Roadmap to ultra-short record high-energy pulses out of laser oscillators

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Abstract

Highly-chirped dissipative solitons of the cubic–quintic Ginzburg–Landau equation found in this work may provide a roadmap to design passively mode-locked laser oscillators that generate pulses of extremely high energy. We provide a region in the space of the system parameters where high-energy dissipative solitons are found, along with their typical spectral and temporal features.

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Ultra-short optical pulses with extremely high energy provide modern experimental physics with new tools. They can be used to generate exceptionally wideband, or supercontinuum, light sources \cite{1} that find numerous applications in biomedical imaging, such as coherence tomography \cite{2}. They can also produce coherent X-ray flashes via high-order harmonic generation in noble gases \cite{3} and offer promising alternatives to the acceleration of charged particles. Thus, producing shorter and more powerful pulses is vital for further progress in science.

Until recently, the generation of ultra-short high-energy optical pulses required a cascade of delicate amplifiers, based on the concept of chirped-pulse amplification \cite{4}, at the output of a mode-locked laser oscillator. Recently, it was shown that high-energy pulses could be produced by a single solid-state laser \cite{5}, reducing the need for amplifiers. Fiber lasers are also studied vigorously in that respect. When designing a laser oscillator with a given amplifying medium, the starting point is the level of pumping power available along with the choice of the mode-locking mechanism. When an efficient cavity design is set up, the subsequent issue is its operational efficiency at increased pumping powers.

Due to the large number of cavity designs, and to the wide range of possible pulse dynamics, there is no general theory that provides an ultimate laser design with optimal performance in all cases. The tremendous progress made during the past 15 years has been a result of alternative advances in experimental trials and of a succession of laser pulse concepts. Until recently, most attention of scientists was focused on cavity dispersion issues. Although soliton propagation in anomalous dispersion regime can provide neat sech-profiled pulses, the search for higher-energy and wider-bandwidth laser outputs led to cavities designed with strong dispersion management, featuring an average dispersion shifted into the normal regime \cite{6}. The principle of dispersion management eventually evolved into the concepts of parabolic-pulse and all-normal-dispersion lasers \cite{7}. The resulting stationary output pulses are usually strongly chirped all along the cavity, making them wider in the temporal domain, but further compression outside the cavity can be performed to shorten the pulse to the femtosecond range. The question arises whether dispersion is the main issue in designing a laser cavity. The fact is that dispersion is only one among many other parameters that play an important role in the laser dynamics.

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In nonlinear dynamics, all the parameters have to be taken into account to find an optimal range for each of them. In the present Letter, we propose a roadmap to increase the pulse energy using a “master equation” approach for passively mode-locked lasers [8]. This technique can be used for modeling stable pulse train generation by both solid-state [8,9] and fiber ring lasers [10]. Moreover, Wise et al. [11] recently designed a so-called “dissipative soliton laser” that generates pulses which could be directly compared to the pulse solutions of the cubic–quintic Ginzburg–Landau equation (CGLE). Our previous experience shows that the cubic–quintic CGLE allows the prediction of highly complicated pulse dynamics, such as “exploding solitons” for instance [12]. Preliminary modeling is essential to provide rough estimates for the range of parameters that would lead to the generation of pulses with the highest possible energy, out of a single laser oscillator. Moreover, finding qualitatively new solutions of the CGLE broadens significantly the “world of the Ginzburg–Landau equation” [13]. Indeed, new solutions can be useful in all areas of science where this universal equation may be applied.

Thus, our approach is based on a complex cubic–quintic Ginzburg–Landau master equation. This model includes cubic and quintic nonlinearities of both dispersive and dissipative terms. The normalized master equation reads [14,15]:

\[ i\delta \psi_z + \frac{D}{2} \psi_{tt} + \left| \psi \right|^2 \psi + \nu |\psi|^4 \psi = i \epsilon |\psi|^3 \psi + i \beta \psi_{tt} + i \mu |\psi|^4 \psi. \] (1)

The optical envelope \( \psi \) in (1) is a complex function of two real variables, i.e., \( \psi = \psi(t, z) \), where \( t \) is the retarded time in the frame moving with the pulse, and \( z \) is the pulse propagation distance. The left-hand side of Eq. (1) contains the conservative terms with the coefficients \( D \) and \( \nu \). Here, \( D \) is the dispersion coefficient which is positive (negative) in the anomalous (normal) propagation regime while \( \nu \) represents a higher order Kerr nonlinearity. The right-hand side of (1) includes all dissipative terms. The coefficients \( \delta, \epsilon, \beta, \) and \( \mu \) control the averaged linear loss in the cavity (when negative), nonlinear gain (if positive), spectral filtering or gain dispersion, and saturation of the nonlinear gain (if negative), respectively.

This master equation can be applied to the modeling of passively mode-locked lasers in the short pulse regime of operation. In principle, changes of the pulse in a round trip should remain relatively small, but we have experienced that the validity of the predicted dynamics can be qualitatively good even in the case of stronger internal pulse dynamics. The laser is assumed to be in stationary regime when all transient effects after starting the laser have vanished. Dissipative terms describe the averaged gain and loss processes taking place when the pulse moves in the cavity. Higher-order dissipative terms describe the cumulative nonlinear transmission characteristics of the cavity, including the passive mode-locking mechanism.

