Eigenfunction analysis of stochastic backscatter

Aberration correction in medical ultrasound imaging

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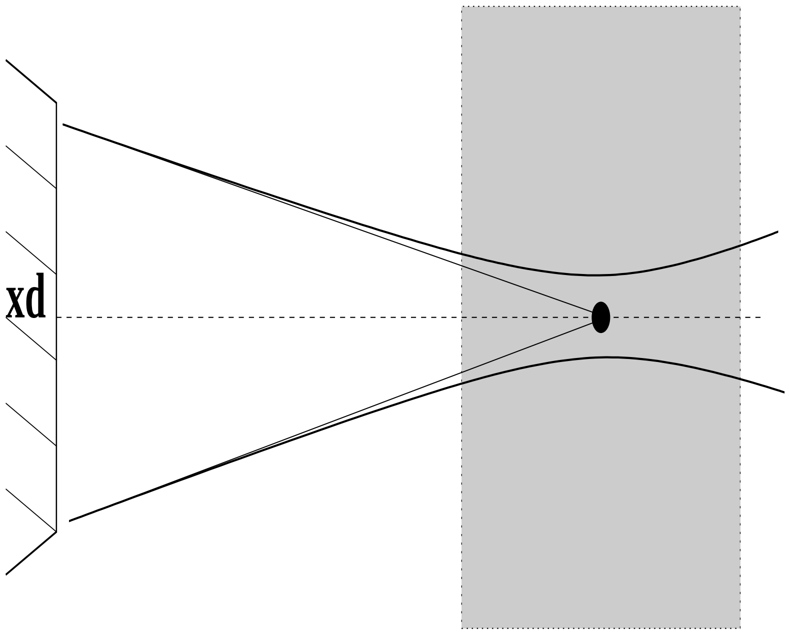
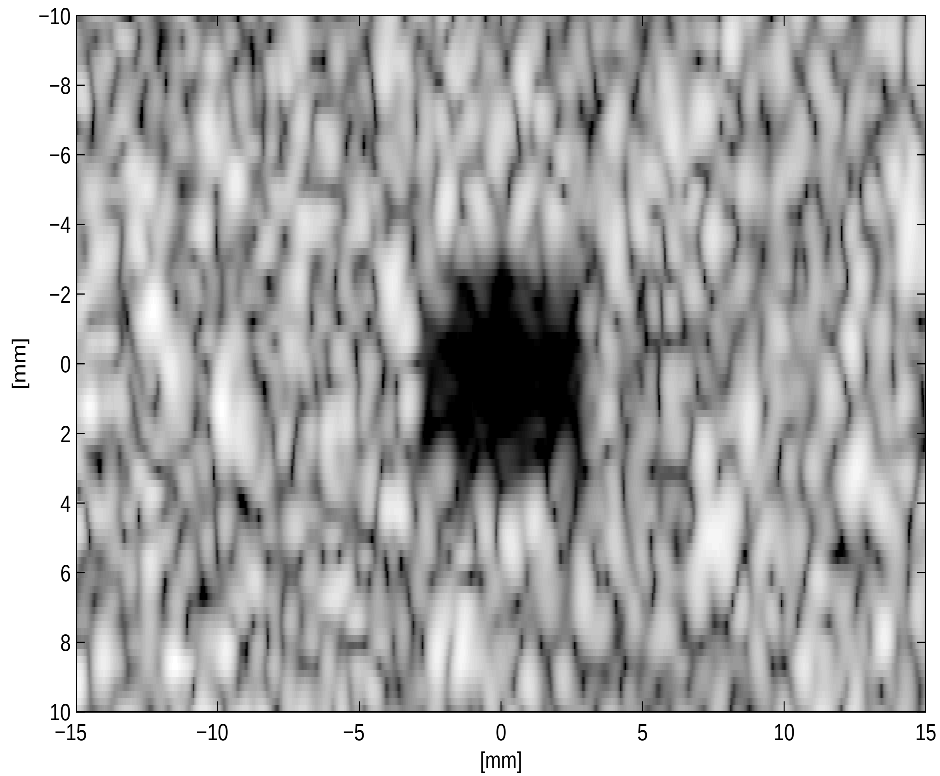
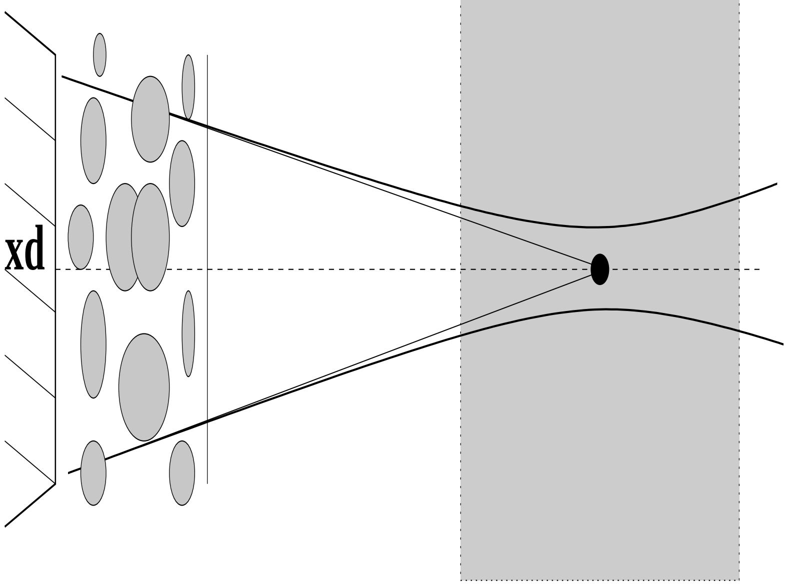
Motivation

