The effect of displacement rate on imbibition relative permeability and residual saturation

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Abstract

A dynamic network model for imbibition based on a physically realistic description of the complex dynamics of film flow, film swelling and snap-off is described. The model shows that film swelling is a capillary driven nonlinear diffusive process and that the competition between snap-off and frontal displacements is rate dependent resulting in rate dependent relative permeability curves and residual saturations. In contrast to existing quasi-static network models where snap-off is suppressed by contact angle alone, the dynamic model introduces displacement rate as an additional snap-off inhibiting mechanism. The network model is used to analyse the complex interaction between displacement rate, contact angle, aspect ratio and pore and throat shapes on relative permeability. Computed relative permeabilities and residual saturations are compared with laboratory measured data for strongly water-wet Berea sandstone. It is concluded that the magnitude of the rate effect on relative permeability for a particular rock and wetting state depends largely on the aspect ratio. Higher aspect ratios produce larger rate effects than smaller aspect ratios.

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1. Introduction

Relative permeability and residual saturation are critical parameters in the evaluation of the recovery performance of petroleum reservoirs. Laboratory tests to measure these parameters are often carried out at high rates to minimise capillary end effects (Rapoport and Leas, 1953). The rates are significantly higher than typical reservoir displacement rates and in applying the measurements to reservoir conditions it is assumed that relative permeability and residual saturation are independent of displacement rate (Odeh and Dotson, 1985). The assumption that relative permeability is independent of rate may be valid for drainage displacements (Akin and Demiral, 1997; Virmovsky et al., 1998) but it is not clear if this is true for imbibition displacements. Labastie et al. (1980), Chen and Wood (2001), Odeh and Dotson (1985) and Qadeer et al. (1988) report that laboratory imbibition water–oil relative permeabilities are independent of rate after accounting for capillary end effects. On the other hand, Heaviside et al. (1987), Kamath et al. (1995), Mohanty and Miller (1991), Ringrose et al. (1996), Skauge et al. (2001) and Wang and Buckley (1999) conclude that displacement rate is important. To confuse matters further, Fulcher et al. (1985) conclude that relative permeability depends on capillary number but that the effect of displacement rate

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is unimportant. Most of the measurements reported in these studies are for sandstone cores and cover a wide range of wetting conditions.

A similar lack of consensus appears to exist in the network modeling literature. Blunt (1997), Constantinides and Payatakes (2000), Hughes and Blunt (2000) and Mogensen and Stenby (1998), amongst others, describe numerical network models in which flow in wetting films and the subsequent snap-off of non-wetting fluid ahead of the displacement front result in rate dependent imbibition relative permeabilities. The network models are based on pore-scale displacement mechanisms observed in transparent micromodel experiments (e.g., Lenormand et al., 1983; Vizika et al., 1994) and show that rate effects remain important down to very low displacement rates—capillary numbers \( \text{Ca} = 10^{-6} - 10^{-8} \) where the capillary number is the viscous to capillary force ratio written as,

\[
\text{Ca} = \frac{\mu v}{\sigma}
\]

\( \mu \) is viscosity, \( v \) is the displacement velocity and \( \sigma \) is the interfacial tension. In contrast, elaborate three-dimensional quasi-static network models, based on realistic representations of actual sandstone morphology, which ignore rate effects have been successfully used to predict laboratory measured imbibition relative permeabilities (e.g., Blunt et al., 2002; McDougal et al., 2001; Øren et al., 1998; Øren and Bakke, 2003; Patzek, 2001; Valvatne and Blunt, 2003).

The effect of displacement rate on residual saturation is less controversial. Chatzis and Morrow (1984) showed that waterflood residual oil saturation for a wide range of water-wet consolidated sandstones decreases with increasing capillary number. The reductions in residual saturation occurred at significantly smaller capillary numbers than those required for mobilisation of discontinuous oil. Maldal et al. (1997) report similar results for reservoir cores from a number of North Sea fields displaying a range of wettabilities. Residual saturations for carbonate rocks appear to display a greater sensitivity to flooding rate. Kamath et al. (1995, 2001) showed that corefloods on cleaned and restored state heterogeneous carbonate cores displayed large reductions in residual oil saturations with increase in capillary numbers from \( 2 \times 10^{-8} \) to \( 4 \times 10^{-7} \). The reductions could not be attributed to capillary end effects. In a recent study Tie and Morrow (2005) report the results of a rate sensitivity study for three outcrop limestones—homogeneous and heterogeneous grainstones and a high porosity/permeability boundstone. Waterflood residual saturations for all three limestones displayed a sensitivity to flooding rate at capillary numbers much lower than measured for the mobilisation of oil both for strongly water wet and mixed wettability conditions.

Snap-off of non-wetting fluid ahead of the displacement front is an important trapping mechanism in imbibition displacements (Chatzis et al., 1983; Chatzis and Morrow, 1984; Kamath et al., 2001; Lenormand and Zarcone, 1984; Mohanty et al., 1980) and the competition between snap-off and frontal displacements determines the pattern of the displacement and therefore the shape of the relative permeability curves and the value of residual saturation (Hughes and Blunt, 2000). Dynamic effects resulting from flow through wetting films can affect the competition between snap-off and frontal displacements and a number of dynamic network models have been proposed to account for this (Blunt and Scher, 1995; Constantinides and Payatakes, 2000; Mogensen and Stenby, 1998; Hughes and Blunt, 2000, 2001). All of the models are based on simplified treatments of flow through films—constant film conductivities, steady-state flow and ad hoc applications of the snap-off mechanism. None of these models capture the complex dynamics of film flow, swelling, snap-off and slow cluster growth by pore filling ahead of the displacement front. Moreover, no attempt was made to compare model predictions with measured relative permeability and residual saturation data.

