Fast 3D simulation of 2nd Harmonic Ultrasound Field from Arbitrary Transducer Geometries

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***Abstract*— A method for fast numerical simulation of nonlinear wave propagation based on a quasi-linear approximation has previously been presented. In the current study this method has been further developed to yield correct levels of the second-harmonic wave. The method can be used for 3D simulations of second-harmonic fields from arbitrarily transducer geometries. The method has been validated by comparing simulations to results produced by a conventional nonlinear simulation model and to experimental measurements. The reference simulation model was a numerical solution of the KZK equation for a forward-propagating pulse using a operator splitting approach. Experimental verifications were performed with hydrophone measurements in a water tank. Results showed a good match between the simulation models and measurements for MI up to 0.4 for an annular array probe and for MI up to 1 for a rectangular probe. For higher MI values the quasi-linear method showed a gradual increased over-estimation of the second harmonic field.**

I. Introduction

Major improvements have been achieved in medical ultrasound imaging by utilizing the second harmonic field due to non-linear wave propagation. To optimize image quality detailed knowledge of the second harmonic field, as a function of the three dimensional transducer geometry and the transmitted pulse, is required.

Numerical simulations of the nonlinear wave equation in three dimensions are time consuming. Several methods have been presented, see e.g. [1]–[5]. For circular symmetrical fields the symmetry can be exploited to reduce the complexity to a two dimensional, but still quite time consuming, problem [4]. The latter method is also limited to annular probes.

A method based on the quasi-linear approximation has previously been presented [6]. It is a method where the second harmonic field is calculated using the first harmonic field as a source. The method is proven to be a fast way to calculate the second harmonic beam profiles accurately when the pulse amplitude is relatively low [7].

This study is a further verification of the quasi-linear approximation. Shapes and levels of the fields are studied as well as pulses, specters, and beam profiles since all aspects of the field may contribute to the image quality. The validation is done against hydrophone measurements and a reference simulation model. Hydrophone measurements of the near field are propagated to the focal plane using the two simulation methods and the results are compared to measurements from the focal plane. The reference simulation model is a solution of the KZK equation using an operator splitting approach [4].

II. Theory

The second harmonic field can be approximated as a function only of the linear field, sometimes referred to as a quasi-linear approximation. In [6] an expression for the second harmonic field in k-space was derived.

$$P_2(\boldsymbol{f}|z_2) = C \int P_1(\boldsymbol{f}-\boldsymbol{f}'|z_1) P_1(\boldsymbol{f}'|z_1) H_p(\boldsymbol{f},\boldsymbol{f}')\,\mathrm{d}^3\boldsymbol{f}' \tag{1}$$

where

$$C = \frac{\beta_n \kappa 2\pi^2 f^2}{iK(\boldsymbol{f})c^4}, \tag{2}$$

$$H_p(\boldsymbol{f},\boldsymbol{f}') = z_2 e^{-iK(\boldsymbol{f})(z_2-z_1)} e^{i\Lambda(\boldsymbol{f},\boldsymbol{f}')(z_1-\frac{z_2}{2})} \operatorname{sinc}(\Lambda(\boldsymbol{f},\boldsymbol{f}')\frac{z_2}{2\pi}), \tag{3}$$

$$\Lambda(\boldsymbol{f},\boldsymbol{f}') = -K(\boldsymbol{f}) + K(\boldsymbol{f}') + K(\boldsymbol{f}-\boldsymbol{f}'), \tag{4}$$

$$K(\boldsymbol{f}) = \frac{2\pi}{c}\sqrt{f^2 - f_x^2 - f_y^2}, \tag{5}$$

and

$$\boldsymbol{f} = (f_x, f_y, f). \tag{6}$$

In these equations;

- $P_1(\boldsymbol{f}|z_1)$ is the fundamental field in depth z_1 and $P_2(\boldsymbol{f}|z_2)$ is the second harmonic field in depth z_2. $P_1(\boldsymbol{f}|z_1)$ and $P_2(\boldsymbol{f}|z_2)$ are Fourier transformed in all dimensions except the range, z. The depths z_1 and z_2 can be arbitrary chosen.
- C is a constant to ensure correct level of the second harmonic field relative to the fundamental field,
- β_n is the nonlinear parameter,
- κ is the compressibility,

- f is the frequency,
- c is the speed of sound in the medium,
- and f_x and f_y are spatial frequencies scaled by c to units Hz.