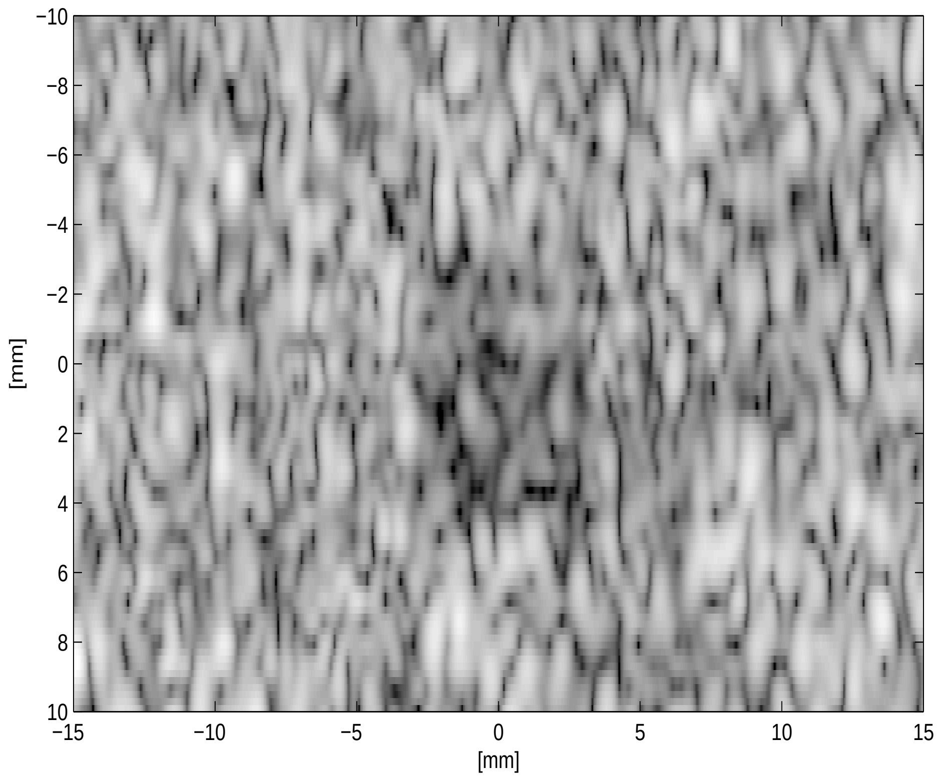




Ideal image

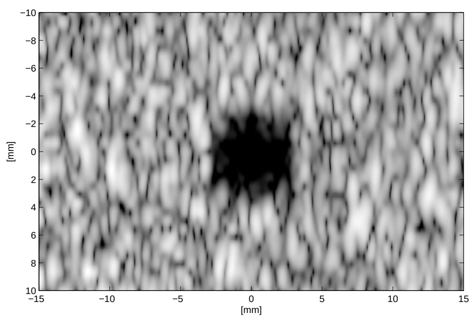
Motivation





Standard image

Ideal



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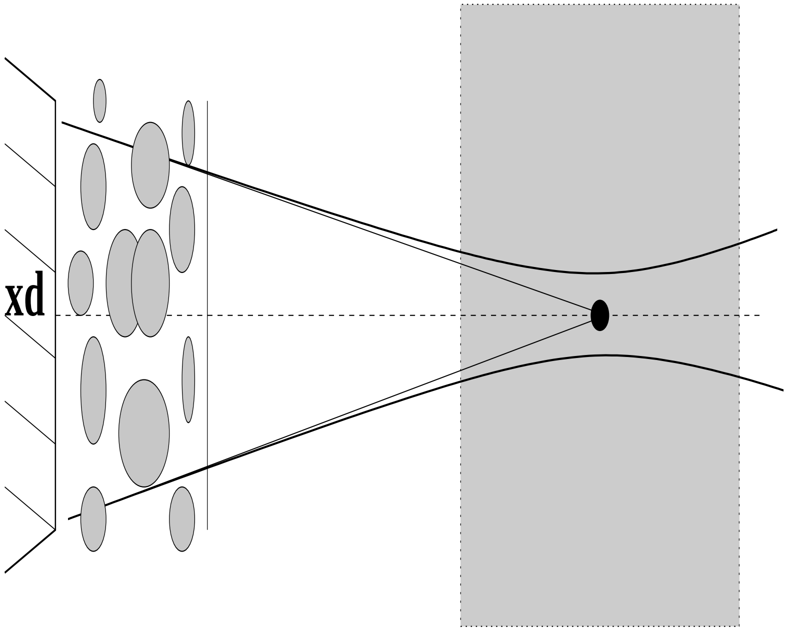
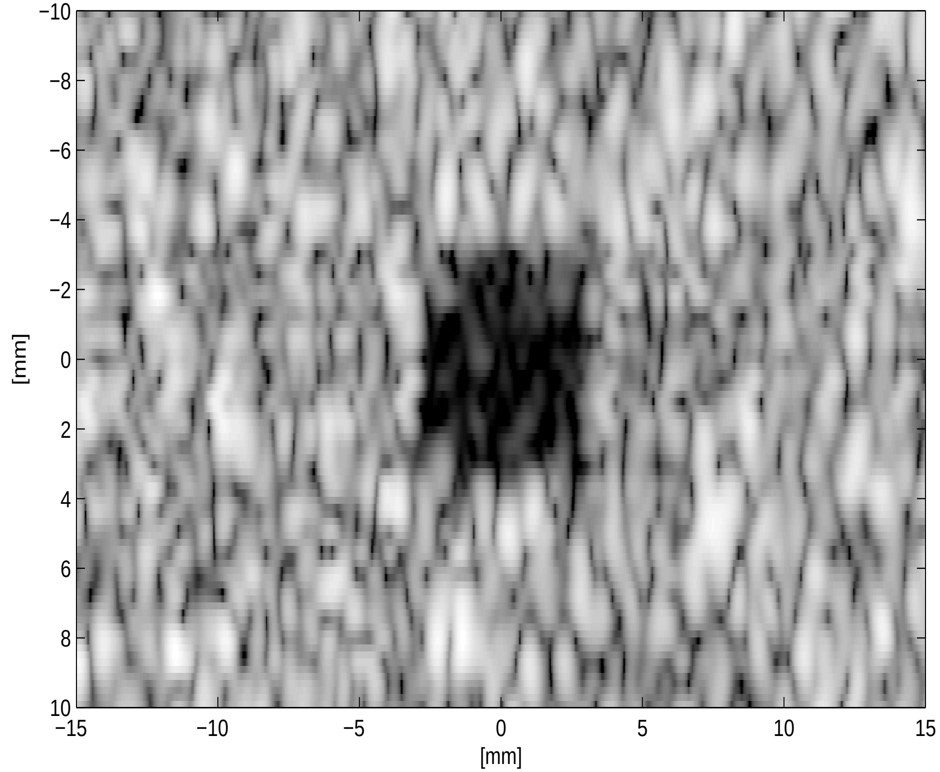