We present a new network model for imbibition displacements which captures the complex dynamics of film flow and the rate-dependent competition between snap-off and frontal displacements. The model displays all the important pore-scale physics observed in micromodel displacement studies—time-dependent swelling of wetting films ahead of the displacement front, local fluctuations in film pressure and thickness, slow filling of snap-off sites ahead of the displacement front, snap-off initiated cluster growth and the suppression of snap-off with increasing displacement rate. The only assumption made is that viscous gradients in bulk fluid (fluid occupying the centres of pores and throats) are negligible. This is a reasonable assumption for capillary numbers \( \text{Ca} = 10^{-4} - 10^{-3} \) (Dullien, 1992).

We use the model to investigate the complex interaction between displacement rate, contact angle, pore–throat aspect ratio and pore and throat shapes on relative permeability and residual saturation and attempt to resolve apparent contradictions regarding the effect of displacement rate on laboratory measured data. We compare predicted relative permeabilities with measurements reported by Oak (1990) for strongly water-wet Berea sandstone. This data has previously been used to
validate quasi-static network models (Blunt et al., 2002; Valvatne and Blunt, 2003). We also compare model predictions with the displacement rate dependent residual oil saturation data for strongly water-wet Berea reported by Chatzis and Morrow (1984). We conclude that the magnitude of the rate effect on imbibition relative permeability and residual saturation, for a particular rock type and wetting state, depends largely on the pore–throat aspect ratio. For cores with high aspect ratios (large pores and small throats) the rate effect can be large. For low aspect ratios (pores and throats of similar size) the rate effect is small. The implications for laboratory relative permeability and residual saturation testing are obvious.

2. Network model for imbibition

2.1. Network representation

We consider the simple case of a uniformly wet network characterised by a single contact angle in order to more clearly demonstrate how the dynamic description of film flow can be incorporated in existing quasi-static network models. These models already contain sophisticated descriptions of complex wettability conditions (Blunt, 1998; Blunt et al., 2002; Øren et al., 1998; Patzek, 2001; Valvatne and Blunt, 2003). Although we do not refer to specific modifications these will be obvious to those familiar with network models.

To allow a direct comparison between our dynamic model and the quasi-static results reported by Valvatne and Blunt (2003) we use their network which is generated from a process based reconstruction of Berea sandstone (Lerdahl et al., 2000). The network represents a rock fragment having a volume of approximately 27 mm$^3$ consisting of 12,349 pores and 26,146 throats. The average coordination number is 4.2. The porosity of the network is 0.24 and the permeability is 2.5 Darcy.

Although Valvatne and Blunt (2003) use a mixture of shapes (circles, squares and triangles) to represent the cross-sections of pores and throats, we assume that pores and throats all have the same shape—regular polygons. This allows us to clearly show the effect of pore shape on relative permeability and residual saturation. The polygon geometry is essential because it allows macroscopic (thick) films of wetting films stabilised by capillary pressure to form in the corners of pores and throats occupied by non-wetting fluid. These films allow wetting fluid to displace non-wetting fluid ahead of the displacement front. Fig. 2 shows the distribution of water wetting films in a triangular capillary for decreasing capillary pressures.

2.2. Pore-scale displacement mechanisms

During imbibition the fluids in the network are separated by a macroscopic interface or connected
displacement front. The wetting fluid is continuous throughout the network. Behind the front, the wetting fluid fully fills all pores and throats other than those containing trapped non-wetting fluid. Ahead of the front, wetting fluid is contained in corner films in pores and throats filled with non-wetting fluid. Imbibition commences at a high capillary pressure and capillary pressure decreases during the course of the displacement. Pore-scale displacement events occur in order of decreasing threshold capillary pressure.

The pore-scale events by which wetting fluid invades the network are based on the micromodel observations reported by Lenormand et al. (1983) and have been described in detail in previous studies (Blunt, 1997; Øren et al., 1998; Patzek, 2001). We refer to them as frontal displacements—piston-type invasion of pores and throats containing non-wetting fluid at the displacement front, and snap-off—invasion of throats ahead of the displacement front by capillary driven swelling and eventual rupture of corner films. Pores ahead of the displacement front can also be invaded from wetting-fluid filled throats produced by snap-off. This results in snap-off nucleated cluster growth ahead of the displacement front. The clusters grow by the same piston-type invasions of pores and throats as for the invasion front, however, cluster growth is relatively slow because it is limited by flow through corner films.

The threshold capillary pressure for piston-type filling of pores depends on the number of connected oil-filled throats, \( n \) (Lenormand et al., 1983). Hughes and Blunt (2000) proposed the following expression,

\[
P_{c,f} = C_i \frac{\sigma \cos \theta}{r_p}
\]

where \( \sigma \) is the interfacial tension, \( \theta \) is the contact angle, \( r_p \) is the inscribed radius of the pore and \( C_i \) are input parameters \( (C_i = 1.7, C_i = 1.15, C_i = 0.7, C_i = 0.5) \). This parametric expression accounts for the fact that the largest radius of curvature that can be achieved for a pore displacement depends on the number of adjacent oil-filled throats (see Fig. 3). Other expressions producing similar results have been proposed by Blunt (1998), Øren et al. (1998) and Patzek (2001).

The capillary pressure for piston-type filling of throats is given by Ma et al. (1996),

\[
P_{c,p} = \frac{\sigma}{r_t} \left\{ \cos \theta + \frac{\tan \alpha}{2} \left[ \sin(2\theta) + \pi - 2(\alpha + \theta) \right] \right\}
\]

where \( r_t \) is the throat radius and \( \alpha \) is the half-angle of the corners.