III. Method

The presented results are obtained using MATLAB implementations of the quasi-linear approximation and the KZK equation. Both implementations can be run on a standard work station.

Two sets of experimental measurements were used in this study, one from an annular array probe (Vingmed Sound Apat 3.25) and one from an 1.5D rectangular phased array probe (GE Vingmed M3S). Dimensions and settings are listed in Tables I and II. The transmit fields were recorded in a water tank using a 0.4 mm needle hydrophone (SEA PVDF-Z44-0400 and Force Instruments MH28-04, respectively).

The circular symmetrical field from the annular array probe was completely described by a line scan from the center of the field perpendicular to the focal axis. The line scans of the acoustic field was recorded in the near field at depth 8,5 mm. To describe the field from the phased array probe one quadrant of the field perpendicular to the focal axis is necessary. This was recorded in the near field at depth 1 mm. All fields were recorded using six different transmit levels—with mechanical index (MI) ranging from 0.1 to 1.4 for the annular array probe and from 0.2 to 1.6 for the phased array probe.

In order to ensure correct symmetry for the simulations, the measured fields were slightly modified; the fields were adjusted to make sure the center of the measurements matched the center of the field, and a tilt in the measurements due to the scan plane not being exactly perpendicular to the focal axis was removed. To avoid discontinuitis at the measurement boundary a window function was applied in space and time as well as in the three frequency dimensions.

In the quasi-linear method a band of 2.5MHz around the fundamental frequency was used as the initial field. It was thus of great importance that the near field measurements were recorded as close to the probe as possible to make sure not too much second harmonic signal would be lost. The line scan from the annular array was used to generate an axisymmetric field, and the quadrant of the field from the rectangular probe was mirrored to generate the entire field. The near-field depth was used as the depth z_1 in Eq. (2) and (4). In the KZK method the full nonlinear field was used. These fields were then propagated to the focal plane using the two simulation methods through a medium with values of β_n and κ for distilled water [8].

In the focal plane line scans were recorded in Azimuth and Elevation. These measurements were compared to the corresponding scans from the simulations.

To evaluate the simulation methods pulse shapes and specters were compared. Beam profiles were found as the RMS of the pressure pulse as a function of time. These profiles and temporal and spatial peak pressures from the focal plane were compared for the fundamental and the second harmonic frequency separately.

