Influence of Acoustic Intensity on the Second-harmonic Beam-profile

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***Abstract* – A method for fast numerical simulation of nonlinear wave propagation based on a quasilinear approximation, has previously been presented [1]. The aim of the current study was to validate this method by comparing the simulation results to the results from a conventional simulation model, and to experimental measurements. The reference simulation model was based on solving the KZK-equation using an operator splitting approach. Experimental measurements were obtained in a water tank. An annular array probe was used to generate a focused transmit pulse. Hydrophone measurements of the ultrasonic field were used for experimental verification of the simulation. Beam-profiles from the two simulation methods and the hydrophone measurements were compared for pulses with a mechanical index (MI) ranging from 0.1 to 1.9. The results showed an almost perfect match between the two simulation models for MI less than 0.5. For higher MI values, the KZK-based method showed a gradual increase in the side-lobe level, whereas the main-lobe width and shape were preserved. These effects were also demonstrated by the hydrophone measurements.**

I. Introduction

The introduction of harmonic imaging in medical ultrasound has led to major improvement in image quality. In order to take full advantage of this in ultrasound transducer design, the ability to predict the beam-profile of a particular transducer geometry is important. At present, this is most efficiently done using computer simulations; most commonly by solving the full nonlinear wave equation [2; 3], or by applying a parabolic approximation which leads to the KZK-equation [4; 5].

To compute the beam-profile accurately, the three dimensional geometry of the transducer, and sound propagation need to be taken into account. For an annular probe, the circular symmetry may be exploited to describe the problem in cylinder coordinates. This effectively reduces the computational problem size to two dimensions [4]. In general, however, this is not possible. As a result, these simulations may only be performed on large-scale computers [3; 5].

In situations where the transmitted pulse amplitude is relatively low, or the attenuation is sufficiently strong to suppress the generation of higher harmonic frequencies, the nonlinear propagation will introduce very little modification to the fundamental field. This may be exploited to introduce a quasilinear approximation of the wave propagation [1]. Presented here is a validation of this quasilinear simulation approach.

The KZK-equation is well suited for modelling the nonlinear sound propagation found in ultrasound beams [5]. Therefore, a simulation model which solves the KZK-equation using an operator splitting approach was used as a reference simulation in this study [6].

A situation where an annular array probe transmits an ultrasound pulse through water was simulated using the quasilinear and the KZK-bases simulation methods. The simulation results were compared for various MIs. Hydrophone measurements in a water tank were recorded and used to verify how well the simulations compare to real sound propagation.

II. Theory

A nonlinear equation which describes the propagation of sound in soft tissue is

$$\nabla^2 p(r,t) - \frac{1}{c^2}(p)_{tt} + L(p) = \epsilon \left(p^2\right)_{tt}. \tag{1}$$

Here c is the speed of sound, L is a linear operator which accounts for loss and ϵ is a small parameter [7]. For soft tissue, ϵ is very small, typically $\sim 10^{-3}$.

The quasilinear simulation method is based on a perturbation solution of (1)

$$p(r,t) = p_1(r,t) + \epsilon p_2(r,t) + O(\epsilon^2).$$

Thus

$$\begin{aligned} \nabla^2 p_1(r,t) - \frac{1}{c^2}(p_1)_{tt} + L(p_1) &= 0 \\ \nabla^2 p_2(r,t) - \frac{1}{c^2}(p_2)_{tt} + L(p_2) &= \epsilon \left(p_1^2\right)_{tt}. \end{aligned}$$

If p_1 is the fundamental field, then p_2 becomes the second-harmonic field.

An underlying assumption for using the perturbation solution is that the remaining terms should be $O(\epsilon^2)$. In particular, this means that

$$\epsilon^2 p_1(r,t) = O(\epsilon^2),$$

i.e. the amplitude of the fundamental field should be small. The quasilinear simulation is therefore expected to give reasonable results for transmit pulses with low MI. However, for a higher MI, this assumption will not to hold. The quasilinear simulation would therefore yield degraded results.

For a more detailed description of the quasilinear simulation method, the reader is referred to [1].

III. METHOD

A Matlab implementation of the quasilinear simulation approach has been shown to solve 3D ultrasound beam propagation problems on a standard workstation. When considering an axis-symmetric situation, this is also possible for the reference simulation. This is due to the fact that the KZK-equation is reduced to a 2D equation even for a 3D propagation, when considering an axis-symmetric problem. Therefore, an experiment where an annular array probe transmitting an ultrasound pulse, was used both then comparing the two simulation methods, and when comparing the KZK-based simulation to hydrophone measurements.

