Aberration in Nonlinear Acoustic Wave Propagation

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Abstract—Theory and simulations are presented indicating that imaging at the second-harmonic frequency does not solve the problem of ultrasonic wave aberration. The nonlinearity of acoustic wave propagation in biological tissue is routinely exploited in medical imaging because the improved contrast resolution leads to better image quality in many applications. The major sources of acoustic noise in ultrasound images are aberration and multiple reflections between the transducer and tissue structures (reverberations), both of which are the result of spatial variations in the acoustic properties of the tissue. These variations mainly occur close to the body surface, i.e., the body wall. As a result, the nonlinearly generated, second harmonic is believed to alleviate both reverberation and aberration because it is assumed that the second harmonic is mainly generated after the body wall. However, in the case of aberration, the second harmonic is generated by an aberrated source. Thus the second harmonic experiences considerable aberration at all depths, originating from this source. The results in this paper show that the second harmonic experiences similar aberration as its generating source, the first harmonic.

I. INTRODUCTION

In this paper, theory and simulations of three-dimensional (3-D) forward nonlinear propagation of ultrasonic fields with absorption in heterogeneous media are presented. The goal is to clarify the issue of second-harmonic imaging and ultrasonic wave aberration.

Harmonic imaging (HI), also known as tissue harmonic imaging or second-harmonic imaging, is today a standard choice of imaging modality in many medical ultrasound applications. In HI, a pulse is transmitted and received at different frequencies. In this paper, these pulses are denoted the first harmonic and second harmonic, respectively. Fundamental imaging is defined here as transmitting and receiving at the same frequency.

Christopher [1], [2] performed a comprehensive study of second-harmonic generation in homogeneous and heterogeneous tissue. He showed that the lateral resolution limits of the second-harmonic beam profile were slightly narrower than the same-frequency fundamental for a heterogeneous medium in all the investigated cases. He furthermore showed that the sidelobe level was substantially lower. This result is confirmed by Shen and Li [3].

In medical applications, reported improvements from the use of HI are typically improved contrast resolution, lateral resolution, and reduced acoustic noise [4]–[8].

However, there is still some debate as to why HI yields better images [9]. The amplitude for any reflected signal is much lower than that of the forward-propagating transmit field. As the second harmonic is generated at a rate proportional to the square of the acoustic pressure, the amount of second harmonic generated by reflected signals is much lower than that generated by the nonlinear, forward-propagating transmit field; reverberation is substantially reduced in the second harmonic. Other effects such as suppression of grating lobes, lower side lobes, and reduced aberration due to the second harmonic being generated behind the aberrating body wall, are also mentioned [4]–[9].

Fedewa et al. [10], [11] showed that, for low-transmit amplitudes, the introduction of an effective apodization led to reproduction of the spatial coherence for HI using only fundamental fields. Wallace et al. [12] performed 1-D experimental measurements using porcine abdominal aberrators, but were not able to reproduce an effective apodization for heterogeneous tissue. However, they show that the second-harmonic transmit field is aberrated, but somewhat less severely than the first harmonic.

Varslot et al. [13] showed that the aberration of the second harmonic is strongly related to the aberration of the first harmonic; both possess focal beam profiles of similar shape. This led to the conclusion that aberration affects the second harmonic in the same way as the first harmonic.

This paper is an extension of the work presented in [13], and it includes a more detailed theoretical description and more simulation results to substantiate the conclusions drawn. This paper is organized as follows. Section II describes the theoretical foundations for understanding aberration of the second-harmonic component. A nonlinear wave equation describing forward propagation with absorption is developed and separated into two equations describing linear and nonlinear propagation in a heterogeneous medium, respectively. In Section III, a description of the 3-D simulations performed is given, and Section IV presents the results of these simulations. In Section V, the results are discussed, and conclusions are drawn in Section VI.
II. Theory

Many nonlinear wave equations are available that aim to model acoustic wave propagation. In the case of soft biological tissue, in which shear waves are negligible, the generalized Westervelt equation provides a good model [14], [15]. Assuming that the length scale for spatial variations of mass density is large compared to the wavelength of the ultrasound pulse, the Westervelt equation may be expressed in Lagrangian coordinates as:

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{1}{c^2} h \frac{\partial^2 p}{\partial t^2} = -\epsilon_n \frac{\partial^2 p^2}{\partial t^2},$$

(1)

where $p$ is the acoustic pressure, $c$ is the speed of sound, $\epsilon_n$ is a nonlinearity parameter, and $h$ is the kernel of a temporal convolution operator that accounts for absorption. It is such an equation that is solved in the 3-D forward simulations [16], [17].