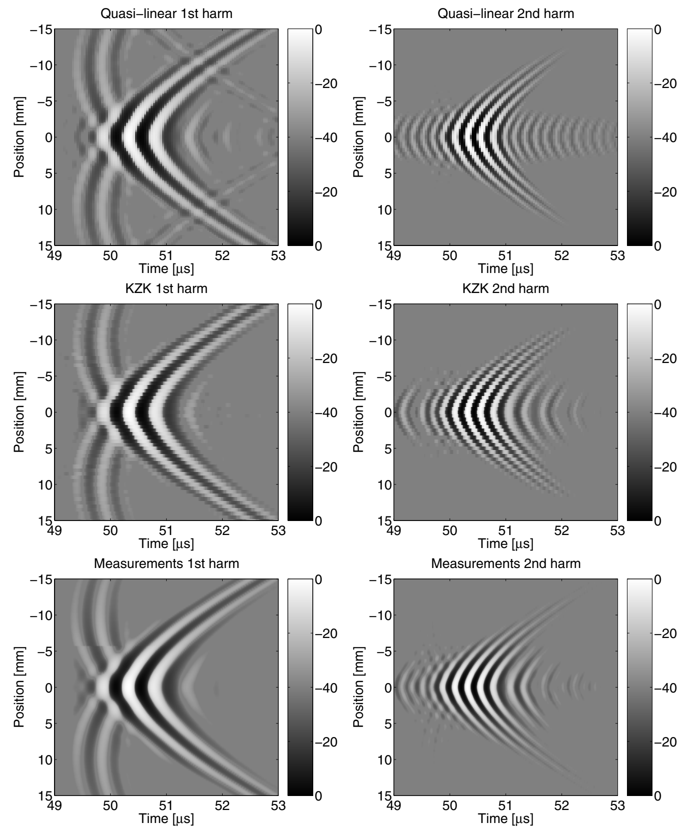

Fig. 1. Azimuth scans of the first and second harmonic fields propagated using the quasi-linear approximation, fields propagated using the KZK method, and measured fields with MI = 1. The gray scale shows the levels in dB where positive values correspond to the light end of the specter and negative values correspond to the dark end of the specter. Top panels: the quasi-linear approximation, middle panels: the KZK equation, lower panels: measurements.

TABLE I

Dimensions and settings for the annular probe.

Diameter:	14.7 mm
Radius of curvature (ROC):	78 mm
Transmit frequency:	2.9 MHz

TABLE II

Dimensions and settings for the rectangular probe.

Azimuth:	22 mm
Elevation:	13 mm
Focal depth:	70 mm
Transmit frequency:	2.2 MHz

IV. Results and discussion

Azimuth scans of first and second harmonic measured fields with MI $= 1$ from the rectangular probe and the corresponding fields propagated using the two simulation tools are shown in Fig. 1. These results show that both simulation models give detailed fields that match the measurements. Weak reflections

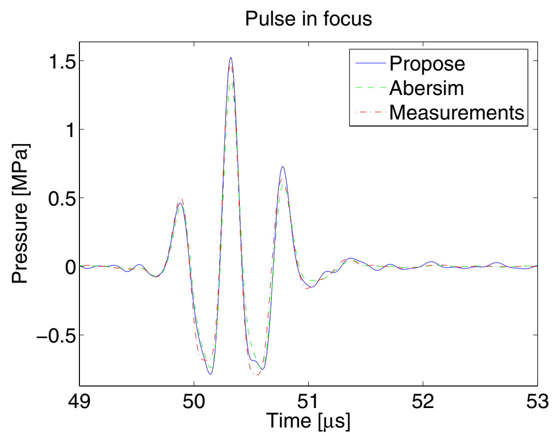

Fig. 2. Pulses in focus from the quasi-linear method, the KZK equation, and measuremets from the rectangular probe, MI = 1.

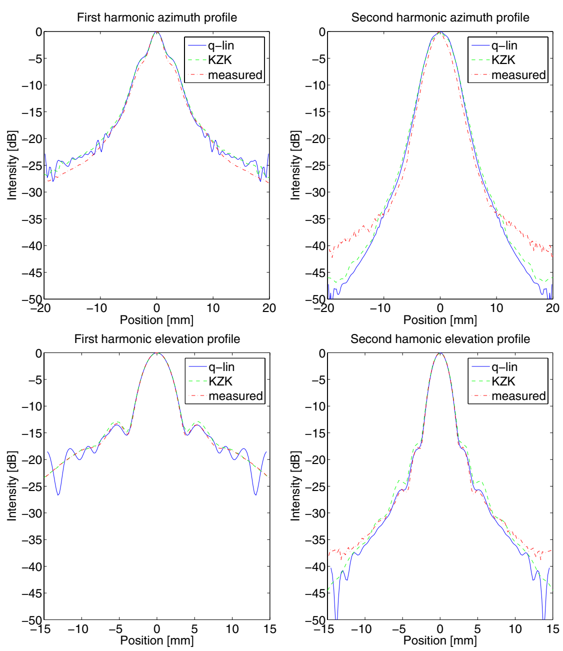

Fig. 3. First (left panels) and second (right panels) harmonic azimuth and elevation beam profiles from the two simulation methods and measurements in the focal plane from the rectangular probe with MI = 1. Upper panels: azimuth profiles, lower panels: elevation profiles.

can be seen on the linear field from the quasi-linear method. They are due to the periodicity of the FFT. They can be reduced by expanding the computation window.

Fig. 2 shows the pulses in focus from the same data set. The pulses are filtered in a band around the first and second harmonic. They show good agreement with both the phase angle of the second harmonic relatively to the first harmonic and the levels. The second harmonic from the quasi-linear is slightly overestimated. This can be seen as sharper positive peaks and some deviation in the negative peaks.

