PHYS3001 Classical Mechanics

Examination: 9:00 am – 11:15 am, Monday 26 June 2000
Study Period 15 minutes duration
Writing period: 2 hours duration
Permitted Materials: Printed course notes, including those for Lecture 12; calculators
You must attempt to answer both sections

Section A

Write short paragraphs (or list of dot points) on each of the following (10% each, 50% total).

1. Compare and contrast the Newtonian and Lagrangian (or Hamiltonian) approaches with respect to ONE of the following (illustrating your points using an example of a non-dissipative physical system):
   - force (vector) vs. energy (scalar) methods
   - geometric vs. analytic methods
   - detailed mechanical description vs. economy and generality
   - cause and effect.
2. Indicate how the dynamics of a system with n degrees of freedom is described in (a) configuration space and (b) phase space. (Some issues: What are the dimensions of the respective spaces? How many time evolution equations are there and what is their order? How many arbitrary constants are there in the general solution of the dynamical equations?)
3. What is the relation between symmetry and conservation relations? Give an example.
4. What is a canonical transformation and what is it good for? Give an example.
5. When is phase space volume conserved and when is it not? Give a specific example of each, sketching the phase plane portraits.

Section B

Attempt all parts of the following problem, preferably, but not necessarily, in the order given (50% total)

With respect to an inertial frame $\mathbf{K}_0$ the Lagrangian function of a particle is

$$L = \tfrac{1}{2}m\dot{\mathbf{r}}_0^2 - V(\mathbf{r}) \ ,$$

where $\mathbf{r}$ is the position vector and $\dot{\mathbf{r}}_0$ is the velocity with respect to $\mathbf{K}_0$.

The frame of reference $\mathbf{K}$ coincides with $\mathbf{K}_0$ at $t = 0$, has the same origin always, but rotates with constant angular velocity $\boldsymbol{\omega}$ with respect to $\mathbf{K}_0$. Assume for definiteness that $\boldsymbol{\omega}$ is parallel to the z axis.
(a) Draw a diagram showing the x and y axes of the two frames and the projection in the x, y plane of the position vector $\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z = x_0\mathbf{e}_x^0 + y_0\mathbf{e}_y^0 + z_0\mathbf{e}_z^0$ (where $\mathbf{e}_{x,y,z}$ and $\mathbf{e}_{x,y,z}^0$ are the unit vectors in the directions of the x, y and z axes of $\mathbf{K}$ and $\mathbf{K}_0$, respectively).

(b) Express $\mathbf{e}_{x,y,z}(t)$ in terms of the fixed unit vectors $\mathbf{e}_{x,y,z}^0$ and show that $\dot{\mathbf{e}}_{x,y,z} = \boldsymbol{\omega}\times\mathbf{e}_{x,y,z}$.

(c) Show that the Lagrangian in the frame $\mathbf{K}$ is

$$L = \tfrac{1}{2}m\dot{\mathbf{r}}^2 + m\dot{\mathbf{r}}\cdot\boldsymbol{\omega}\times\mathbf{r} + \tfrac{1}{2}m(\boldsymbol{\omega}\times\mathbf{r})^2 - V(\mathbf{r}) \ ,$$

where $\dot{\mathbf{r}}$ is the velocity vector with respect to the frame $\mathbf{K}$, i.e. $\dot{\mathbf{r}} \equiv \dot{x}\mathbf{e}_x + \dot{y}\mathbf{e}_y + \dot{z}\mathbf{e}_z$.

(d) Derive the equations of motion in the rotating frame from this Lagrangian and identify the centrifugal force and the Coriolis (velocity dependent) force.

(e) Define the canonical momenta conjugate to the rotating frame coordinates x, y, z. Are they the same as the kinematic momenta? Derive the corresponding Hamiltonian.