For the purpose of analysis, solutions to (1) are further explored here. In the temporal frequency domain, this wave equation is written as:

$$\nabla^2 \hat{p} + \frac{\omega^2}{c^2} [1 + H(r, \omega)] \hat{p} = \epsilon_n(r) \omega^2 \hat{p} \ast \hat{p}.$$  

(2)

In the latter equation, * indicates convolution with respect to frequency, and $H(r, \omega)$ is the frequency-domain representation of the absorption operator. Functional dependence on frequency and spatial coordinates has been written out explicitly in this equation, but it mostly will be omitted in the rest of the paper.

It is constructive to define a linear part and a nonlinear part of the acoustic pressure, denoted $p_l$ and $p_{nl}$, respectively. They are defined to satisfy the following equations:

$$p = p_l + p_{nl},$$

(3)

$$\nabla^2 \hat{p}_l + \frac{\omega^2}{c^2} [1 + H] \hat{p}_l = 0,$$  

(4)

$$\nabla^2 \hat{p}_{nl} + \frac{\omega^2}{c^2} [1 + H] \hat{p}_{nl} = \epsilon_n \omega^2 \hat{p} \ast \hat{p}. $$

(5)

Aberration of the linear part has been studied by many authors. A list of some relevant references is found in M˚asøy et al. [18].

In the following, the Cartesian coordinates of a point $(x, y, z)$ will be denoted $(\xi, z)$, where $\xi = (x, y)$ is considered to be a vector in $\mathbb{R}^2$. Furthermore, $d\xi$ will denote the 2-D surface measure, i.e., $d\xi = dx\, dy$. In this notation, the pressure at a point $(\xi, z)$ with $z > 0$ emanating from an array surface $S$ is computed by means of a modified Rayleigh-Sommerfeld diffraction integral.

Assuming the array surface is embedded in the plane $z = 0$ and an array baffle directivity function $b$, the forward radiated field from the array may be expressed as [19]:

$$\hat{p}_l(\xi, z) = i\omega \int_S 2g(\xi, z, \xi_0, 0) b(\xi - \xi_0, z) \rho U_n(\xi_0) d\xi_0,$$

$$= \int_S g(\xi, z, \xi_0, 0) f(\xi_0) d\xi_0,$$  

(6)

where:

$$f(\xi) = i\omega 2\rho U_n(\xi),$$

(7)

here $g$ is the Green’s function for a heterogeneous medium, satisfying (4). The baffle directivity function and absorption are included in the Green’s function by choosing a Green’s function that satisfies the appropriate boundary conditions [19]. The variable $\rho$ represents the mass density of the medium, and $U_n$ is the normal velocity of the array.

It has been shown that a good description of the aberration is supplied by the generalized frequency-dependent screen, or its approximation by a time-shift and amplitude screen [20]:

$$\hat{p}_n(\xi, z) = \int_S g_n(\xi - \xi_0, z) s(\xi, z, \xi_0, 0) f(\xi_0) d\xi_0, $$

(8)

Thus, in this case the integration domain is not only the array surface, but also the full extent of the acoustic beam. Furthermore, as $\epsilon_n$ is generally spatially variable, it is kept inside the integral.

If $|\hat{p}_{nl}| \ll |\hat{p}_l|$, then the quasilinear approximation is reasonable giving:

$$\hat{p}_{nl}(\xi, z) = \omega^2 \int_0^\infty \int_{\mathbb{R}^2} g(\xi, z, \xi_0, 0) \epsilon_n \hat{p}_l \ast \hat{p}(\xi_0, 0) d\xi_0 d\xi_0. $$

(9)

If $\hat{p}_l$ is a narrow-band pulse centered around the frequency $\omega_0$, then $\hat{p}_{nl}$ will be a narrow-band pulse centered around $2\omega_0$. The aberrated linear part, therefore, represents a source for the nonlinear part.