A. Quasilinear vs. KZK-based simulation

An initial pressure pulse was prescribed in the transducer plane, mimicking a planar annular array probe. The diameter of this probe was 14.7 mm with a 70 mm focal length. Focus was aquired by delaying the pressure pulse appropriately at each position (see Fig. 1). The centre transmit frequency was 3.0 MHz with a 6 dB bandwidth of 1.0 MHz.

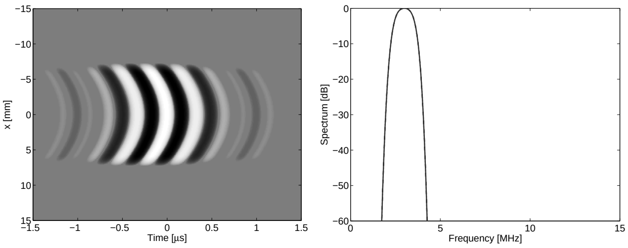

Fig. 1. Transmit field at the simulated annular array probe used for comparing the quasilinear method to the conventional KZK-based method.

The specified transmit pulse was then propagated to the focal plane using the quasilinear and the KZK-based simulation methods. In the focal plane, the RMS-value of the pressure pulse as a function of time was computed for the fundamental field and the second-harmonic field separately. These beam-profiles were compared for transmit pulses with MI ranging from 0.1 to 1.9.

Since the fundamental and second-harmonic fields are computed separately in the quasilinear approach, no additional effort was required to obtain the separate beam-profiles. For the KZK-based simulation, all harmonics are computed at the same time. Thus, filtering out the fundamental and the second-harmonic was achieved by using a

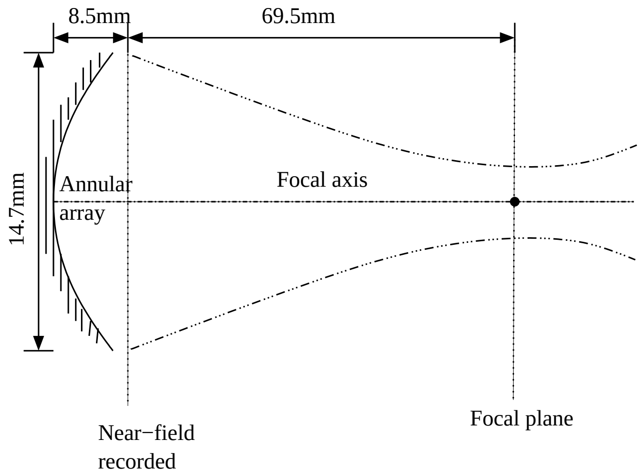

Fig. 2. Experimental setup for comparing the KZK-based simulation to hydrophone measurements.

1.0 MHz bandpass filter centred at 3.0 MHz and 6.0 MHz respectively.

The implementation of the KZK-based simulation is described in [6].

B. Hydrophone measurements vs. KZK-based simulation

The experimental measurements used in this study were recorded in a water-tank using a hydrophone (SEA PVDF-Z44-0400). A pulse with centre frequency of 2.9 MHz was transmitted from an annular array probe (Vingmed Sound APAT 3.25) with a diameter of 14.7 mm and 78.0 mm radius of curvature. This results in an approximate *f*-number of 5.2. In order to obtain an initial condition for the KZK-based simulation, measurements of the near-field were recorded 8.5 mm away from the centre of the probe, perpendicular to the focal axis (see Fig. 2). By doing so, the problem of modelling the physical characteristics of the transducer, for example the curved surface and element sizes, was avoided. However, the near-field measurements contain errors. They are not, therefore, axis-symmetric. Thus, the near-field measurements were modified slightly. Any tilt in the measurements due to the hydrophone scanning not being perpendicular to the focal axis was removed. A representative half axis of the measurement was then selected and rotated around the focal axis to produce the desired axis-symmetric initial condition (see Fig.3).

The rectified near-field was then propagated another 69.5 mm to the focal plane of the annular array. The fundamental component and the second-harmonic component were extracted using a filter with 1.0 MHz bandwidth centred at 2.9 MHz and 5.8 MHz respectively. RMS-values of the simulated fundamental component and the second-harmonic component were then computed. The corresponding beam-profiles for the measurements in the focal plane were created in the same manner.