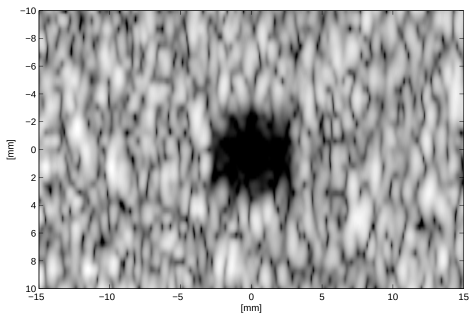


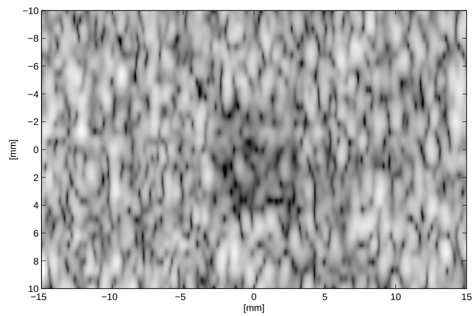


Corrected image

Ideal



Standard



Simplifications

- Scattering model: first-order

$$\hat{p}_1(r,\omega) = \int g(r-\xi,\omega)\Psi(\xi,\omega)\hat{p}_0(\xi,\omega)\mathrm{d}\xi.$$