For a pure-tone transmitted signal with frequency $\omega_0$:

$$\omega^2 \hat{p}_l \ast \hat{p}_n(\xi, z) = \frac{\omega^2}{4} \delta(|\omega| - 2\omega_0) \Lambda(\xi, z, \omega_0), $$

(11)

where:

$$\Lambda(\xi, z, \omega) = \hat{p}_l^2(\xi, z, \omega) = \left( \int_S g(\xi, z, \xi_0, 0, \omega) f(\xi_0, \omega) d\xi_0 \right)^2. $$

(12)

Inserting (11) into (10), and using the generalized frequency-dependent screen, the nonlinear part may be expressed as:

$$\hat{p}_{nl}(\xi, z, 2\omega_0) = \omega_0^2 \int_0^\infty \int_{\mathbb{R}^2} g_n(\xi - \xi_0, z - z_0, 2\omega_0) \times s(\xi, z, \xi_0, 0, \omega_0) \epsilon_n \Lambda(\xi_0, z_0, \omega_0) d\xi_0 d\xi_0, $$

(13)
where:

\[ \Lambda(\xi, z, \omega_0) = \left( \int_S g_h(\xi - \xi_0, z - z_0, \omega_0) s(\xi, z, \xi_0, 0, \omega_0 f(\xi_0, \omega_0) d\xi_0 \right)^2. \] (14)

Eq. (14) shows that the source for the nonlinear pressure is aberrated according to \( \omega_0 \). The nonlinear part is aberrated further as it propagates through the tissue with angular frequency 2\( \omega_0 \).

Assume that the human body wall is of thickness \( d \) (see Fig. 1), and that the aberration occurring outside the body wall is negligible in comparison to the aberration by the body wall, i.e., \( s = 1 \) for \( z > d \). Then (13) may be separated into the sum of two integrals on the form:

\[
\hat{p}_{nl}(\xi, z, 2\omega_0) = \omega_0^2 \left[ \int_0^d \int_{\mathbb{R}^2} g_h(\xi - \xi_0, z - z_0, 2\omega_0) \times s(\xi, z, \xi_0, z_0, 2\omega_0) \epsilon_n \Lambda(\xi_0, z_0, \omega_0) d\xi_0 d\xi_0 \right. \\
\left. + \int_d^\infty \int_{\mathbb{R}^2} g_h(\xi - \xi_0, z - z_0, 2\omega_0) \epsilon_n \Lambda(\xi_0, z_0, \omega_0) d\xi_0 d\xi_0 \right].
\] (15)

Now, the first integral describes heterogeneous propagation of the nonlinear field at 2\( \omega_0 \) with aberration at the same frequency, and the source is aberrated at \( \omega_0 \). The second integral describes homogeneous propagation of the nonlinear field at 2\( \omega_0 \), of the source aberrated at \( \omega_0 \). This relation may be used to interpret the results presented in Section IV.

In the special case in which \( d = 0 \) (i.e., no aberration) (14) is simplified to:

\[ \Lambda(\xi, z, \omega_0) = \left( \mathcal{F}_\xi^{-1} \left\{ \hat{g}_h(k, \omega_0) f(k, \omega_0) \right\} \right)^2, \] (16)

where \( \mathcal{F}_\xi \) is the Fourier transform with respect to \( \xi \), and \( k \) is the corresponding wave number. This may be used to explicitly compute the integral:

\[
I(\xi, \xi_0, z, 2\omega_0) = \int_0^z g_h(\xi - \xi_0, z - z_0, 2\omega_0) \Lambda(\xi_0, z_0, \omega_0) d\xi_0 \\
= \mathcal{F}_z^{-1} \left\{ \hat{g}_h(\xi, k_z, 2\omega_0) \Lambda(\xi, k_z, \omega_0) \right\}, \quad (17)
\]

where the functions have been appropriately extended by zero for \( z < 0 \) before Fourier transformation. It is now seen that:

\[ \hat{p}_{nl}(\xi, z, 2\omega_0) = \omega_0^2 \epsilon_n \int_{\mathbb{R}^2} I(\xi, \xi_0, z, 2\omega_0) d\xi_0. \] (18)

This approach was used in order to perform fast numerical simulations of ultrasound fields [21].