As capillary pressure decreases wetting films in the corners of pores and throats swell (see Fig. 2). When the film swells to the point where it is no longer in contact with the walls the non-wetting fluid becomes unstable and snaps-off. The threshold capillary pressure decreases with increasing number of connected throats filled with oil \((I_1-I_3)\).
pressure for snap-off is given by Hughes and Blunt (2000),

\[ P_{cs} = \frac{\sigma}{r_1} (\cos \theta - \sin \theta \tan \alpha) \quad (4) \]

Snap-off in a throat results in the creation of two piston-type interfaces and an increase in capillary pressure from \( P_{cs} \) given by Eq. (4) to \( P_{cp} \) given by Eq. (3). The local reduction in wetting fluid pressure causes wetting fluid to flow into the throat and the piston interfaces to retract to the adjoining pores. The speed with which this occurs depends on the volume of the throat and the conductivity of the surrounding films.


2.2.1. Competition between snap-off and frontal displacements

Snap-off is an important bond breaking mechanism and is responsible for large-scale trapping of oil in the network. The order in which frontal and snap-off displacements occur determines the pattern of the displacement and therefore the shape of the relative permeability curves and the value of the residual saturation. If the displacement is dominated by snap-off, relative permeabilities are low and residual saturations are high. If the displacement is dominated by frontal events, trapping is low (low residual saturation) and relative permeabilities are high. The balance or competition between snap-off and frontal displacements is central to the description of imbibition displacements.

The ratio of threshold pressures for snap-off in throats and frontal displacements of pores is given by Eqs. (2) and (4)

\[ \frac{P_{cs}}{P_{cf}} = \frac{1}{C_{n}} \left( \frac{r_p}{r_1} \right) \left( 1 - \tan \theta \tan \alpha \right) \quad (5) \]

Eq. (5) shows that snap-off is favoured by high aspect ratios (large pores and small throats), small contact angles (strongly water-wet rocks) and small corner half-angles (more angular pores and throats). Snap-off is also favoured by lower values of \( C_{n} \), i.e., snap-off is more competitive with frontal pore invasions when the pore is invaded from a smaller number of adjoining throats—high \( n \).

Eqs. (2)–(5) also show the crucial role of contact angle and pore shape on the competition between snap-off and frontal displacements. The threshold capillary pressure for frontal displacements is always positive for \( \theta < 90^\circ \), Eq. (2). This means that frontal displacements can always occur. In contrast, Eq. (4) shows that the threshold capillary pressure for snap-off falls to zero when

\[ \tan \theta = \frac{1}{\tan \alpha} \quad (6) \]

When this condition is satisfied, snap-off is entirely suppressed and the displacement is purely frontal.

2.3. Dynamic effects

Quasi-static models ignore viscous effects so the capillary pressure is constant throughout the network and the competition between snap-off and frontal displacements is determined by Eq. (4). For a particular network (topology, geometry and pore shape) this is determined by the value of the contact angle. However, viscous gradients in corner films can introduce gradients in capillary pressure across the network which significantly alter the order of displacement events and the competition between frontal displacements and snap-off. In order to model this, it is necessary to consider the dynamic effects introduced by flow through corner films.

2.3.1. Fluid flow in networks

We model fluid flow in networks on the basis of the following assumptions:

(i) Viscous gradients in bulk fluid (fluid occupying the centres of pores and adjacent throats) are negligible. This is a reasonable assumption for \( Ca < 10^{-3} - 10^{-4} \) (Dullien, 1992) which covers a wide range of capillary numbers normally encountered in reservoir displacements and laboratory tests.

(ii) Local capillary equilibrium—the curvature of the interface in any pore or throat at any time is determined by the local capillary pressure.

Assuming that the cross-sectional shapes of pores and throats are regular polygons, the cross-sectional area is given by,

\[ A = \left( \frac{n}{\tan \alpha} \right) r^2 \quad (7) \]

where \( n \) is the number of corners and \( r \) is the radius of the inscribed circle (\( r_p \) or \( r_t \)).
The area of corner films is given by
\[ A_w = nr_w^2 \left( \frac{\cos(\cos(\theta + \alpha))}{\sin\alpha} - \frac{\pi}{2} + \theta + \alpha \right) \] (8)
where \( r_w = \sigma/P_c \) is the radius of curvature of the corner films and \( P_c \) is the local capillary pressure.

The wetting fluid saturation in a pore or throat, \( S_w \), is assumed to be the fraction of the cross-section occupied by wetting fluid (Blunt et al., 2002; Øren et al., 1998; Hughes and Blunt, 2000),
\[ S_w = \frac{A_w}{A} \] (9)

In order to realistically model snap-off—slow accumulation of wetting fluid in throats and the abrupt increase in capillary pressure at the point of snap-off—we calculate wetting film pressures in both pores and throats (see Fig. 4). The film flow rate from a pore or throat, \( i \), to a throat or pore, \( j \), may be written as,
\[ q_{i,j} = g_{i,j}(P_i - P_j) \] (10)
where \( g_{i,j} \) is the effective film conductance and \( P_i \) and \( P_j \) are the corresponding pore or throat pressures.

The effective film conductance is assumed to be the harmonic mean of the film conductances of the pore and throat,
\[ \frac{1}{g_{i,j}} = \frac{1}{g_i} + \frac{1}{g_j} \] (11)
where \( g_i \) and \( g_j \) are the corresponding pore and throat conductances.