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$$p_m(r,\omega) = s(r,\omega)p_1(r,\omega)$$

- Scatterer model: δ-correlated

$$\begin{aligned} R_{\hat{p}_1}(r_1,r_2) &= E\left[\hat{p}_1(r_1)\overline{\hat{p}_1(r_2)}\right] \\ &= \sigma^2 \int e^{i\frac{\omega}{c_0}\frac{(r_1-r_2)\cdot r}{|r_f|}} |\hat{p}_0(r)|^2 \mathrm{d}r \end{aligned}$$

Expected energy

Define stochastic linear functional

$$\mathcal{L}x = \int_T \hat{p}_m(r)\overline{x(r)}\mathrm{d}r$$

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"Expected energy" (variance):

$$E\left[|\mathcal{L}x|^2\right] = \int_T \overline{x(r_1)}x(r_2)R_{\hat{p}_m}(r_1,r_2)\mathrm{d}r_1\mathrm{d}r_2$$

Eigenfunction interpretation

Expected energy is

$$E\left[|\mathcal{L}x|^2\right] = \langle Ax, x\rangle$$

where

$$Ax(r) = \int_T x(\xi) R_{\hat{p}_m}(r,\xi)\mathrm{d}\xi.$$

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Maximum expected energy:

$$\lambda_1 = \int_T \int_T R_{\hat{\mathrm{p}}_m}(r,\xi) x_1(\xi)\mathrm{d}\xi \overline{x_1(r)}\mathrm{d}r$$

Focusing properties

In order to investigate the properties of the eigenfunctions, write

$$\lambda(x) = \langle Ax, x \rangle = \iint_{T\times T} R_{\hat{p}_m}(\xi_1, \xi_2) x(\xi_2) \overline{x(\xi_1)} \mathrm{d}\xi_2 \mathrm{d}\xi_1$$

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Use $R_{\hat{p}_m}$:

$$R_{\hat{p}_m}(r_1, r_2) = s(r_1)\overline{s(r_2)}\sigma^2 \int_T e^{i\frac{\omega}{c_0}\frac{(r_1-r_2)\cdot r}{|r_f|}}|\hat{p}_0(r)|^2\mathrm{d}r$$

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Define $\alpha(x, r)$:

$$\alpha(x, r) = \int_T s(\xi) \overline{x(\xi)} e^{i \frac{\omega}{c} \frac{\xi \cdot r}{|r_f|}} \mathrm{d}\xi,$$