In a region in which \( I(\xi, \xi_0, z, \omega)/g_h(\xi - \xi_0, z, \omega) \) is relatively constant with respect to \( \xi \) and \( z \) [i.e., well approximated by a function \( f_s(\xi_0, \omega) \)], this corresponds to an equivalent radiating source at the array surface. This source will incorporate an apodization, which in this case is equivalent to the effective apodization defined by Fedewa et al. [10], [11].

A similar computation also may be performed in the case of aberration, and an equivalent radiating source with an associated equivalent aberration screen may be defined. However, in order to compute this screen, knowledge about the aberration between all pairs of points \( (\xi_0, z_0) \) and \( (\xi, z) \) between the array and the plane \( z \) is required. The practical use of such a computation is limited as the aberration can be investigated only on the surface.

III. Method

A 3-D numerical experiment was conducted using a simulation setup capable of capturing nonlinear, forward propagation and absorption in heterogeneous tissue [16]. The simulation setup is presented in Fig. 1. In order to investigate the effect of aberration on the second-harmonic component, an ultrasound field was generated at the array surface and propagated through a body wall model and beyond the focal point of the array.

In these forward simulations, an ultrasound transmit pulse of 1.75 MHz, using an aperture of 20\( \times \)20 mm focused at \( F = 60 \) mm, was propagated through a body wall model of thickness \( d = 20 \) mm, with aberration characteristics comparable to published measurements of the human abdominal wall. To understand how different transmit pressures affect the aberration of the second-harmonic component, a series of transmit pressures \( p_0 = \{0.3, 0.6, 1.0\} \) MPa was investigated. As a comparison, the fundamental—a linear simulation performed with a 3.5 MHz transmit pulse—also was propagated through the body wall model.

The body wall model was generated using a set of 2-D, time-delay screens, filtered and tuned to obtain characteristics according to abdominal wall measurements [22]. In this instance, eight time-delay screens were used, equally spaced \( (\Delta z = 2.5 \) mm) over the thickness of the body wall model (see Fig. 1). A more detailed description of the construction is found in Måsøy et al. [23].
TABLE I
PARAMETERS CHARACTERIZING THE PROPAGATION MEDIUM AND
THE ABERRATING BODY WALL MODEL.

<table>
<thead>
<tr>
<th>Quantity</th>
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<td>Time-delay rms fluctuation</td>
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<td>Amplitude rms fluctuation</td>
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<td>Attenuation</td>
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<tr>
<td>Nonlinearity (B/A)</td>
<td>—</td>
<td>5.8</td>
</tr>
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</table>

A. Simulation Parameters and Data Processing

The simulations were implemented in Matlab using the Abersim simulation package developed at the Department of Circulation and Imaging, Norwegian University of Science and Technology (NTNU) in Trondheim, Norway. Abersim allows full 3-D, nonlinear simulations, including absorption of forward propagating ultrasound fields [17].

The simulation domain was 5.12 cm in the x- and y-direction (see Fig. 1) with a resolution of 0.2 mm. The sampling frequency was 70 MHz, providing a time window of 7.3 μs. All transmit pulses from the array had a range resolution of 2.5 periods, and a −6 dB relative bandwidth of 50%. The spatial grid does not resolve individual transmit elements of the array. Hence, grating lobes due to finite-element size are not observed in the simulations. The medium through which the signals were propagated had acoustic properties equal to that of muscle at 37°C [24] (see also Table I).

First-harmonic and second-harmonic components of the acoustic field were obtained by filtering the signals with a 50% and 40% −6 dB relative bandwidth bandpass filter of about 1.75 MHz and 3.5 MHz, respectively. The reason for the 40% relative bandwidth of the second-harmonic filter was to exclude as much first-harmonic component as possible. In the narrow-band, low-amplitude situation, this is equivalent to separating out the linear part and the nonlinear part of the pressure field as defined in (3)–(5).