The wetting film conductances are given by (Hughes and Blunt, 2000),
\[ g_i = \frac{A_w r_w}{\beta \mu_w l_i} \] (12)
where \( \beta \) is the resistance factor proposed by Ransohoff and Radke (1988), \( \mu_w \) is the wetting fluid viscosity and \( l_i \) is the distance between the pore or throat centre and the pore–throat boundary (Hughes and Blunt, 2000).

2.3.2. Calculation of film pressures
Film pressures in pores and throats are computed using the requirement that the volume of the wetting fluid in the films is conserved. Assuming that the wetting fluid is incompressible this may be written as,
\[ \sum_k q_{i,k} = \frac{\partial V_{w,i}}{\partial t} \] (13)
where \( V_{w,i} \) is the film volume in pore or throat \( i \), \( q_{i,k} \) is the volumetric film flow rate into and out of pore or throat \( i \), and \( k \) is the number of throats connected to the pore (coordination number of the pore or throat). Using Eq. (10), Eq. (13) may be written as,
\[ \sum_k g_{i,k}(P_k - P_i) = V_i \frac{\partial S_{w,i}}{\partial t} \] (14)
where \( V_i \) is the volume of pore or throat \( i \) and \( S_{w,i} \) is the corresponding wetting fluid saturation.

Eq. (14) may be used to update pore and throat film volumes. This is done using a simple explicit finite-difference scheme which is written as,
\[ S_{w,i}^{n+1} = S_{w,i}^n + \Delta t \sum_k g_{i,k}^n \left( P_k^n - P_i^n \right) \] (15)
where \( \Delta t \) is the time-step size and the superscripts \( n \) and \( n+1 \) refer to the known and unknown time levels, respectively. Eqs. (8) and (9) are used to update film curvatures and film pressures.

2.3.3. Numerical algorithm
A schematic of the dynamic network model is shown in Fig. 5. The network is initially static. Non-wetting fluid occupies the centres of pores and throats with wetting fluid in corner films. The thickness of the films is determined by the specified initial capillary pressure. Wetting fluid is injected at a constant rate over the inlet face of the network. The non-wetting fluid is free to leave the network across the opposite face which is held at a constant pressure, \( P_{nw} \). No-flow conditions are imposed on the other boundaries. The outlet face is impermeable to wetting fluid and injection continues until all the non-wetting fluid is trapped. The bulk of the initial water saturation—that which is not contained in
corner films—is not modelled explicitly. It is assumed to be associated with micro-porosity and water saturated clays which remain undrained. Following Bakke and Øren (1997), Blunt et al. (2002) and Valvatne and Blunt (2004) the network porosity is adjusted a priori to match the initial water saturation.

The invasion front is the irregular interface separating regions of interconnected pores and throats full of wetting fluid and interconnected pores and throats filled with bulk non-wetting fluid and corner wetting fluid films. Since we neglect pressure gradients in bulk fluids, the wetting fluid pressure at the displacement front is constant and is equal to the inlet pressure. The non-wetting fluid pressure, $P_{\text{nw}}$, is constant throughout the cluster and is defined as the wetting fluid volume in the pore or throat given by Eq. (2) or Eq. (3) i.e., $P_c^{*,n}$ or $P_c^{c,f}$.

(3) Updated wetting film saturations, $S_w^{n+1}$, are calculated using the simple explicit scheme given by Eq. (15). Eqs. (8) and (9) are used to update capillary radius and this is used to determine updated capillary pressures, $P_c^{n+1}$. Updated film pressures are simply $P_w^{n+1} = P_{\text{nw}} - P_c^{n+1}$.

(4) Wetting fluid pressures in trapped non-wetting fluid clusters connected to the displacement front by films are updated using a modified form of Eq. (15) to express the fluid conservation equation for the cluster—the volume of fluid in a trapped cluster is constant. Since we assume that pressure gradients in the bulk fluid are negelible the wetting fluid pressure throughout the cluster is constant and Eq. (15) is written as,

$$\sum_K g_{I,K}^n (P_{w,I}^{n+1} - P_{w,K}^n) = 0$$

where the subscripts $I$ and $K$ refer to the cluster $I$ and the $K$ throats connected to the cluster. Since we assume negligible pressure gradients in bulk fluid, the film pressures in a trapped cluster which is completely surrounded by water filled pores and throats are constant and there is no film flow in the cluster.

(5) The total flow into films at time level $n$ is,

$$q_{I}^n = \sum_{ij} g_{ij}^n (P_{w,I}^n - P_{w,j}^n)$$

where the summation is for all pore–throat pairs connected to the displacement front. The flow into the frontal pore or throat being invaded is $q_{I}^n = q - q_{I}^f$ and the wetting fluid volume in the pore or throat is updated by the amount $q_{I}^n \Delta t$. 

Fig. 5. Schematic of dynamic network model.
For all time steps after the first snap-off event a check is made to determine if piston-like invasions from wetting fluid filled throats (snap-off sites) and pores ahead of the invasion front are possible. A check is made to identify trapped non-wetting fluid clusters. A cluster is trapped if it is no longer connected to the outlet.

Since we employ an explicit solution, selection of time-step size, $\Delta t$, is an important consideration. We find that a variable time-step size based on an estimate of the time required to complete the next displacement event works well. The time required to complete a displacement event is estimated from the minimum saturation change required to complete the event. This time is then multiplied by a factor less than 1. We have employed a factor of 0.95 for most of the computations reported in this paper. Reducing the factor to 0.5 increases the computation time but has little effect on the solution. Increasing the factor above 0.95 usually results in instability. A check is made at the end of each time-step to determine if the capillary pressure in any pore or throat is within a specified tolerance of a threshold. When this occurs the site occupancy is changed, capillary pressures are reset as necessary and a new time-step commences.

