Stability Analysis of Part Time Investor Strategy in 10 Player Minority Game.

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The performance and stability of the part time investor strategy in the ten player minority game was investigated. For trials ranging in length from 10 rounds to 10 000 rounds, a parameter value was found that was able to provide a significant improvement in global wealth to the case where all agents picked at random. The part time investor strategy was found to be stable once it had become established, but not in the initial period of the trials.

1 Introduction.

Since their introduction by Arthur in the form of the El Farol Bar problem [1], minority games have received considerable interest due to their potential to model complex systems such as the stock market.[2] The simplest minority game involves $N_a$ agents, that each round must choose between two options, 0 and 1. The agents are then awarded one point if they choose the minority number, that is the number picked by the least number of agents. The agents are also provided with the minority numbers for the previous $M$ rounds, providing a feedback mechanism whereby results of previous rounds may influence future behaviour.

This form of minority game may be extended by allowing the agents to choose between $n_o$ different options, denoted by the integers 1 to $n_o$. While in the 2 option minority game it is possible to ensure there is a unique minority number by only allowing an odd number of agents, there is no comparable mechanism for games with more options. Having a unique minority outcome is desirable as it places an upper bound on the global winnings per round; the number of agents picking the minority number must be less than the average number of agents picking a number. Hence:

$$w_g \leq \frac{N_a}{n_o}$$

where $w_g$ is the global winnings per round. Therefore, in the following analysis, if there is more than one possible minority number, a single minority number will be chosen from this group at random, and only agents who had chosen that number will be awarded points.

Most studies of the minority game, including those which have extended the game to allow the agents to choose between multiple options[3], have allocated agents a fixed number of strategies from an abstract strategy space. Agents then evaluate the past performance of all the strategies allocated to them, and choose their number for the next round according to their best performing strategy.
**Part Time Investor Strategy.**

1. In the first round, the 100 agents each pick a number between 1 and 10 at random.
2. Before each subsequent round, the agents have a probability $\beta$ of doing a strategy evaluation.
3. In a strategy evaluation, they will check if they have won in more than 10% of the rounds since their last strategy evaluation. If this is not the case, they will change to the number that did best in the previous round.
4. If they are satisfied with how their strategy has performed, or if they did not do a strategy evaluation, they will keep their number from the previous round.

Table 1: Strategy used by each agent in the part time investor simulation.

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<th>Strategy used by each agent in the part time investor simulation.</th>
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| Such models show a key relationship between the diversity of strategies allocated to each agent, and the value of $w_g$ obtained. If the agents all have similar strategies, then, as they all choose the strategy that has performed best in the past, the agents will act in similar ways in future rounds and therefore the number of agents choosing the minority number will be lower than if the agents all picked at random. [4][5] As the diversity increases, fewer agents have the same strategy, and a better than random global wealth is achieved. If the number of strategies in the space is too large however, the agents are no longer allocated a sufficiently diverse set of strategies, and therefore do not gain much from choosing amongst them.[4]

Recently however, a model was proposed in the Complex Systems Project by Lal et al [5] that relies on each agent using the same strategy but also achieves high global wealth. The details of this strategy, which will hereafter be referred as the part time investor strategy, are shown in table (1). In a simulation involving 100 agents, 10 options, 50000 rounds and $\beta$ set to .02, an average total winnings per agent of $4440 \pm 20$ was achieved.[5] This corresponds to an average global winnings per round of $w_g = 8.88 \pm .04$, which compares favourably to the theoretical maximum of 10. The results are also a significant improvement over the case in which the agents pick at random, for which the average global winnings per round was found to be $w_g = 5.46 \pm .01$, again from a simulation involving 100 agents and 50 000 rounds.[5]

This strategy is worthy of further attention, not only because of its success in maximising global wealth, but also because it resembles the strategies used by part time investors in the stock market. Such investors will choose companies in which to invest, and will remain with these companies for extended periods of time, only occasionally checking on how their investments are performing. In comparison, traders follow the market on a minute by minute basis. The success of the part time investor strategy therefore poses an interesting question: given that in the ten player minority game, being a part time investor is so successful, why is there a place for traders in the actual stock market? This project will go part way towards answering this question, by examining the optimum values of $\beta$, and how stable these strategies are.

### 2 Optimum Values of $\beta$.

Under the part time investor strategy, investors change the number they are picking relatively infrequently, meaning the distribution from the previous round is a very
good predictor of the distribution in the following round. The few agents that change are therefore able to use this information to flatten the distribution, that is approach the case where equal numbers of agents pick each number.

While lower values of $\beta$ mean less people are changing each round, and should therefore achieve a flatter eventual distribution, it also takes longer to reach this point. This relationship is shown in figure (1), which shows the average wealth per player per round as a function of $\beta$, for trials involving different numbers of rounds. As can be seen, for trials lasting only ten rounds, the global winnings are optimised by setting $\beta$ around .03. Such a bias against small $\beta$ values continues until about 5000 rounds, at which point global wealth it is globally best to choose the lowest value of $\beta$ considered, which in this case was .001. Also shown is the average winnings achieved when the agents are choosing at random, which performed worse in the range of $\beta$ considered for all numbers of rounds.