One-dimensional and 2-D beam profiles in the focal plane of the array, and along the xz- and yz-axes, were acquired as the rms value of the temporal signal at each spatial position. These profiles were used for the visual evaluation of the effect of the body wall model.

In order to determine where the nonlinear part of the forward propagating energy was generated, the following quantity was used:

$$E(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |p(\xi, z, t)|^2 dt d\xi.$$  \hspace{1cm} (19)

This is proportional to the total acoustic energy of the field in a plane at depth z, and, therefore, is denoted the total acoustic energy at depth z in this paper. The slope of this function is a measure of the rate at which energy is generated for each depth.

IV. Results

Fig. 2 shows the beam profiles for the first-harmonic, second-harmonic, and fundamental in the focal plane for y = 0 and x = 0. In order to compare the shape of beam profiles at different frequencies, the axes should be scaled to account for the different opening angle of the beams. In this case, the axes are scaled by a factor of 2 for the 1.75 MHz profiles to account for the different opening angle at the higher frequency. All profiles are normalized to its respective maximum value.

Fig. 3 shows the 2-D beam profiles in a plane through the beam axis, perpendicular to the array surface, the xz-plane and yz-plane. Aberration is clearly visible for both the first-harmonic and the second-harmonic parts of the field.

The peak pressure amplitude for the first and second harmonics along the array center axis is plotted in Fig. 4. It is seen that aberration reduces the amplitude by 1–2 dB for the first harmonic. A 4–6 dB reduction is observed.
Fig. 3. Beam profiles in the xz-plane and yz-plane. The first row shows the unaberrated case in the xz-plane, which is equal in the yz-plane due to the symmetric aperture. The second row shows the aberrated case in the xz-plane. The third row shows the aberrated case in the yz-plane. The first column shows the beam profiles for the first-harmonic frequency at 1.75 MHz. The second column shows the beam profiles for the second harmonic at 3.5 MHz. The third column shows the beam profiles for the fundamental at 3.5 MHz. All plots are normalized relative to the maximum value of the unaberrated 1.75 MHz first harmonic. The transmit amplitude was 1 MPa for this simulation.

for the second harmonic. Although the peak value varies between the different simulations, the shape of the curves is almost identical for all transmit pressures.

In Fig. 5 the total acoustic energy for the second harmonic as a function of depth is plotted [see (19)]. The second-harmonic energy increases toward the peak pressure of the first harmonic in the unaberrated case (see Fig. 4). For the aberrated case, only a weak increase in second-harmonic energy is displayed from about 3.0 cm, well before the peak pressure location. In the second column of Fig. 5, the energy has been compensated for attenuation. As a consequence, it indicates how much of the energy, generated at a particular depth, is present in the focal plane. It is seen that, when transmitting a 1 MPa pulse in a homogeneous medium, 20% of the energy in the focal plane was generated in the first 3 cm of propagation.

V. Discussion

The literature has often argued that the second-harmonic component is less aberrated because it is mainly generated after the body wall [5]–[7]. The results presented here indeed show that between 85–90% of the second-harmonic energy is generated beyond the body wall (see right column of Fig. 5) for the setup and transmit pressures used. Still, the second-harmonic beam is aberrated.

The aberrated beam profiles in the focal plane, displayed in Fig. 2, exhibit similarities in shape and sidelobe level for the first and second harmonic. Note that, for the first plot in the third row of Fig. 2, the sidelobe level to the right side of the center axis is almost the same for the first and second harmonic. Fig. 3 indicates that the similarities exist throughout the whole acoustic beam. The aberration of the second harmonic may be understood from (15). The equation shows that, inside the body wall (the first integral), the second harmonic is aberrated both at \(\omega_0\) and \(2\omega_0\). Propagation of the second harmonic may now be viewed as a two-step process. First, the first harmonic is propagated to the plane \((\xi_0, z_0)\), where it acts as a source for the second harmonic. Note that the first harmonic is aberrated according to \(\omega_0\). The second-harmonic component in the plane \((\xi, z)\) then is found by heterogeneous propagation from \((\xi_0, z_0)\). In this step, the second harmonic is aberrated at \(2\omega_0\).