Relative permeabilities are calculated for the network in the same manner as for quasi-static models (Hughes and Blunt, 2000; Øren et al., 1998). Wetting fluid conductances are calculated using Eq. (12). Analogous equations are used to calculate full pore and throat wetting fluid and non-wetting fluid conductances. A constant pressure difference is imposed across the network and the steady-state flow is calculated. The fluid flow rates together with Darcy’s law are used to calculate relative permeabilities.

2.4. Limiting behaviour

It is instructive to show the character of the discretized form of the film conservation equation, Eq. (14). Consider a one-dimensional network with identical pores and throats which we will refer to as elements. Eq. (14) may be written as

$$g_{i-1,j}(P_{w,i-1} - P_{w,j}) - g_{i,j+1}(P_{w,i} - P_{w,i+1}) = l \frac{\partial A}{\partial t}$$

(19)

where $A$ is the total cross-sectional area of the film and $l$ is the length of an element.

For $t \rightarrow 0$ we can write the equation in differential form as

$$\frac{\partial}{\partial x} \left( g \frac{\partial P_w}{\partial x} \right) = \frac{\partial A}{\partial t}$$

(20)

This equation is identical to the conservation law for the imbibition of a wetting fluid along the corners of a square capillary (Dong and Chatzis, 1995).

Since $P_w = P_w(A)$ and $g = g(A)$ we can write,

$$\frac{\partial}{\partial x} \left( f(A) \frac{\partial A}{\partial x} \right) = \frac{\partial A}{\partial t}$$

(21)

where $f(A) = gdP_w/dA$.

Eq. (21) shows that the swelling of wetting films is a nonlinear diffusive process. Diffusive processes are slow and the extent to which swelling occurs in a displacement depends on the total displacement time—the time available for diffusion. The displacement time, in tum, is inversely proportional to displacement rate—the higher the displacement rate the shorter the displacement time. This is the essence of the competition between piston-like frontal displacements and snap-off and can introduce strong rate effects in imbibition displacements. Low rates favour snap-off because long displacement times allow significant film swelling and consequent snap-offs. High displacement rates favour piston-like displacements at the connected front because there is insufficient time for films to swell. At sufficiently high displacement rates the time for film swelling may be so short that snap-off is completely suppressed.

Fig. 6 shows the effect of displacement rate on the competition between snap-off and frontal displacements for a two-dimensional network with a high pore–throat aspect ratio (6.8) and a zero contact angle—conditions strongly favouring snap-off. For the lowest displacement rate ($Ca=10^{-8}$), pressure gradients in the films are very small and the displacement front is dominated by snap-off which occurs throughout the network. Snapped-off sites nucleate cluster growth but this is limited by the relatively high thresholds for pore invasions when compared to snap-off. This is a direct consequence of the high aspect ratio. The displacement pattern corresponds to the bond-percolation regimes observed by Lenormand and Zarcone (1984) in micro-model displacement experiments and the snap-off dominated displacement regimes predicted with quasi-static models (Blunt and Scher, 1995).

As displacement rate increases (Fig. 6, $Ca=10^{-7}–10^{-6}$), pressure gradients in corner films become sufficient to suppress snap-off and the region of the
network affected by snap-off progressively contracts towards the connected displacement front. Further increases in displacement rate \((Ca=10^{-5} - 10^{-4})\) produce wetting fluid pressures which are sufficient to initiate significant frontal displacements and limit the extent of snap-off to the immediate vicinity of the displacement front. For the highest displacement rate shown \((Ca=10^{-3})\) snap-off is completely suppressed and the displacement is purely frontal. This corresponds to the site invasion percolation regime observed with quasi-static models when it is assumed that there is no corner film flow (Blunt and Scher, 1995).

3. Results and discussion

We first show the sensitivity of computed relative permeability and residual saturation to displacement rate, contact angle, aspect ratio, pore shape and initial capillary pressure. We then show comparisons between our computations and the measured relative permeabilities reported by Oak (1990). This data has previously been used to validate quasi-static network models (see Jackson et al., 2003; Valvatne and Blunt, 2003, and references therein). We also compare model predictions with the displacement rate dependent residual oil.

Fig. 6. Displacement patterns for a high aspect ratio (6.8) two-dimensional network showing the effect of displacement rate on the competition between snap-off and frontal displacements.
saturation data for strongly water wet Berea reported by Chatzis and Morrow (1984). Finally, we discuss the implications to laboratory measured relative permeability data.

3.1. Sensitivity analysis

The sensitivity analysis is based on the conditions for the laboratory core flood experiments using strongly water wet Berea sandstone reported by Oak (1990). We use the Berea network with an aspect ratio of 1.7 with a regular triangular (equilateral) pore shape. The experimental displacement rate corresponds to a capillary number of $3.5 \times 10^{-6}$. The initial capillary pressure is estimated to be 300 kPa.

3.1.1. Contact angle and displacement rate

Fig. 7(a) shows the effect of contact angle on relative permeabilities for a relatively low displacement rate corresponding to $Ca = 3.5 \times 10^{-7}$. Relative permeabilities increase and residual saturations decrease with increasing contact angle. This is due to the increasing suppression of snap-off and trapping with increasing contact angle. Snap-off is highly favoured for a contact angle of 0° and completely suppressed for 60°. The effect has been noted in previous studies concerned with quasi-static models (Blunt et al., 2002; Hughes and Blunt, 2000; Øren et al., 1998; Øren and Bakke, 2003; Patzek, 2001; Valvatne and Blunt, 2003).