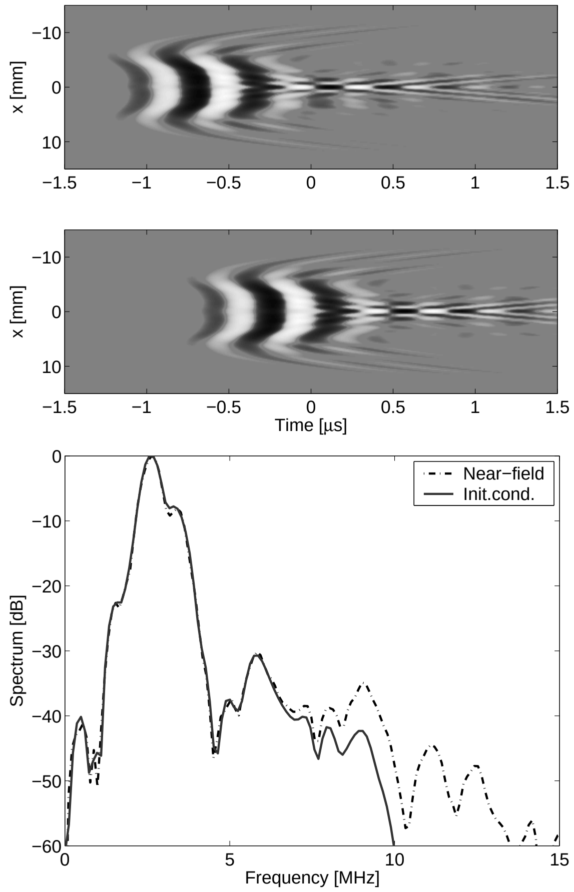

Fig. 3. Measured near-field at 8.5 mm depth (top) and rectified near-field measurement used as initial condition for the KZK-based simulation (middle). Their respective spectra along the focal axis are displayed in the bottom figure.

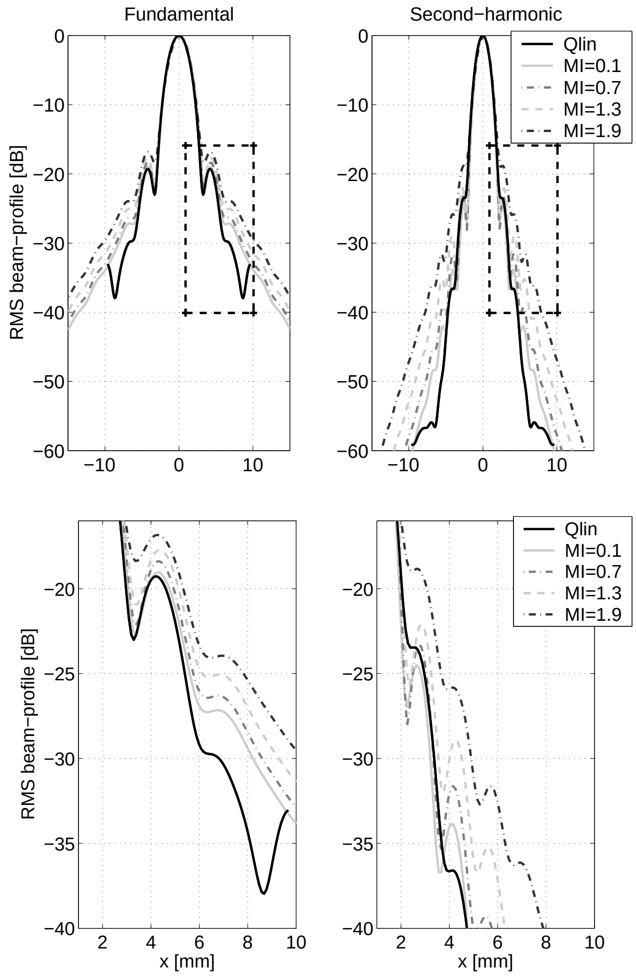

Fig. 4. Beam-profiles obtained by the quasilinear method (Qlin) and the KZK-based method for MI between 0.1 and 1.9. The comparison was made at the focal depth of 70 mm.

IV. Results

Figure 4 shows the beam-profiles in the focal plane when transmitting a pulse with MI ranging from 0.1 to 1.9. The beam-profiles for the quasilinear method do not depend on the MI, and are thus represented only once. The KZK-simulation, however, is a true nonlinear simulation, and displays a change in the beam-profile as the MI increases. The two simulation methods yield almost identical results with MI up to approximately 0.5. For higher MI, the quasilinear simulation gradually under-estimates the side-lobe levels compared to the KZK-simulation.