The second step is most important when the plane \((\xi_0, z_0)\) is close to the array surface. When it is close to \(d\) (the end of the body wall) the effect of the aberration at \(2\omega_0\) will be small due to the short propagation distance. This is due to the fact that, for a short propagation distance (relative to the body wall thickness), the accumulated effect of the aberrator is much less than for a large propagation distance. Still, the second harmonic generated close to \(d\) will experience the cumulative aberration of the aberrator at \(\omega_0\) from its source; the first harmonic.

The general shape of the beam profiles of the first harmonic at 1.75 MHz and the fundamental at 3.5 MHz also are similar, although the 3.5 MHz fundamental has a higher sidelobe level. This again is depicted in Figs. 2
In the absence of aberration, Fig. 4 clearly shows that the peak pressure of the first harmonic increases gradually and reaches its maximum value approximately 7 mm before the geometric focal point. Then it decreases rapidly in the far-field. The associated second harmonic mimics this behavior in that it increases steadily toward the peak location of the first harmonic.

Looking carefully, a dip in the axial pressure is observed for the aberrated transmit beam around 2.5 cm. Comparing this with Fig. 3, it is seen to be caused by local destructive interference in the aberrated beam. Aberration may cause significant local interference phenomena, as demonstrated by Fig. 3. The axial pressure, therefore, is not a reliable indicator for the generation of the second harmonic.

An alternative indicator for the generation of second harmonic is the acoustic energy as a function of depth. This is defined in (19). Fig. 5 shows that the generation of second-harmonic energy is weakened by the aberration. As a consequence, the second-harmonic energy does not increase significantly from about 3.0 cm (Fig. 5, first col-
umn). It is seen that the aberration reduces the amount of second-harmonic energy by 40–60%.

The second column of Fig. 5 contains additional information about the amount of harmonic energy propagated to the focus. Interestingly enough, as aberration reduces the total amount of energy generated, almost 30% of the energy is generated in the first 3 cm for the 1 MPa case. For lower transmit amplitudes, a larger fraction of the energy was generated close to the focal plane. As a consequence, the reduction due to aberration is greater in these cases; they rely more on a sharply focused beam.

In a medium with higher absorption, the effect will be to move the attenuation-compensated curve downward; the second-harmonic energy, which is generated early, will be attenuated more. Thus, aberration due to the first integral of (15) will be less significant. This effectively means that aberration according to $2\omega_0$ will be less of a problem, and the benefit from lowering the transmit frequency is higher.

The second-harmonic component is generated by an effective volume source. For low acoustic pressure amplitudes, the strength of this source per unit volume is proportional to the square of the first-harmonic component. This is described by (10). Christopher [1] presented a simple argument for the generation of second harmonic. If the amplitude of the fundamental beam is approximately constant across the beam width, the second harmonic will be generated at a constant rate in the extreme near-field. This is because the reduced cross section of the beam is balanced out by the increased amplitude; both as a result of focusing.

Aberration distorts the wavefront of the beam and reduces the focusing. As a result, the effective volume source for the second harmonic is weakened. For an unaberrated beam, the source strength as a function of depth is almost proportional to the acoustic energy as defined by (19); in a homogeneous medium, the source from a focused beam is an almost coherent sum across the beam width. Wavefront distortion associated with aberration will reduce the coherency and thus the source strength. The reduced amplitude will further contribute to this weakening.

When the acoustic pressure is low, (10) shows that scaling of the amplitude for the linear part will only imply a scaling of the nonlinear part. Hence, the transmit beams should be similar for different pressure amplitudes. This is often referred to as the quasilinear regime. For higher pressure values, this approximation does not hold, and a difference in the beam profiles will be observed. The 0.3 MPa and 0.6 MPa transmit pressures used in this study are within the range of pressure amplitudes in which the quasilinear approximation is reasonable. The highest pressure amplitude is strictly not within this regime. However, the beam profiles still do not deviate much from the others, indicating that (15) provides a valid description of second-harmonic aberration for this pressure. Henceforth, only beam profiles for one pressure amplitude are presented in Figs. 2 and 3.

