Complex Matter
HOW THINGS PACK (CLOSELY) IN SPACE?
Huygens
_Trattè de la lumière_ (1690)

Kepler
_De Nive Sexangula_ (1611)

Haüy
_Essai d'une théorie sur la structure des cristaux_ (1784)

Wollaston (1737-1815)

Barlow,
_On the probable nature of the internal symmetry of crystals_ (1883)
the laws of nature tend to produce order
Disorder is NOT randomness

Order is everywhere

A certain amount of defects can be introduced and classified

C₄

C₆

Structures can be classified in a compact way

Order is efficient

There are local motifs which *repeat* (a-periodically) in space.

They are organized hierarchically.

They results in efficient configurations both geometrically and topologically.

\[ \rho = 0.740... \]

\[ \rho = 0.55-0.64 \]
La Ricerca
dell’Impaccamento Perfetto

Quale è la massima densità ottenibile con un impaccamento di sfere identiche?

1591  T Harriot
1611  J Kepler
1840  K Gauss
1900  D Hilbert
1959  C Rogers
1964  F Toth
1988  J Conway, N Sloane
1998  T Hales

"It’s one of those problems that tells us that we are not as smart as we think we are"  D J Muder
hcp
fcc
Configurazioni locali e impaccamenti globali

\[ \rho = 0.7405... \]

\[ \rho = 0.754... \]
Dear colleagues,

I have started to distribute copies of a series of papers giving a solution to the Kepler conjecture, the oldest problem in discrete geometry. These results are still preliminary in the sense that they have not been refereed and have not even been submitted for publication, but the proofs are to the best of my knowledge correct and complete.

Nearly four hundred years ago, Kepler asserted that no packing of congruent spheres can have a density greater than the density of the face-centred cubic packing. This assertion has come to be known as the Kepler conjecture. In 1900, Hilbert included the Kepler conjecture in his famous list of mathematical problems.

In a paper published last year in the journal "Discrete and Computational Geometry", (DCG), I published a detailed plan describing how the Kepler conjecture might be proved. This approach differs significantly from earlier approaches to this problem by making extensive use of computers.

(L. Fejes Toth was the first to suggest the use of computers.)

The full proof appears in a series of papers totalling well over 250 pages. The computer files containing the computer code and data files for combinatorics, interval arithmetic, and linear programs require over 3 gigabytes of space for storage.

Samuel P. Ferguson, who finished his Ph.D. last year at the University of Michigan under my direction, has contributed significantly to this project.

The papers containing the proof are:

An Overview of the Kepler Conjecture, Thomas C. Hales
A Formulation of the Kepler Conjecture, Samuel P. Ferguson and Thomas C. Hales
Sphere Packings I, Thomas C. Hales (published in DCG, 1997)
Sphere Packings II, Thomas C. Hales (published in DCG, 1997)
Sphere Packings III, Thomas C. Hales
Sphere Packings IV, Thomas C. Hales
Sphere Packings V, Samuel P. Ferguson
The Kepler Conjecture (Sphere Packings VI), Thomas C. Hales

Postscript versions of the papers and more information about this project can be found at

http://www.math.lsa.umich.edu/~hales

Tom Hales
POLIDISPERSIÀ

\[ \rho = 1 - \left( \frac{r_{\text{min}}}{r_{\text{max}}} \right)^{d-d_f} \]

\( \rho \rightarrow 1 \)

quando

\[ \frac{r_{\text{min}}}{r_{\text{max}}} \rightarrow 0 \]
‘IL BACIO PRECISO’

Four pairs of lips to kiss maybe
Involves no trigonometry.
(…)

Four circles to the kissing come,
The smaller are the bender,
The bend is just the inverse of
The distance from the center.
Though their intrigue left Euclid dumb.
There’s now no need for the rule of thumb.
Since zero bends a straight line
And concave bends have minus sign,
The sum of the squares of all four bends
Is half the square of their sum.