Fig. 7(b) shows the effect of contact angle for a much higher displacement rate of $Ca = 3.5 \times 10^{-2}$. In contrast to the low displacement rate cases shown in Fig. 7(a), for high displacement rates the relative permeabilities and residual saturations are insensitive to contact angle. Displacement times at high displacement rates are short and there is insufficient time for films to swell and snap-off. Increasing displacement rates result in increasing suppression of snap-off and trapping. Increasing displacement rate and increasing contact angle have a similar effect—both suppress snap-off and trapping.

The similarity between the overall effects of contact angle and displacement rate is further illustrated Fig. 8. Fig. 8(a) shows that increasing rate from $Ca = 3.5 \times 10^{-7}$ to $3.5 \times 10^{-2}$ for a contact angle of $\theta = 0^\circ$ results in more favourable relative permeabilities and lower residual saturation. Fig. 8(b) shows that the same rate change has no effect on relative permeability and residual saturation for a contact angle of 60° where snap-off is completely suppressed by contact angle alone.

3.1.2. Aspect ratio

The effect of aspect ratio on relative permeabilities and residual saturations is shown in Fig. 9. The computed relative permeability curves are for two aspect ratios (AR = 1.7 and 6.8) and two displacement rates ($Ca = 3.5 \times 10^{-7}$, Fig. 9(a) and $Ca = 3.5 \times 10^{-2}$, Fig. 9(b)). The contact angle is 0, which strongly favours snap-off, for all cases.

Fig. 9(a) shows that for low displacement rates, where pressure gradients in corner films are small, snap-off is suppressed for the low aspect ratio case resulting in higher relative permeabilities and lower residual saturations when compared to the high aspect ratio case where snap-off events dominate. At high displacement rates (Fig. 9(b)), where the rate effect suppresses snap-off for both high and low aspect ratios, the relative permeabilities and residual saturations are almost independent of aspect ratio.
3.1.3. Pore shape

Fig. 10 shows the effect of pore/throat shape on relative permeabilities and residual saturations. Three different shapes are considered—equilateral triangle with a half angle of 30°, square with a half angle of 45° and regular hexagon with a half angle of 60°. The three different pore shapes all have the same capillary radii and pore volumes (Berea network). The computations were made with an initial capillary pressure of 300 kPa and a contact angle of 0°. The aspect ratio is 1.7 for all the cases considered. Again, results are shown for two displacement rates—Ca = 3.5 × 10\(^{-7}\) and Ca = 3.5 × 10\(^{-2}\).

Fig. 10(a) shows that at low displacement rates, where snap-off dominates, relative permeabilities and residual saturations are influenced by pore shape. The relative permeability of the wetting fluid is particularly sensitive to pore shape with triangular pores displaying the highest relative permeabilities and hexagonal pores the lowest. This is consistent with triangular pores having the highest film conductivities and hexagonal pores having the lowest for the same capillary pressure. The trend in residual saturation is somewhat more complex. Although the residuals are all very similar, the square pores display the highest residual (0.40), hexagonal pores the lowest (0.37) and triangular pores an intermediate value (0.38). This is a clear example of the difficulty in interpreting relative permeability and residual saturation data.

Fig. 11(a) and (b) show the relative permeabilities plotted against the number of frontal and distant pores invaded, respectively. Frontal pores refer to the pores invaded from the connected displacement front and distant pores to pores invaded ahead of the front from snapped-off sites. The displacement is clearly dominated by frontal events for the hexagonal pore shape.
3.1.4. Initial capillary pressure

The effect of initial capillary pressure (initial film thickness) on relative permeability curves is shown in Fig. 12. Computed relative permeabilities are shown for initial capillary pressures of 30 kPa, 300 kPa and 3000 kPa and two displacement rates; $Ca=3.5 \times 10^{-7}$ (Fig. 12(a)) and $Ca=3.5 \times 10^{-2}$ (Fig. 12(b)). The pore shape is triangular and the contact angle is zero for all cases. The initial water saturation is in corner films for capillary pressures of 300 kPa and 3000 kPa with a small number of the smallest throats water filled for a capillary pressure of 30 kPa.

Fig. 12(a) shows that for low displacement rates there is sufficient time for all films to swell sufficiently for the competition between frontal displacements and snap-off to be largely unaffected by initial film thickness. This is certainly true for capillary pressures of 30 kPa and

![Fig. 10. Computed relative permeabilities as a function of pore shape for (a) $Ca=3.5 \times 10^{-7}$ and (b) $Ca=3.5 \times 10^{-2}$. Aspect ratio is 1.7, contact angle is 0 and initial capillary pressure is 300 kPa for both cases.](image)

where snap-off is suppressed by significant pressure gradients in the lower conductivity films. For triangular pores, where film conductivities are higher, the displacement is dominated by snap-off as indicated by the large number of distant pores. Square pores with intermediate conductivities display intermediate behaviour. The trend in residual saturations shown in Fig. 10(a) is not the result of a change in the nature of the displacement pattern but is due to the highly nonlinear relationship between capillary radius and pore volume characteristic of real porous media.