Looking carefully at the uncompensated total energy plots in Fig. 5, a slight shift of the maximum energy point is observable in the 1 MPa case compared to the other cases. This shift is probably related to a close-to-acoustic-shock situation. Fig. 4 shows that the peak pressure amplitude for the unaberrated case is $-10$ dB down. This is a quite high level; still the total energy curve is only slightly modified, indicating that the effect of the shock is negligible. Introducing aberration reduces the peak pressure amplitude of both the first and second harmonic. Thus the reduction of generated second-harmonic energy for the 1 MPa transmit pressure is related to the introduction of aberration, and not to an acoustic shock.

By introducing an effective apodization, Fedewa et al. [10], [11] were able to reproduce the profile of the second-harmonic beam using only fundamental fields. Eq. (18) gives an explicit expression for an equivalent radiating source array with an effective apodization. An equivalent source also may be defined for the heterogeneous tissue. However, in this case the source structure is more complicated and will in essence be found by linearly propagating the second-harmonic field back to the transducer array. Note that this, in general, will lead to an apodization in the form of an amplitude as well as to an effective waveform distortion due to the aberration.

A necessary requirement for this effective apodization to make sense is that the quasilinear approximation in (10) holds. For higher transmit amplitudes, a full nonlinear backpropagation would yield a similar result. In this case stability problems pertaining to backpropagation through an attenuating medium would be alleviated by bandlimited backpropagation. This is similar to the prebiasing method suggested for cancellation of tissue-generated harmonic energy [25], [26].

If the aberration at both the first-harmonic and the second-harmonic frequencies are the same, the procedure for defining an equivalent source will by construction be an amplitude modification of the original source. This is the case for homogeneous tissue. The equivalent source may in turn be viewed as an aberration correction scheme in which the aberration incurred at the second-harmonic frequency is removed, and it is replaced by the aberration as a result of the first-harmonic field. As such, the extent of the region in which this approximation is valid is limited by the iso-planatic patch.

In order to illustrate the validity of the effective apodization, an additional simulation of the fundamental was performed in which the effective apodization was used (for the unaberrated case and the aberrated case). This was compared to the second-harmonic simulations with 0.3 MPa transmit amplitude.

Fig. 6 shows cross sections of the effective apodization function computed for the unaberrated case and the aberrated case, together with the effective waveform distortion required to obtain the equivalent source. It is seen that, for the unaberrated case, the waveform distortion is zero; the waveforms arriving at each point on the array are equal. The aberration, however, introduces a waveform distortion that is displayed in the right lower panel of Fig. 6. By applying this apodization and distortion to a 3.5 MHz

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transmit pulse, a fundamental beam is obtained that will have the same characteristics as a second-harmonic beam generated from a 1.75 MHz first harmonic. In this respect, the distortion accounts for aberration differences between the second harmonic and the fundamental frequency.

Fig. 7 shows beam profiles averaged over circles centered at the beam axis, as a function of radius:

\[ f(r) = \frac{1}{2\pi} \int |p(r, \theta, t)|^2 dt d\theta. \]

Note that, by construction, the fundamental field from the equivalent source is almost equal to that of the second harmonic—also in the presence of aberration. By the time-reversal principle, the only deviations are the result of limiting the aperture size. Plots of the beam profiles resulting from the full equivalent source, therefore, has been omitted in the plots.

It is clearly seen that, in homogeneous tissue, the fundamental beam with an effective apodization exhibits much the same beam profile as the second harmonic. This is indeed expected, as the equivalent source in this case is obtained by amplitude apodization only. Also in the unaberrated situation, some improvement of the fundamental profile is observed when the effective apodization is used (only amplitude part from the effective source). However, there is still a significant deviation between the fundamental and the second harmonic. It is observed that the averaged first-harmonic beam profile has much the same shape as the second harmonic, when the frequency difference has been accounted for by scaling the axis. Indeed, the difference is comparable to that of the first harmonic and the second harmonic in the unaberrated case. This confirms the impressions from Fig. 4; aberration is indeed similar for the two fields.