- 1643 Principessa Elisabetta di Boemia
- 1842 Philip Beecroft
- 1936 F. Soddy, (“The Kiss Precise”, Nature 137)
- 1937 T. Gosset, (“The Kiss Precise”, Nature 139)

\[
d \left[ \left( \frac{1}{r_1} \right)^2 + \left( \frac{1}{r_2} \right)^2 + \left( \frac{1}{r_3} \right)^2 + \ldots + \left( \frac{1}{r_{d+2}} \right)^2 \right] = \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \ldots + \frac{1}{r_{d+2}^2}
\]
Imagine a circle; inscribe within it three circles congruent to each other and of maximum radius; proceed similarly within each interval within them, and imagine that the process continues to infinity.

G. Leibniz (1790 c.a) to Brosse

\[ \frac{1}{r_i} \quad \frac{r_i}{r_{i+1}} \rightarrow 2.89 \]

Invarianza di Scala $\rightarrow$ Progressione Geometrica

\[ x = \frac{r_i}{r_{i+1}} \]

\[ x > 1 \quad \text{in generale} \]

Packing di Apollonio

\[ d = 2 \quad x = \tau + \sqrt{\tau} = 2.89 \]
\[ \tau = \frac{1 + \sqrt{5}}{2} = 1.618 \]

\[ d = 3 \quad x = \frac{1}{2} + \frac{1}{\sqrt{2}} + \sqrt{\frac{1}{\sqrt{2}} - \frac{1}{4}} = 1.8832 \]
Ricoprimento progressivo

\[ R_{\nu} = x^{-\nu} R_0 \]

\[ N_{\nu} = \left( 1 + \frac{n_{d,0}}{n_{0,d}} \right)^{\nu} N_0 \]

- \( n_{3,0} \) numero medio di sfere in contatto con una data sfera centrale
- \( n_{0,3} \) numero medio di sfere attorno ad un interstizio

Apollonio:

<table>
<thead>
<tr>
<th>( n_{2,0} )</th>
<th>( n_{3,0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>10.66...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( n_{0,2} )</th>
<th>( n_{0,3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Legge di Eulero:

\[ V - E + F = 1 \quad d=2 \]
\[ V - E + F - C = 1 \quad d=3 \]
Distribuzione Granulometrica

\[ N(R) = \left( \frac{R_0}{R_V} \right)^\alpha N_0 \]

\[ \alpha = \frac{\ln\left( \frac{n_{d,0}}{n_{0,d}} \right)}{\ln(x)} \]

D Bentz J. Am. Ceram. Soc. 80 1997

Dimensionalità Frattale

struttura che si ripete simile a se stessa su varie scale di grandezza

\[ d_f = \alpha \]

(Sierpinski 1915)

\[
\begin{align*}
    n_{2,0} &= 6 \\
    n_{0,2} &= 3 \\
    x &= 2 \\
    d_f &= 1.58
\end{align*}
\]

Apollonio

\[
\begin{align*}
    d_f &= 1.305 \quad \text{per } d=2 \\
    d_f &= 2.50 \quad \text{per } d=3
\end{align*}
\]

Max Pack

\[ d_f < d \]

de Gennes 1973

\[
\begin{align*}
    n_{2,0} &= 6 \\
    n_{0,2} &= 3 \\
    x &= 3 \\
    d_f &= 1
\end{align*}
\]

BB Mandelbrot “The Fractal Geometry of Nature” 1977
\[ R_{V} = x^{-\alpha} R_{0} \quad N(R) = \left( \frac{R_{0}}{R_{V}} \right)^{\alpha} N_{0} \quad \alpha = \frac{\ln \left( \frac{n_{d,0}}{n_{0,d}} \right)}{\ln(x)} \]

\[
\rho = \frac{4\pi}{3} \sum_{\nu=0}^{V^*} N(R_{\nu})R_{\nu}^d = 1 - \left( \frac{R_{V^*}}{R_0} \right)^{d-d_f}
\]

\[
p = 1 - \rho = \left( \frac{R_{\text{min}}}{R_{\text{max}}} \right)^{d-d_f}
\]

**Equazione Empirica**


\[ p = \left( \frac{r_{\text{min}}}{r_{\text{max}}} \right)^{\frac{1}{5}} \]

\[ d_f = 2.8 \]