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Then

$$\lambda(x) = \sigma^2 \int |\hat{p}_0(r)|^2 |\alpha(x, r)|^2 \mathrm{d}r.$$

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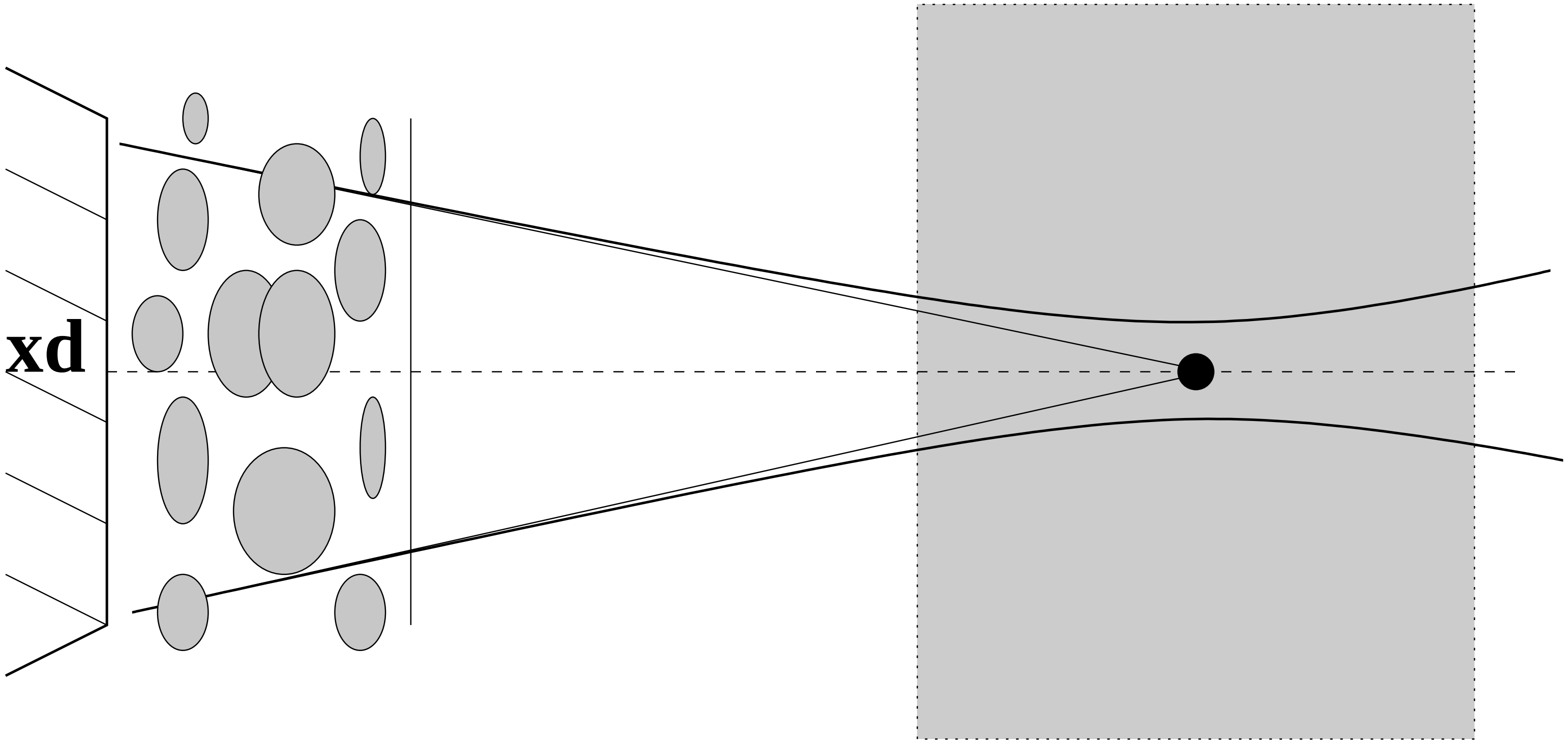
Define $\alpha(x, r)$:

$$\alpha(x, r) = \int_T s(\xi) \overline{x(\xi)} e^{i\frac{\omega}{c}\frac{\xi\cdot r}{|r_f|}} \mathrm{d}\xi,$$