Fig. 10(b) shows the effect of pore shape on relative permeability and residual saturation at high displacement rates where snap-off is entirely suppressed by rate and the displacement is completely frontal. For these conditions the computed relative permeabilities and residual saturations are insensitive to the shape of pores and throats.
Computed relative permeability curves are shown for three displacement rates—the experimental rate of $Ca = 3.5 \times 10^{-6}$, a lower rate of $Ca = 3.5 \times 10^{-7}$ and a much higher rate of $Ca = 3.5 \times 10^{-2}$. Snap-off is completely suppressed for the higher rate case and the fit to the prediction of the end-point relative permeabilities and residual saturations is poor. The experimental displacement rate produces a satisfactory match given that the Berea network is based on a core plug having a permeability of approximately 2500 md and a porosity of 24% whilst the experimental data is for a core having a permeability of approximately 185 md and a porosity of 18%. Decreasing the rate by a factor of 10 has only a slight effect on the relative permeabilities but the prediction of the relative permeability end-point is not as good.

Although there are differences in the end-points the overall shape of the relative permeability curves for the different rates are similar. The relative insensitivity of the shape of the relative permeability curves to displacement rate is largely due to the low aspect ratio of the Berea network as discussed above.

Blunt et al. (2002) and Valvatne and Blunt (2003) report good agreement between the measured relative permeabilities of Oak (1990) and the predictions of a quasi-static model which ignores rate effects. It is significant that they required contact angles in the range 30–90° with an average contact angle of approximately 60°. Fig. 14 shows that the dynamic model with different combinations of lower contact angles and higher displacement rates can produce similar relative permeability curves. A contact angle of 0 and a rate of

300 kPa and is approximately true for the case of 3000 kPa where the initial film thickness is smallest. For the high displacement cases shown in Fig. 12(b), where snap-off is completely suppressed by rate effects, films play an insignificant role and the relative permeabilities and residual saturations are insensitive to initial capillary pressure.

3.2. Network model predictions of measured data for Berea sandstone

3.2.1. Relative permeability

Fig. 13 shows a comparison between measured oil–water relative permeabilities for strongly water-wet Berea sandstone reported by Oak (1990) and the predictions made with the network model using the Berea network with triangular pore shape. The contact angle which gives the best fit to the data is 45° and the initial capillary pressure is 300 kPa.

![Fig. 13. Comparison between computed relative permeabilities and measurements reported by Oak (1990) for strongly water-wet Berea sandstone. The measured displacement rate is $Ca = 3.5 \times 10^{-6}$, triangular pores, aspect ratio is 1.7, contact angle is 45° and initial capillary pressure is 300 kPa.](image)
Ca=3.5×10^{-4} produces a similar match to that obtained with a contact angle of 45° and the experimental displacement rate of Ca=3.5×10^{-6}. The higher contact angles required to match the quasi-static model used by Blunt et al. (2002) and Valvatne and Blunt (2003) are consistent with the results shown in Fig. 14 in that higher contact angles are required to suppress snap-off in the absence of the suppressive effect of displacement rate.

Fig. 15 shows the sensitivity of the dynamic model predictions to pore shape. Three regular shapes are considered—triangular, square and hexagonal. The displacement rate is Ca=3.5×10^{-6} for all the cases and the contact angle corresponds to the best fit to the measured data. As the half angle increases the contact angle for the best fit decreases. This is consistent with the effects of half angle and contact angle on snap-off—the increasing suppressive effect due to increasing half angle is countered by the decreasing effect due to decreasing contact angle. The end-point relative permeability for the wetting fluid is particularly sensitive to pore shape and we conclude that pore shape is an important factor in network model predictions. This is consistent with the findings of Blunt et al. (2002) and Valvatne and Blunt (2003) in their studies of quasi-static models.

### 3.2.2. Residual saturation

Chatzis and Morrow (1984) report measurements of residual saturations for the displacement of initially continuous oil from water-wet sandstone cores as a function of capillary number. Their data for three of their higher permeability Berea cores (CQ-5-ϕ=0.20, k=772 md; CQ-11-ϕ=0.21, k=859 md and CQ-A7-ϕ=0.21, k=852 md) are plotted in Fig. 16. The data shows that the normalised residual oil saturation, So/Sor⁎ (ratio of residual oil saturation, So, divided by the waterflood residual oil saturation at low capillary number, Sor⁎) reduces with increasing displacement rate. The reduction in residual oil saturation occurs at capillary numbers which are several orders of magnitude lower than capillary numbers for the onset of mobilisation. Chatzis and Morrow suggest that the reduction of residual oil saturation with displacement rate is due to a reduction in trapping with increasing rate.

![Fig. 14. Comparison between measured relative permeabilities reported by Oak (1990) for strongly water-wet Berea sandstone and computations for different combinations of contact angle and displacement rate. Triangular pores, aspect ratio is 1.7 and initial capillary pressure is 300 kPa.](image)

![Fig. 15. Comparison between measured relative permeabilities reported by Oak (1990) for strongly water-wet Berea sandstone and computations for different combinations of pore shape and contact angle. Aspect ratio is 1.7, capillary number is Ca=3.5×10^{-7} and initial capillary pressure is 300 kPa.](image)

![Fig. 16. Comparison between residual oil saturations for water-wet Berea cores reported by Chatzis and Morrow (1984) and the dynamic network for Berea shown in Fig. 13.](image)
Fig. 16 also shows a plot of computed residual oil saturations using the same Berea network model which was used to match the relative permeability data of Oak (1990) shown in Fig. 13. Again, given the variations in residuals displayed by the experimental data for different cores with similar porosities and permeabilities, the agreement between the measured and computed data is encouraging.

Tie and Morrow (2005) have recently reported the results of a rate sensitivity study for three outcrop limestones. They show that residual oil saturations for limestones display sensitivity to rate at capillary numbers much lower than that observed for consolidated sandstones. Their findings are consistent with the earlier studies of Kamath et al. (1995, 2001) for carbonate reservoir cores. Since no network information is available for the carbonate cores studied it is not possible to make meaningful comparisons between the measured residual saturation data and the dynamic network model. However, an important geometrical difference between carbonates and sandstones is pore–throat aspect ratio with carbonates displaying significantly larger aspect ratios than sandstones (Frank et al., 2005).