Also recorded in this simulation was the standard deviation of the agents winnings, which is shown in figure (2). As may be expected lower values of $\beta$ have a higher standard deviation, as due to the low level of change each round, agents that win in one round, are very likely to win in the next. The standard deviation is also of interest, as it is supposed that if a large number of agents are doing much worse under a given strategy, they would be better off using a different strategy, hence affecting the stability of the solution.


Guided by the optimum values calculated in the previous section, the stability of these solutions were investigated from a Nash Equilibrium perspective. In the following simulations, 99 agents, referred to as the main group, act according to the part time investor strategy with a fixed value of $\beta$. The 100th agent also acts according to the part time investor strategy, but their choice for the value of the parameter $\beta_{100}$ is varied. The average winnings per round of agent 100 were then compared to the average winnings per agent per round for the main group.

Figure (3) shows such a simulation where $\beta = .01$ for the main group, and each
trial consisted of 10 000 rounds. For each value of $\beta_{100}$, 10 trials were run and the plotted values are averages. Somewhat surprisingly, agent 100 performs better than the main group when $\beta_{100} < .01$ and worse otherwise.

The reason for this performance can be seen if we consider the following distribution of choices: eleven people choose 1, nine people choose 2 and 10 people choose each of the numbers 3 to 10. Such a distribution is somewhat flatter than those observed for $\beta = .01$, however was observed when $\beta = .001$. An agent who has picked 3, will not be satisfied with their number, as it is not the minority number, however they can not improve their position by changing. The position of such an agent will only improve if another agent who has picked the same number changes. It is therefore difficult to conceive of a ‘trader’ strategy, that is one involving frequent changing, which will yield higher winnings than the part time investor strategy once the flat distribution has been reached.

It is however possible to find strategies that improve the performance of the strategy over short time periods, as can be seen in figures (4) and (5). When the main group has a $\beta$ value of .001, much less than the optimum value of about .03 from figure (1), there is a clear incentive to take advantage of the slow rate of change in the distribution and to perform a strategy evaluation as frequently as possible. When $\beta = .03$ however, this trend is reversed, and it is disadvantageous to change more frequently.

Figure (5) does not imply there is no strategy that is able to better the part time investor strategy if $\beta > .03$. If agent 100 decides to change to the second least picked number, but otherwise behaves according to the part time investor strategy, they are able to outperform the part time investor strategy with $\beta = .04$, as can be seen in figure (6).

4 Conclusions and Future Areas for Exploration.

From simulations of the part time investor strategy, it was demonstrated that for trial sizes ranging from 10 to 10 000 rounds, there was a value of $\beta$ for which the part time investor strategy represented a significant improvement over the case where agents pick at random. Once these strategies become established, they were also seen to be
Figure 3: Nash Equilibrium style stability calculation for $\beta = .01$ and trials lasting 10 000 rounds. In the simulation, 99 agents used the part time investor strategy with $\beta = .01$, while agent 100 varied their choice of $\beta_{100}$. For each value of $\beta_{100}$, ten trials were performed, and the plotted values are the averages over these trials.

Figure 4: Nash Equilibrium style stability simulation for $\beta = .001$ and trials lasting 10 rounds. In the simulation, 99 agents used the part time investor strategy with $\beta = .001$, while agent 100 varied their choice of $\beta_{100}$. For each value of $\beta_{100}$, one thousand trials were performed, and the plotted values are the averages over these trials.
very stable, with little opportunity for a trader to achieve an increase in their winnings by changing more regularly.

In the initial period, there is, however, the opportunity for traders to significantly outperform the part time investors. It would therefore be interesting to see whether the strategy could be modified to reduce this initial instability. Possible modifications include making the value of $\beta$ vary with time, starting with a higher value around .03 and eventually reaching .001, and allowing the agents to choose between the less picked numbers when they change, rather than always choosing the minority number. The results of these simulations would provide some indication as to whether there is a stable strategy like the part time investor strategy, or whether the initial instability means it is unlikely to become the dominant strategy in a minority game where agents are able to choose between different types of strategies.

There are also a number of features of the minority game considered that are not present in real world examples such as the stock market. Firstly, each of the 100 agents participate in every one of the rounds, while in real world minority games there would be some agents who are forced to sit out of some rounds, and others that stop playing at some point and are replaced by new agents. Such changes reduce the predictability of the game, which should then reduce the performance of the part time investor strategy. This could be investigated by assigning the agents a probability that they will not choose a number (agent is unable to participate in a round) or choose a new number at random (agent leaves game and is replaced).

The stock market, and other systems that may be modelled by minority games, are also affected by external events, outside the collective control of the agents. In the presence of such events, it seems likely that traders would have the advantage, as they are able to adapt more rapidly to changing conditions. It is proposed that such
Figure 6: Nash Equilibrium style stability simulation for $\beta = .04$ and trials lasting 10 rounds. In the simulation, 99 agents used the part time investor strategy with $\beta = .04$, while agent 100 varied their choice of $\beta_{100}$, and changed to the second least picked number instead of the minority number. For each value of $\beta_{100}$, one thousand trials were performed, and the plotted values are the averages over these trials.
conditions may be investigated using a weighted minority game. In this game, each number is assigned a weighting, and the winning number is the one for which the ratio of the number of people choosing the number to the number’s weighting is smallest. The weightings could then be changed during a trial, thereby altering the conditions under which the agents are operating.

References


