-1</sup> in the vapor, via 3280, 3490 cm<sup>-1</sup> in the liquid to 3085 and 3220 cm<sup>-1</sup> in ice Ih [29]. In addition, the free rotations and translations present in a *vapor* turn into librations and vibrations in the liquid and solid phases. The net effect of these opposing trends is to keep the calculated value of  $K_e(H)$  in water almost the same between 5 K and 300 K as may be seen in Fig. 3 which also shows the value,  $K_e(H) = 149 \pm 5$  meV measured for ice at 118 K using the e-scattering method [5]. The figure also displays several other measured values obtained using inelastic neutron scattering (INS) and deep inelastic neutron scattering (DINS). Note also in Fig. 3 the nearly *continuous* behavior of  $K_e(H)$  when passing between the various phases as was also found for ammonia and was discussed in detail in Ref. [29]. Table 3 summarizes the ECS measured and SE calculated  $K_e(H)$ ,  $K_e(D)$  and  $K_e(O)$  values in H<sub>2</sub>O and D<sub>2</sub>O.

## 3.4. Scattering intensities from the H- and D-atoms in water

One of the first high momentum transfer studies of electron scattering from solid H<sub>2</sub>O and D<sub>2</sub>O was published in Ref. [32] but no clear separation of H and O was possible. An important feature studied in the present work using the ECS method is the scattering intensities from H/D atoms relative to the O-atom in the molecule. This problem arose because of strong reported anomalies in the NCS method [11] when comparing the *neutron* scattering intensities from H:O in liquid H<sub>2</sub>O with D:O in liquid D<sub>2</sub>O at incident n-energies,  $E_n = 10-200$  eV. An anomalous ~30% drop of the known n-H scattering intensity relative to the n-D scattering intensity was reported [11]. Because of the similarity between the NCS

#### Table 3

Average measured and calculated  $K_e(H)$ ,  $K_e(D)$ , and  $K_e(O)$  (in meV units) for ice lh at 118 K in both H<sub>2</sub>O and D<sub>2</sub>O where errors are enclosed in parentheses. The incident electron energies were between 1.5 and 6.0 keV [5]. The calculation utilized optical frequencies together with the measured translational and librational spectra of solid [30] and liquid [31] water.

<i>T</i> (K)	H <sub>2</sub> O				D <sub>2</sub> O			
	$K_e(H)$		$K_e(O)$		$K_e(D)$		<i>K</i> <sub>e</sub> (O)	
	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.
118	149(5)	152.1(3)	34(2)	39.2(2)	97(3)	108.4(3)	43(2)	41.4(2)



**Fig. 4.** Calculated  $K_e(H)$  values versus *T* for methane (CH<sub>4</sub>) between 5 K and 300 K, covering the gaseous, liquid and solid phases. ECS measured value at 295 K is indicated. It was deduced by averaging over 6 measured values using different incident energies between 0.75 keV and 2.5 keV [4].

and ECS methods, it was interesting to test this anomaly using ECS [5]. Thus, the e-scattering intensity ratio of H to O in H<sub>2</sub>O ice at 118 K was compared to the intensity ratio of D to O in  $D_2O$ . The results showed that the two ratios are equal [5], thus ruling out the existence of such anomaly with the ECS method. Another anomaly using NCS was reported in which the n-H scattering intensity was found to be smaller by ~30% compared to that of n-D scattering [13] in a sample of liquid HD at 20 K. However, the measured scattering intensities of n-H to n-D using the ECS method and a gaseous HD sample were found to be the same [23], thus no anomaly was revealed [14,33]. Another point worth noting in Ref. [14] concerns an additional drop in the scattering intensity ratio  $I_{\rm H}/I_{\rm D}$  of  ${\sim}30\%$  was reported in a gas mixture of  $H_2 + D_2$ . However, this was very probably caused by the different velocities of the two molecules H<sub>2</sub> and D<sub>2</sub> at the same temperature and not due to any real effect as explained in Ref. [34].

# 3.5. The CH<sub>4</sub> molecule

The  $K_e(H)$  values of CH<sub>4</sub> were calculated in a similar manner to that of NH<sub>3</sub> by accounting for its 12 internal vibrational modes [16] and for the translational and librational frequencies of the solid phase [35]. The results vs T are plotted in Fig. 4 together with one measured value at 295 K obtained [4,36] using the ECS technique,  $K_e(H) = 151 \pm 4$  meV, revealing a good agreement with the calculations. This value is in fact an average over 6 measured values using different incident energies between 0.75 keV and 2.5 keV. Here again the ZPKE of the H-atom is also high but less pronounced than in H<sub>2</sub>O and NH<sub>3</sub> because of the smaller contribution of its lattice frequencies revealing nearly free rotations at low T [37]; this is due to the highly spherical shape of CH<sub>4</sub>.

#### 4. Discussion

The electron scattering technique is now established as an important tool for studying a variety of topics including the measurement of the kinetic energies of H-atoms in several cases. It is of interest to note that the values of  $K_e(H)$  in the three molecules: NH<sub>3</sub>, H<sub>2</sub>O and CH<sub>4</sub> are practically the same at ~295 K. This is probably because the heavy partner has nearly the same mass in all three cases. The situation is different in the diatomic gases because the two partners are very light and the kinetic energy is divided

accordingly between the two atoms. Note that in Ref. [38] the zeropoint *internal* vibrational energies of H<sub>2</sub>, HD and D<sub>2</sub> were calculated by including anharmonic effects. It is interesting to compare the findings of [38] with the present results. Here, we assumed the harmonic approximation, calculated the *kinetic* part of ZPE per atom and included the effect of the *external* lattice vibration and libration. In [38], the *total* internal ZPE was calculated which is nearly twice the kinetic part; the inclusion of the anharmonic effects increased the ZPKE by ~4.4%. Here we included the contribution of the *external* lattice vibration and libration which amounted to ~10% of the ZPKE and was deduced by using the Debye temperature of the solid (see Table 1). This effect was not included in [38].

It is of interest to note that the ECS measured  $K_e(H)$  values are close to those obtained using the NCS. In H<sub>2</sub>O, the  $K_e(H)$  value is practically constant between 5 K and 300 K as it is contributed by two compensating effects: one due to internal stretch vibrations which increases with *T* between 5 and 300 K and the second is due to the external vibrations which decreases with increasing *T* in the above T-range.

It was also shown that the calculation of  $K_e(H)$  using the SE approach can reproduce most experimental results measured by both the ECS and the NCS techniques. Moreover, it was possible to test some of the anomalies in neutron scattering from water reported using the NCS method.

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