Figures 5 and 6 show a comparison between the KZK-simulation using a recorded near-field as the initial condition, and hydrophone measurements in the focal plane at 78,0 mm depth. The beam-profiles for the simulated field concur with the corresponding measurement.

V. Conclusion

This study shows that the quasilinear simulation approach yields reasonable results for both fundamental and second-harmonic beam-profiles for low MI. However, when the MI increases beyond approximately 0.5, the predicted side-lobe levels are too low compared to a simulation based on the KZK-equation. This is in concurrence with the underlying assumption for using the first-order perturbation.

When using near-field measurements as initial conditions, the KZK-based simulation produces beam-profiles which are virtually identical to those obtained from the corresponding measurements in the focal plane. Some discrepancy is to be expected, as the near-field was modified to be axis-symmetric before it could be used as an initial condition. The KZK-based simulation may, therefore, be used as a realistic reference for comparison with the quasilinear simulation.

The study presented here was conducted with reference

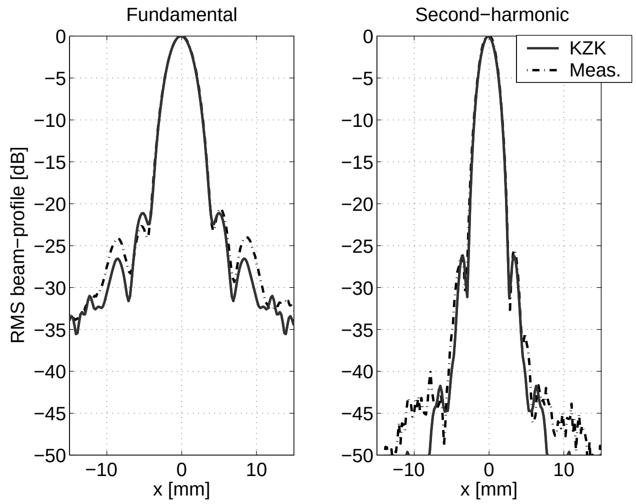

Fig. 5. Beam-profiles for a field simulated using the KZK-equation, and the corresponding measured field compared at the focal depth of 78 mm. Near-field measurement of a transmit field with MI=0.3 was used as the initial condition for the KZK-based simulation.

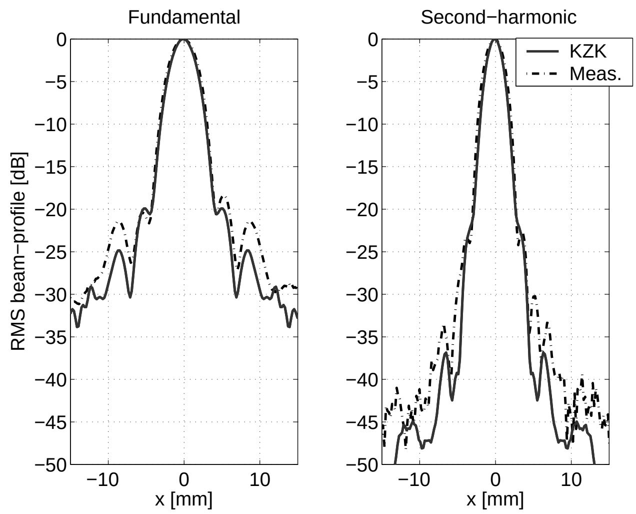

Fig. 6. Same as Fig. 5, but for a transmit field with MI=0.9.

to propagation in water. However, propagation in water is more nonlinear than soft tissue such as muscle and fat, where the absorption is more dominant. The result is substantial generation of harmonic frequencies and shock formation. In tissue, where the generation of harmonics is lower, the quasilinear approximation can be expected to perform better, thus making this simulation method valid also for higher pressures.

In order to make the KZK-based simulation feasible on a standard workstation, the axis-symmetric properties of the problem were utilised to reduce the computation size to a 2D problem for a 3D wave propagation. The additional computational cost for considering a 3D problem without this symmetry property is prohibitive with regard to the KZK-based simulation method. The axis-symmetry was not exploited to any significant degree when performing the 3D quasilinear simulation. Performance would, therefore, be comparably unaffected when considering different transducer geometries. The 3D simulations presented here took approximately 5 minutes on a standard workstation for the quasilinear simulation. By comparison, the KZK-based simulation took 10 minutes.

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