Nonlinear Dynamics: From Lasers to Butterflies

Lectures from the 15th Canberra International Physics Summer School, 2002

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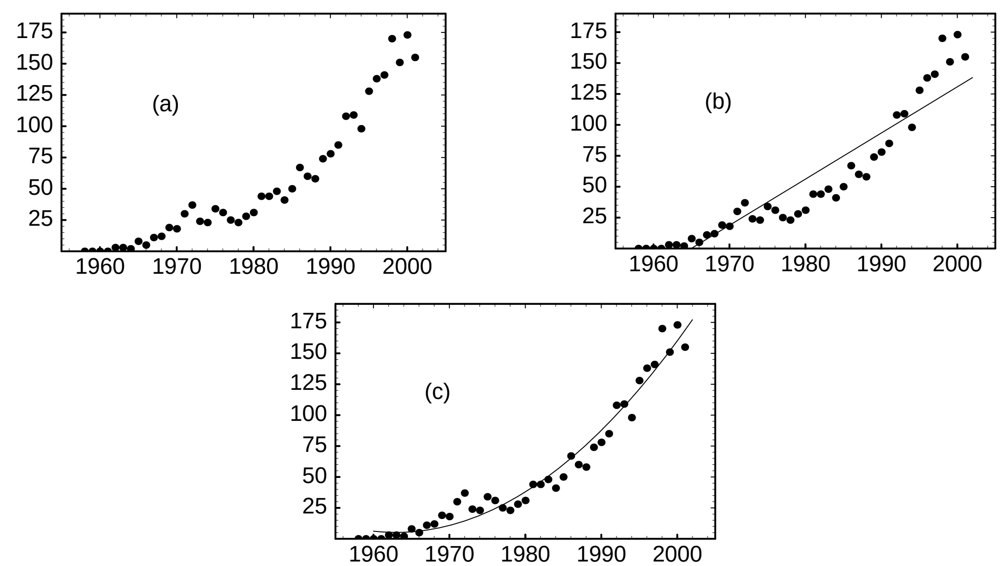
Preface

Do you sometimes hear or read grand pronouncements such as “The world is essentially nonlinear”? Or maybe you have heard that famous smart remark, to the effect that nonlinear dynamics is “like defining the bulk of zoology as the study of of non-elephants”, attributed to the mathematician Stanislaw M. Ulam[1] (who, however, failed to provide convincing proof that elephants are linear) — the point being that linearity is the exceptional case.

Over the last three or four decades interest in nonlinear phenomena has certainly grown nonlinearly, as you can see in the graphs. The data represent the number of times the



Frequency of the word “nonlinear” (vertical axis) versus year (horizontal axis)

[1] David K. Campbell: Nonlinear Science from Paradigms to Practicalities. In: From cardinals to chaos: reflections on the life and legacy of Stanislaw Ulam, ed. Necia Grant Cooper, Cambridge University Press, 1989, p. 218.

word "nonlinear" occurs in the titles and abstracts of articles in Physical Review Letters, a leading broad-spectrum physics journal, for each of the years between 1958 and 2001. The data do not look very linear (a). In fact a linear fit is really not very good (b), and a nonlinear fit is obviously much better (c). (Of course a least-squares fit does not model fluctuations in the data.)

Another inference that can be made from these data is that nonlinear science is increasingly multi-field and multi-disciplinary. A selection of keywords from these articles from 1999–2001 includes terms such as solitons, earthquakes, neural nets, photorefractive materials, laser cavity resonators, lattice dynamics, plasma transport, optical communications, chaos, information encryption and decryption, patterns, Bose-Einstein condensates, superconductivity, gravitational waves, black holes, quasicrystals, thin liquid films, the cochlea, quantum computing, and carbon nanotubes.

Many of the leading-edge research fields associated with these keywords have grown up synergistically with advances in the theory and mathematics of nonlinear dynamics. In disciplines such as the bio-medical sciences and earth sciences many fields have been given new impetus and direction from lateral applications of the paradigms of nonlinear dynamics that are described by the authors of the chapters in this volume.

In a sense this exponential growth of interest in nonlinear phenomena is unique in the history of physics. It is not driven by defence and cold war imperatives, as for example was nuclear physics in the 1950s and 60s, or industrialization, as was thermodynamics in the 19th century. What we are seeing now is the reverse: nonlinear theory and mathematics is actually driving developments in a wide range of very diverse fields, from lasers to butterflies, optics to ecosystems, neuroscience to climatology to engineering.

Such marvellous universality of nonlinear dynamics is all very well, but if, like most graduate students and researchers, you have emerged from the undergrad subspace of simple pendulums and first-order reactions, you might ask: how, exactly, does one begin to describe the dynamics and physics of a nonlinear world?