Previous studies of aberration for linear propagation has shown that correcting for the delays is most significant, and that amplitude correction will contribute to further improvement. Therefore, it is not unexpected that an effective apodization is not able to reproduce the second-harmonic field. This is furthermore confirmed by the experimental study of Wallace et al. [12]—the effect of nonlinear propagation is more complex through an aberrator. However, if aberration is relatively independent of frequency, it is expected that the second-harmonic field could be reproduced by a fundamental field with an effective apodization.

The thickness of the body wall, \( d \), related to the focus \( F \) of the array, is a factor of relevance for the results presented. Here, \( F = 3d \), that is, one-third of the propagation occurs inside the body wall. Having \( F \leq d \) is possible in several different situations. First, the focus of the array is inside the body wall, which for the setup presented here also corresponds to a reduction in the \( f \)-number. Second, the entire object to be imaged is heterogeneous, e.g., the female breast. In the latter situation, large \( f \)-numbers are possible. In both these situations, the second integral in (15) is zero. This indicates that the second harmonic will
experience a larger cumulative mix of aberration at both $\omega_0$ and $2\omega_0$ as given by the first integral.

In the simulations presented here, the $f$-number of the array is 3. Changing the $f$-number of the array will alter its focusing properties. This in turn will affect the generation of second-harmonic energy.

An additional effect that comes into play when the aberration is distributed over the whole propagation length, or altering the $f$-number, is the correlation length of the aberration. If the correlation length of the aberration is larger than the width of the beam, the aberration will not impact greatly on the coherence of the propagating wave. Hence, the effect of aberration is expected to be lower for a narrow propagating beam.

It is beyond the scope of this article to study this effect, and a topic for further research to investigate the effects of beam-width, $f$-number, and extent of the aberrator.

VI. Conclusions

This article presents a theoretical description of nonlinear generation and propagation in a heterogeneous medium. This is accompanied with results from 3-D simulations of a heterogeneous medium with absorption.

The results show that, relative to the the focal plane of the array, close to 90% of the second-harmonic generation occurs beyond the body wall. Nonetheless, the second-harmonic beam is aberrated. The shapes of the aberrated first-harmonic and second-harmonic beam profiles show clear similarities, and in some cases they have close to equal sidelobe levels when scaled for the frequency difference. This indicates that aberration of the second harmonic is not simply obtained by squaring the first harmonic. Eq. (15) provides a theoretical description of this process. The introduction of an equivalent source further illustrates the difference between HI in homogeneous medium and in a heterogeneous medium. The equivalent source effectively lowers the frequency at which the fundamental field is aberrated.

In an imaging situation, the acoustic backscatter received at the transducer array is subject to the same aberration as the transmitted wave. However, due to the much lower amplitude of the backscatter, the generating source term for the second harmonic is negligible on the way back. Harmonic imaging, therefore, does not have an effect on the aberration of the received backscatter, beyond that of modifying the insouffled scattering region.

Aberration introduces a defocusing of the beam. As a result, the growth in total energy for the second harmonic is reduced by 40–60% for the aberrated case compared to the unaberrated case. In the presented simulations, a reduction of 1–2 dB in peak pressure value for the first harmonic implied a 4–6 dB reduction in peak pressure values for the second harmonic, depending on the transmit pressure.

The peak pressure curves of the second harmonic along the transducer axis do not adequately display where the second-harmonic energy is generated. The introduction of absorption-compensated total energy plots provide new information of where second-harmonic energy is generated.

When using second-harmonic imaging in modern ultrasound scanners, the first-harmonic frequency is reduced compared to standard fundamental imaging. The increased wavelength associated with this frequency reduction will imply that a given propagation delay will be smaller relative to the wavelength for the first harmonic than for the fundamental. A consequence of this is that the effect of aberration will be less severe at the lower frequency. As demonstrated in this article, the aberration of the second harmonic mimics that of the first harmonic, which again is less aberrated than the fundamental beam (reduced sidelobe level). Therefore, harmonic imaging suffers less from aberration compared to fundamental imaging at the same frequency.

The strong link between aberration of the first harmonic and the second harmonic suggests that performing aberration correction on the first-harmonic transmit-beam will improve the second-harmonic imaging system. Based on the results presented here, this is at least true in situations in which the main source for aberration of the second harmonic is the second integral of (15), the aberrated first harmonic.

References


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