Cohen, Erez, ben-Avraham, and Havlin Reply: In our Letter [1] we studied the resilience of scale-free networks to intentional attack (deletion of the most highly connected nodes). Our main result is a formula for $p_c$—the fraction of most connected sites that must be removed before the network collapses—which follows from Eqs. (8) and (11) [1]:

$$p_c^{\frac{2-a}{2}} - 2 = \frac{2 - \alpha}{3 - \alpha} m(p_c^{\frac{2-a}{2}} - 1).$$

(1)

This was derived under the assumption that $P(k)$, the probability that a site has $k$ connections, is modeled by the continuous distribution

$$P(k) = ck^{-\alpha}, \quad m \leq k \leq K,$$

(2)

where $c$ is a normalization constant, and $m$ and $K$ are lower and upper cutoffs for the site connectivity, respectively. In practice, though, a site may have only an integer number of connections. Indeed, in our simulations [1] we have used the discrete distribution

$$P_1(k) = \int_{k-1/2}^{k+1/2} P(q) \, dq.$$

The analytical formula of Eq. (1) provides an excellent approximation to results from simulations performed with the distribution $P_1(k)$; see Fig. 1 in [1].

The Comment’s [2] main claim is that in [1] we did not compare our results to the discrete distribution [3,4]:

$$P_{11}(k) = k^{-\alpha}/\xi(\alpha), \quad k = 1, 2, \ldots.$$  

(4)

Following our theory, the authors of the Comment show that $p_c$, for the distribution $P_{11}(k)$, is given by the solution to the set of equations:

$$\sum_{k=1}^{K} k^{3-a} = \xi(\alpha - 1) + \sum_{k=1}^{K} k^{1-a}, \quad (5a)$$

$$p_c = 1 - \sum_{k=1}^{K} k^{-\alpha}/\xi(\alpha). \quad (5b)$$

Because the authors of the Comment regard the distribution $P_{11}(k)$ as “genuine” compared to $P_1(k)$, they view with alarm the differences in $p_c$ obtained from the two distributions.

We observe that (a) $P_1$ and $P_{11}$ are equal, asymptotically, in the limit of large $k$ and (b) the differences are most pronounced for $k = m$, where $P_1(1)$ is quite smaller than $P_{11}(1)$. The difference in $p_c$ between our approach and Ref. [4] is mainly due to the values of $P(k)$ for small $k$ and is not related to the type of approximation, continuous or discrete. More details will be forthcoming [5].

Moreover, we strongly disagree that, in the context of the Internet, $P_{11}$ is more original than $P_1$. While it has been firmly established that $P(k) \sim k^{-\alpha}$ for large $k$ [6], which is valid for both $P_1(k)$ and $P_{11}(k)$, the distribution for small $k$ has not been explored. In this limit, the distribution is most fluid, due to computers connecting and detaching from the net. Our aim in [1] has been merely to explore the effect of the scale-free tail (at large $k$).

Surely, the simplicity of Eq. (1), vis-à-vis Eqs. (5), more than makes up for any conceivable aesthetic advantage of $P_{11}$ over $P_1$. The use of the distribution $P_1$ (and its continuous analog) is more than worthwhile.

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[3] Note that the Comment specializes to lower cutoff $m = 1$.