Fig. 17 shows the effect on residual oil saturation of increasing the aspect ratio from 1.7 to 6.8 for the Berea network. The higher aspect ratio case displays a rate sensitivity at capillary numbers several orders of magnitude lower than for sandstones where the aspect ratio is smaller. This is qualitatively similar to the differences between sandstones and carbonates measured by Chatzis and Morrow (1984), Kamath et al. (1995, 2001) and Tie and Morrow (2005), amongst others.

3.3. Relevance to laboratory measured relative permeabilities

Laboratory measured relative permeabilities for Berea sandstone have been used to validate the predictions of quasi-static network models (e.g., Blunt et al., 2002; Øren and Bakke, 2003; Øren et al., 1998; Patzek, 2001; Valvatne and Blunt, 2003). Our calculations show that the relatively low aspect ratio for Berea sandstone may make imbibition relative permeability for this type of rock insensitive to displacement rate and quasi-static and dynamic network models will therefore produce similar results for these conditions.

The laboratory studies which clearly conclude that imbibition relative permeabilities are independent of rate (Chen and Wood, 2001; Fulcher et al., 1985; Labastie et al., 1980; Odeh and Dotson, 1985; Qadeer et al., 1988) appear to have been conducted on strongly to intermediate water-wet Berea or similar high permeability homogeneous sandstones. The combination of low aspect ratio and moderate to high contact angles make relative permeabilities for these cores largely insensitive to displacement rate.

Kamath et al. (2001) and Mohanty and Miller (1991) measured rate-dependent relative permeabilities and end-point saturations for low permeability highly heterogeneous preserved state cores. The overall trend is for relative permeabilities to increase and residual saturations to decrease with increasing displacement rate. The same trend is clearly evident in the computed curves discussed above. Although the pore/throat aspect ratio for the cores tested is unknown, it is not unreasonable to assume that it would be higher than for Berea given the low permeability and heterogeneity of the cores. The higher aspect ratio would suggest a greater dependence of relative permeability and residual saturation on displacement rate than for Berea. This is particularly true for heterogeneous carbonate cores where aspect ratios are higher than for typical consolidated sandstones.

Relative permeability data of the type reported by Heaviside et al. (1987), Skauge et al. (2001) and Wang and Buckley (1999) is a little more difficult to resolve with the present network model. These data, mainly for sandstones, display little or no dependence of relative permeability and end-point values for strongly water-wet conditions but do show rate sensitivity for mixed-wet conditions. The present network model assumes uniform wettability conditions and does not include the additional pore-scale physics introduced by mixed wetting conditions which can result in the simultaneous existence of both water and
oil films (e.g., Blunt et al., 2002; Øren and Bakke, 2003; Patzek, 2001).

Finally, in attempting to reconcile differences between different studies it is important to recognise the real experimental difficulties in measuring reliable relative permeability data in the laboratory. McPhee and Arthur (1994) report a comparative study of four service laboratories using similar methods and cores. Measured relative permeability end-points for imbibition and swelling is readily incorporated into existing quasi-static models. The model was used to study the complex dependence of imbibition relative permeability on displacement rate. On the basis of this study we make the following conclusions:

1. The swelling of wetting corner films is a capillary pressure driven nonlinear diffusion process which is largely independent of displacement rate. The time for filling of pores and throats depends directly on displacement rate. This is the essence of the rate-dependent competition between frontal displacements and snap-off ahead of the displacement front.
2. Displacement rate and contact angle affect the shape of imbibition relative permeability curves in a similar manner. Increasing rate and increasing contact angle suppress snap-off and result in more favourable relative permeability curves and lower residual saturations.
3. The sensitivity of relative permeability to displacement rate and contact angle depends on the pore/throat aspect ratio. The sensitivity to rate is high for large aspect ratios and low for small aspect ratios.
4. The complex interaction between displacement rate, contact angle, aspect ratio and pore shape makes it difficult to correctly interpret measured imbibition relative permeability and residual saturation data without the aid of a dynamic network model based on a realistic description of film flow.

4. Conclusions

A new network model for imbibition based on a physically realistic description of the dynamic nature of imbibition has been described. The model reproduces the flow regimes observed in micromodel displacement experiments for a wide range of displacement rates and predicts relative permeabilities and residual saturations which compare favourably with measured data for Berea sandstone. The treatment of film flow and swelling is readily incorporated into existing quasi-static models.

The model was used to study the complex dependence of imbibition relative permeability on displacement rate. On the basis of this study we make the following conclusions:

1. The swelling of wetting corner films is a capillary pressure driven nonlinear diffusion process which is largely independent of displacement rate. The time for filling of pores and throats depends directly on displacement rate. This is the essence of the rate-dependent competition between frontal displacements and snap-off ahead of the displacement front.
2. Displacement rate and contact angle affect the shape of imbibition relative permeability curves in a similar manner. Increasing rate and increasing contact angle suppress snap-off and result in more favourable relative permeability curves and lower residual saturations.
3. The sensitivity of relative permeability to displacement rate and contact angle depends on the pore/throat aspect ratio. The sensitivity to rate is high for large aspect ratios and low for small aspect ratios.
4. The complex interaction between displacement rate, contact angle, aspect ratio and pore shape makes it difficult to correctly interpret measured imbibition relative permeability and residual saturation data without the aid of a dynamic network model based on a realistic description of film flow.

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