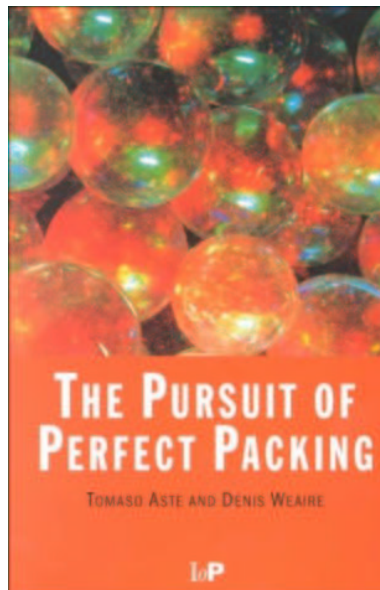


The Subtleties of Space Filling

Book Review by Robert V. Moody, University of Alberta

The Pursuit of Perfect Packing by Tomaso Aste & Denis Weaire

Institute of Physics Publishing, Bristol and Philadelphia, 2000



Given that we are all born and live out our lives in a 3-dimensional world, it is incredible how little we know about it. The number of tantalizingly easy-to-state impossibly-hard-to-solve problems around filling space with similarly shaped objects of various types is probably on a par with the famous puzzles of number theory. Yet packing problems have never had the mystique nor the avid following of number theory. It is for certain that there is no correspondingly elegant theory or canon of sophisticated techniques and ideas. Is this our failure of intuition or is it that these problems are intrinsically less ordered and harder?

What space filling loses in the way of theory it gains in its colourful history. Nature abounds in space filling devices, and from ancient times to the present these have caught the imaginations of scientists, mathematicians, and dilettantes.

To quote the authors of the *Pursuit of Perfect Packing*:

“The history of ideas about packing is peopled by many eminent and colourful characters. An English reverend gentleman is remembered for his experiments in squashing peas together in the pursuit of geometrical insights. A blind Belgian scientist performed by proxy experiments that laid down the ground

rules for serious play with bubbles. An Irishman of unrivaled reputation for dalliance (at least among crystallographers) gave us the rules for random packing of balls. A Scotsman who was the grand old man of Victorian science was briefly obsessed with the parsimonious partitioning of space.

All of them shared the curiosity of the child at the church bazaar: how many sweets are there in the jar?"

None of these characters is more famous, nor more interesting, than Johannes Kepler. Kepler was a scientist of the first order whose feet seemed to stand firmly both in the scientific age, just dawning, and the receding age, possessed with its medieval sense of mystery and awe. It was Kepler's interest in the hexagonal form of snow-flakes that led him to look at the densest packing of spheres and to the now famous Kepler problem. As has often been said, every mathematician believes and every physicist knows that the usual cannonball stacking in hexagonally packed layers is the best. In fact this is not a unique packing: each successive layer has two ways of being placed on the former, but they all have the same density. It is quite astounding that even now, in 2001, the final verdict on this is not in, though there seems to be considerable confidence in Thomas Hales' announced (computer aided) solution to it (<http://www.lsa.umich.edu/hales/countdown>).

In the same work, Kepler uncovers two famous semi-regular figures, the rhombic-dodecahedron and the lovely triacontrahedron with its 30 rhombic faces. Kepler was also the first person to think about tiling the plane with figures based on 5-fold symmetry, and his splendid aperiodic tiling is a 400 year old precursor to the famous Penrose tiling and the world of quasicrystals.

In the *Pursuit of Perfect Packing*, Tomaso Aste and Denis Weaire take us on a tour of this familiar, yet not quite so familiar, world, with a blend of mathematical insight, historical incident, and just plain good fun. This is serious mathematics, written in an engaging and amusing style, that ranks at a high level for expository science writing. The authors are both physicists, which only serves to underline the fact that good mathematics and good science are never far apart.

One of the authors (D. Weaire, FRS) has the distinction of being a main contributor in defeating the 100 year old Kelvin conjecture. It was the blind Plateau who, with the help of his visually unchallenged friends, laid down the geometric principles of soap bubbles. Later Lord Kelvin, in his musings on the (long defunct) ether, speculated that it might be a foam and was led to the problem of determining the shape of the cells, equal in volume but minimizing surface area that would fill space. He quickly came up with the space-filling Kelvin cell (already known by Leonardo) as the solution. Doubt continued however, since in real foams the ideal Kelvin cell is nowhere to be found! Weaire and Phelan, using Brakke's surface evolver (computer software) and some very solid hints from Nature itself, brought an end to the Kelvin conjecture (but the resulting foam is a mixture of 12 and 14 sided cells). (A search of Weaire's web-site provides much more information on this, as well as plenty of examples of his Irish wit).

All this and many other marvelous tales, from honeycombs, lattices, sausages,

dimples on golf balls, squashed peas, Apollonian packings, quasicrystals, buckballs, abound in this little book.

Who would enjoy the of Perfect Packing? I think just about everyone who is mathematically inclined. It is visually appealing (almost every page has some engaging picture) and easy to browse. It is not, however, uniformly easy to read. The authors' mathematical expectations of the reader vary greatly from page to page. Those who want to really understand what is going on will have to go elsewhere. Still, I think that the result is a good compromise.

It conveys the sense of order, structure and wonder in mathematics and Nature that is the life-blood of mathematicians and scientists and a sense of the excitement and humanness that accompanies all research and experiment.

Finally it leaves in no doubt how little we know about the space we live in and the subtleties of Nature.

The book would make a fine gift or prize for anyone who is mathematically inclined, from high school students upwards.