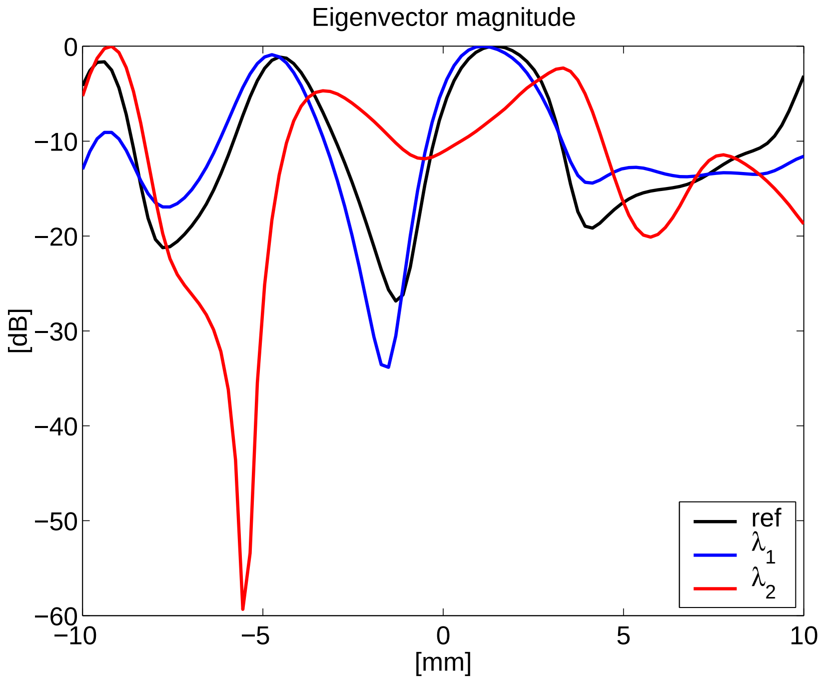
Then

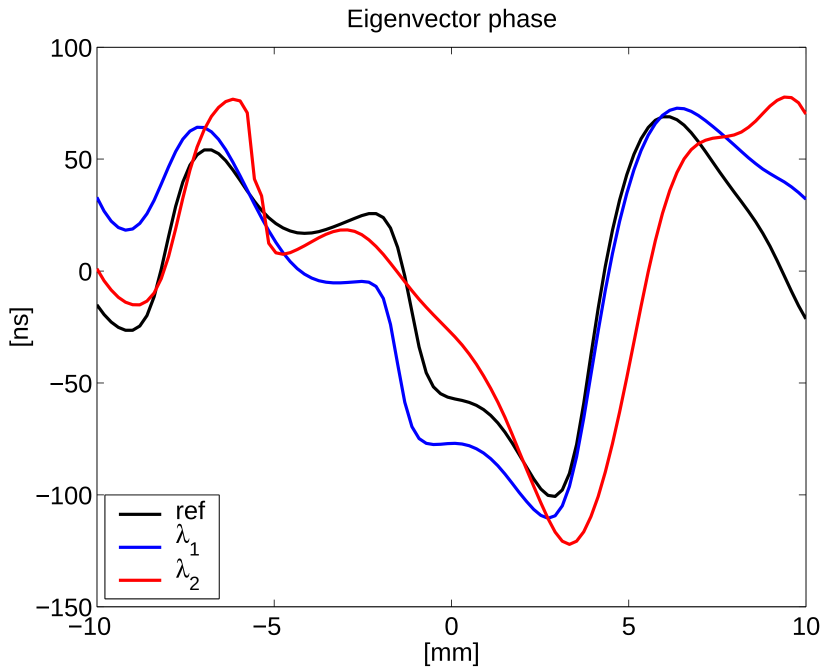
$$\lambda(x) \sim \sigma^2 \int |\hat{p}_0(r)|^2 |\hat{p}_{\mathrm{cor}}(r)|^2 \mathrm{d}r.$$