This book is intended for those students and researchers. It is a collection of lectures that were delivered by nine eminent scientists at DynamicSummer: the 15th Canberra International Physics Summer School, in January 2002. The topics covered in the lectures collectively provide an inspirational introduction to modern research directions and scholarship in nonlinear dynamics, at the same time as guiding the graduate student through the fundamental concepts and methodologies. With each chapter having a comprehensive bibliography, the book will also be a valuable reference for researchers already delving into matters nonlinear. In the overall content a balance is achieved between theory, modelling, mathematics, and experiments. Applications and examples chosen by the authors are drawn from mathematical and physical sciences disciplines.

The association between nonlinear dynamics and zoology noted by Ulam is reflected in several aspects of the book. The butterflies mentioned in the title are of two species. The first kind is introduced in Chapter 1 by Brian Davies. This is the famous Lorenz butterfly, a metaphor for the amazing sensitivity of deterministic nonlinear evolutions to initial conditions. In Chaper 2 this butterfly is also mentioned by Nalini Joshi, who goes on to show that it is but one species of a vast family of unstable and mischievous butterflies, many of whom hide deep within the asymptotic solutions of innocent-looking, ordered, integrable systems that are important in physics, such as Painlevé equations.

A classic problem of Hamiltonian dynamics, the Fermi problem of a particle bouncing between a fixed and an oscillating wall, is described by Mike Lieberman in Chapter 3. With the aid of Poincaré return maps, a useful tool of nonlinear dynamics that was introduced in Chapter 1, it is shown how the Fermi acceleration model descibes collisionless electron heating in industrial plasmas. Hamiltonian dynamics remain the theme of Chapter 4, in which Cathy Holmes compares model and experimental dynamics for two systems, cold atoms in a periodic potential coupled Bose-Einstein condensates of alkali gases.

The next few chapters eludidate various aspects of that perennial problem of nonlinear science: turbulence. In Chapter 5 a new analysis of wave turbulence is presented by Biven, Connaughton, and Newell. Renormalized closure theory is elucidated in Chapter 6 by Jorgen Frederiksen, and its applications for subgrid-scale parameterizations of two-dimensional turbulence simulations are developed. Tony Roberts in Chapter 7 describes a flexible, low-dimensional approach to modelling of flows using centre manifold theory.

In the final chapters there is a more explicit theme-shift to the spatial structures that grow out of nonlinear processes. Weiss and co-workers in Chapter 8 discuss the beautiful and intriguing vortices and solitons found in non-linear optical resonators. Mathematical treatments of solitons are developed in Chapter 9 by Ablowitz, Hirooka, and Musslimani, with applications to an optical beam in a waveguide array and optical fiber communications. In Chapter 10 Ercolani, Indik, Newell, and Passot show that pattern formation can be described in terms of the Cross-Newell phase diffusion equation, and incidentally proving that elephants, which are qualitatively hinted at in figure 9 of Chapter 10, are actually nonlinear.

This book could not have been prepared without the valued input and assistance of a great many people. As Convenor of DynamicSummer I would like to thank them all a thousand times.

To the lecturers, all internationally recognised champions in their research fields, who gave their time and expertise so freely, communicated their lectures with such clarity and inspirational quality, and prepared the generously written and richly illustrated manuscripts that form this book — thank you all.

To the participants of DynamicSummer, the students and researchers whose interest and enthusiasm drove them to come from many parts of the world to learn, special thanks are owed. The tutelage and nurturing of new talent in nonlinear dynamics was the major motivation for this event and for this book, and the liveliness and enthusiasm of the participants ensured that DynamicSummer developed a memorable dynamics of its own.

Sincerest thanks go to the other three members of the DynamicSummer Production Team: Prof. Nail Akhmediev — Associate Editor and Casting Director, Dr Vanessa Robins — Technical Director, Site Coordinator, Schedule Planner, and Continuity

Manager, and Prof. Bob Dewar, Casting Director, Expert Adviser, and Finance and Budget Officer (for those students requesting financial assistance), and Complaints Officer (for those students requesting financial assistance). I am also deeply grateful to Nyssa Gyorgi-Faul and Heli Jackson for their administrative expertise, and Dr Mihajlo Mudrinic, who managed our web site.

Finally I thank, in advance, the readers of this volume. Some of you will have already heard, as DynamicSummer participants, the lectures on which the chapters are based. I have no doubt that these readers will enjoy dipping into the volume, revisiting topics that they so often engaged the lecturers in lively discussions and arguments. That is good — learning should be enjoyable and argumentative. For other readers this book will be a guided but exciting adventure into the nonlinear realm. I am confident that all readers will see that opportunities for students who are trained in methods of tackling nonlinear problems are rich and diverse, that non-elephants are on the increase, and that many more butterflies await discovery.

Rowena Ball, Convenor, DynamicSummer 2002.

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