Normalized beam profiles of the first and second harmonic field in azimuth and elevation from the rectangular probe are

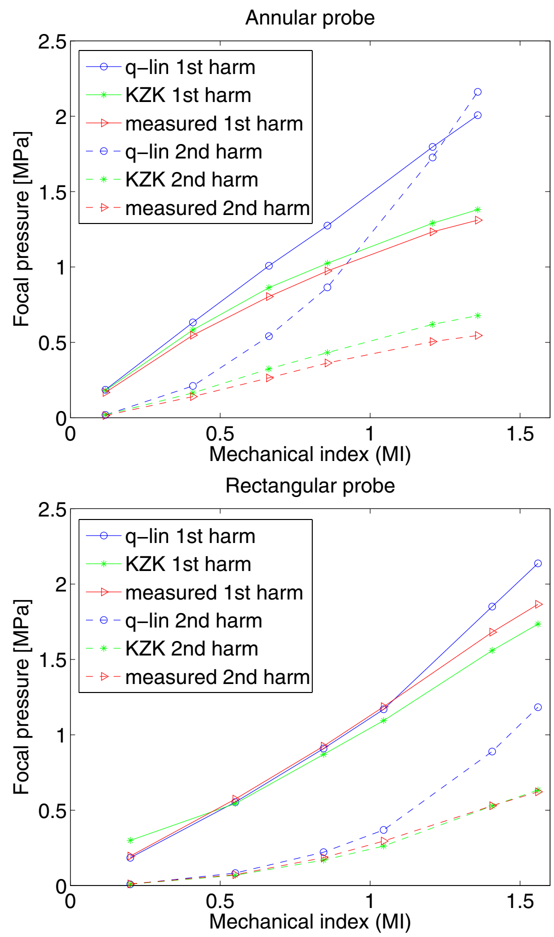

Fig. 4. Levels of the filtered first and second harmonic in the focal plane for the annular array probe (upper panel) and the phased array probe (lower panel).

shown in Fig. 3. In azimuth the profiles are almost identical from the two simulation methods, while the measurements show slightly lower side lobe levels for the first harmonic and a narrower main lobe for the second harmonic. In elevation the KZK method show slightly higher side lobe levels. On the second harmonic profiles noise can be seen in the far side lobes of the measurements.

The levels of the spatial and temporal peaks of the field in the focal plane from measurements and propagated using the two simulation tools are shown in Fig. 4. The upper panel shows the results from the annular probe and the results from the rectangular probe is shown in the lower panel. The first harmonic field is used as a source to generate the second harmonic field in the quasi-linear method. As a result the reduced energy level in the fundamental band due to the generation of higher harmonics is not modeled. Similarly the reduction of the second harmonic due to spectral leakage is not modeled. More importantly the second harmonic will be over-estimated because it is calculated as a function of the already over-estimated first harmonic. The second harmonic level is over-estimated by less than 0.1 MPa for MI up to 0.4 for the annular array probe and for MI up to 1 for the phased array probe. The linear field from the rectangular probe calculated using the KZK approach is slightly under-estimated.

Validation by using near field measurements introduces some error sources which can be the reasons for some of the observed phenomena. The measurements contains errors, and the sampling by the needle hydrophone, which is larger than a point, will introduce some error. The measurements of different axis of the field from the annular array probe showed that these did not match exactly, but one axis was chosen, and a different axis would yield slightly different results. Similarly, for the rectangular probe, choosing a different quadrant would yield slightly different results. Another error source is the choice of β_n and κ for distilled water while the water used might contain impurities. This method was, however, chosen because the problem of modeling the complex coupling between the transducer surface and the water was avoided.

V. Conclusion

Both the quasi-linear simulation method and the KZK-based simulation method are in good agreement with measurements. Second harmonic levels from the quasilinear method are over-estimated by less than 0.1MPa for MI up to 0.4 for the annular array probe, and for MI up to 1 for the rectangular phased array probe. However, these experiments were performed in water where attenuation of harmonic frequency components is less than 0.1% of that of tissue. Thus the results represent a worst-case scenario for the quasi-linear approximation.

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