Simulated scattering



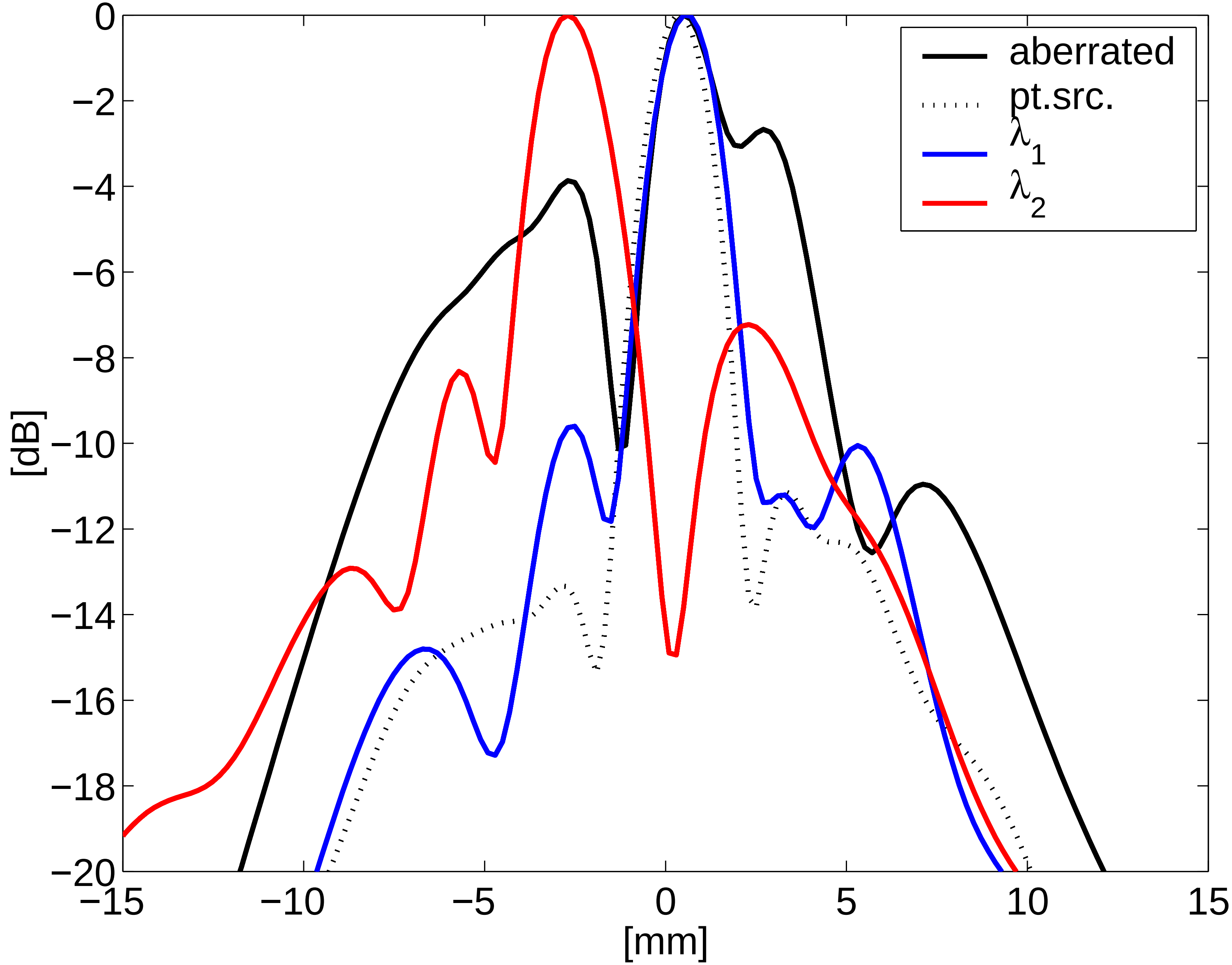
Aberration characterisation



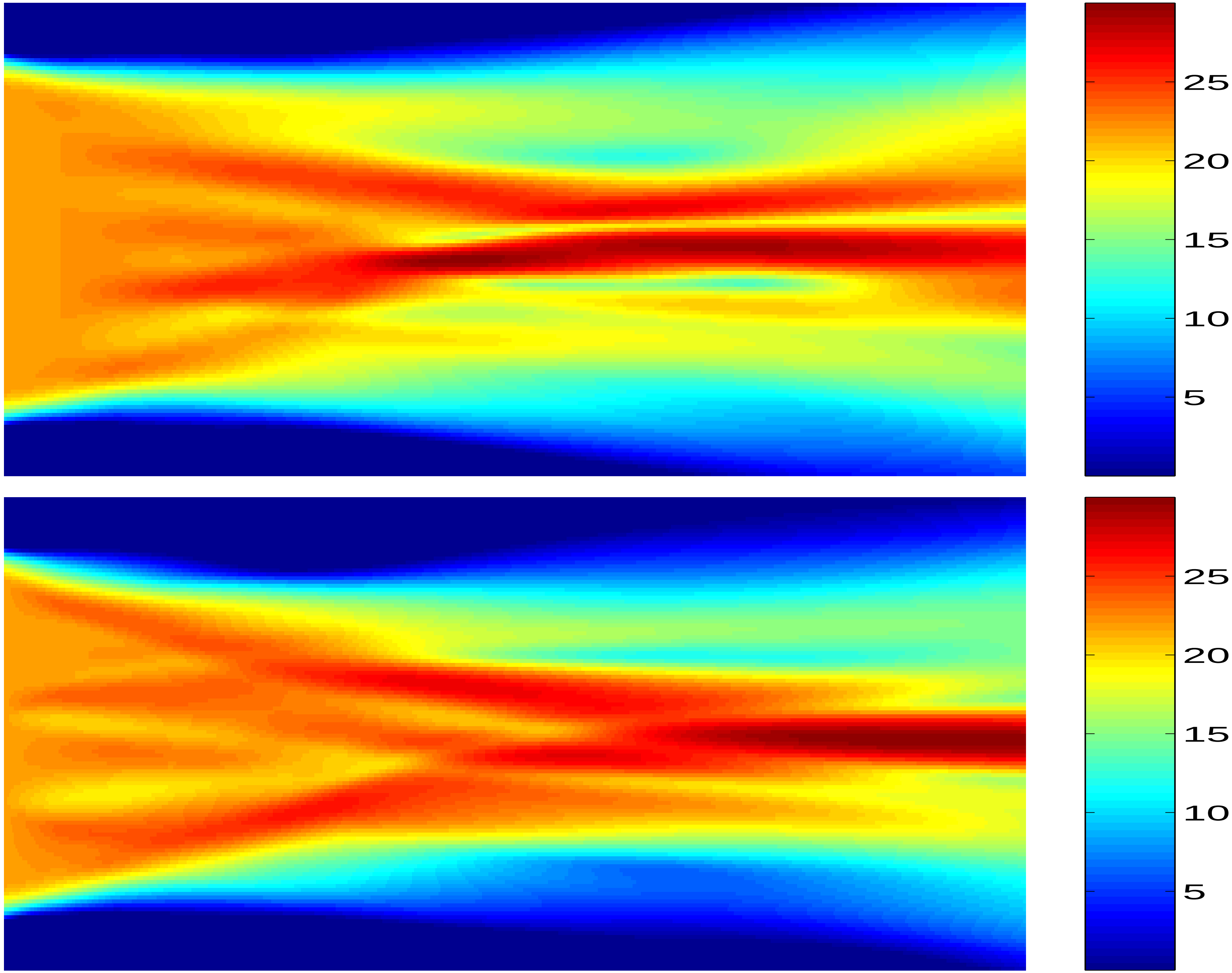


Estimated screen amplitude and arrival-time delay.

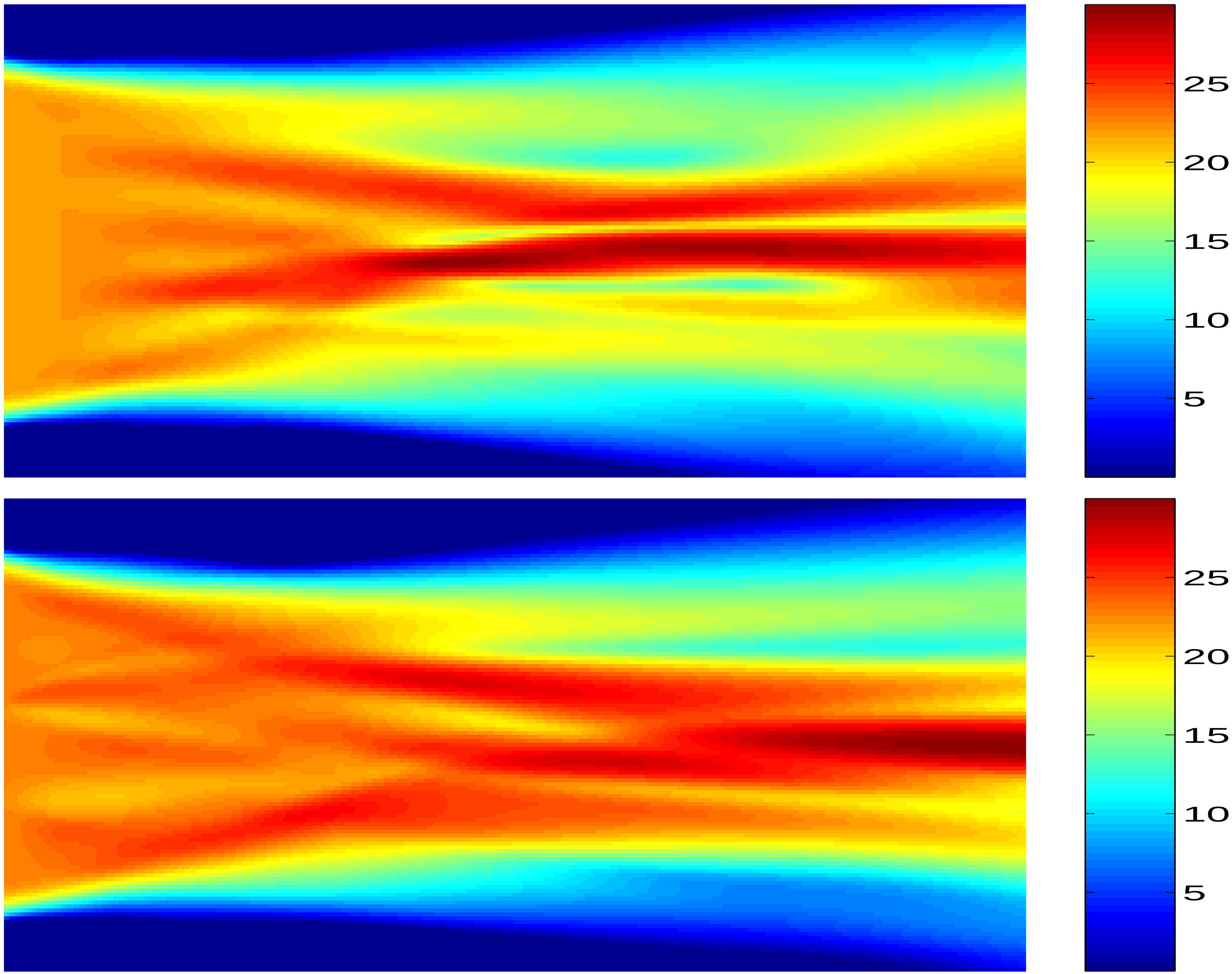
Corrected beam profiles



Reference correction



λ_1 correction



Summary

- Maximum expected energy considerations result in an eigenfunction formulation of aberration estimation/correction.
- Eigenfunctions focus the beam in the direction of high initial transmitted energy, and are hence dependent on the aberration.
- Considerable correction is obtained using eigenfunctions associated with high eigenvalues.

Acknowledgements

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