We have solved Eq. (1) using a split-step Fourier method. The second-order derivative terms in \( t \) are solved in Fourier space. All other linear and nonlinear terms in the equation are solved in real space using a fourth-order Runge–Kutta method. We used various values of step sizes along \( t \) and \( z \) to check that the results do not depend on the mesh intervals, thus avoiding any numerical artifacts.

The cubic–quintic CGLE admits a wide variety of soliton solutions [16–18]. The presence of six parameters in the equation makes it difficult to find all of them at once. The main task is to find a region of parameters of physical significance where stationary stable solitons do exist. Once one stable localized solution is found for a given set of equation parameters, it can serve as the initial condition for finding solutions at other nearby values of the parameters. By moving slowly in the parameter space, we are able to determine regions of soliton existence in a relatively easy way. Our present technique produces exclusively robust stable solitons. In general, we fix four of the parameters, for instance \( \mu, \nu, \delta, \beta, \) and change \( D \) and \( \epsilon \) when looking for stable localized solutions. Indeed, dispersion and gain can be rather easily controlled in any passively mode-locked laser.

An important characteristic of the output pulse is the total energy \( Q \), given by the integral of \( |\psi|^2 \) over \( t \): \( Q(z) = \int_{-\infty}^{\infty} |\psi(t, z)|^2 dt \). For a dissipative system, the energy is not conserved but evolves in accordance with a balance equation [15]. If the solution stays localized, the energy evolves but remains finite. Furthermore, when a stationary solution is reached, the energy \( Q \) converges to a constant value.

In our present study, we found a region in the parameter space where stable dissipative solitons have energies much higher than those reported before. When the quintic nonlinearity is positive (\( \nu > 0 \)), solitons can be observed for both anomalous \( (D > 0) \) and normal \( (D < 0) \) dispersion regimes. We present, in Fig. 1, a region of existence of dissipative solitons for this case. When \( \nu \) is negative, solitons may exist but in a significantly narrower region of values of the parameter space.

Examples of dissipative soliton spectra and pulse shapes are presented in Figs. 2 and 3. They correspond to the thick dots in Fig. 1. Soliton shapes and spectra evolve dramatically with the change of the dispersion \( D \) and the cubic gain parameter \( \epsilon \). When we move \( D \) further into the normal dispersion regime,
the spectrum acquires an approximately rectangular shape with two or three local maxima. At the same time, the pulse becomes highly chirped so that it appears much wider in the temporal domain. Fig. 2 shows that for a fixed $\epsilon$, the width of the soliton spectrum is almost constant. However, the spectral intensity depends strongly on the dispersion parameter, resulting in an increase of the total energy of the pulse when changing the $D$-value towards the left-hand side of the region of existence.

The qualitative shape of the spectrum is defined mainly by $D$, but its spectral width depends predominantly on the cubic gain parameter $\epsilon$. Fig. 3 shows the dependence of the spectra and pulse profiles on the cubic gain parameter $\epsilon$ when $D = -1$. With the increase of $\epsilon$, the pulse becomes narrower and of higher amplitude, while the spectrum becomes wider, keeping its roughly rectangular profile. An interesting feature of the spectra at this value of $D$ is that they all have three maxima rather than two. Moving to even more negative values of $D$, the central maximum in the spectra increases considerably. The pulses shown in Figs. 2 and 3 are clearly far from being Fourier-transform limited. However, the pulse duration can be significantly shortened by de-chirping the pulse in a dispersion compensation line. We verified, numerically, that close-to transform limited, ultra-short pulses, can be obtained this way.

The most remarkable feature of dissipative solitons in the normal dispersion regime is that their energy $Q$ grows indefinitely when the absolute value of $D$ increases. Fig. 4 shows the energy, $Q$, versus $D$ for three values of $\epsilon$. This figure shows clearly that $Q$ depends strongly on the dispersion $D$. In fact, the value of the energy increases almost infinitely when $D$ reaches the edge of stability at the left-hand side of the region of existence in Fig. 1. This result shows that the only practical limits in increasing the values of the pulse energy in the normal dispersion regime are the available pump power and the stability of the pulses with respect to higher-order terms not included in our model.

The region of existence of new solitons can be presented most efficiently on a two-dimensional plot such as in Fig. 1. The four other parameters can be found as a result of massive search in the parameter space, selecting the values that provide the highest energy. Without previous knowledge of at least some of the regions of soliton existence [17], the search could have...
taken years of numerical simulations. In particular, we found that the higher-order nonlinearity coefficient \( \nu \) has to be slightly positive. When \( \nu \) is negative, the region of existence of normal dispersion solitons shrinks significantly (see Fig. 8 of [17]). In particular, for previously chosen values of \( \beta \) and \( \mu \), high-energy solitons cease to exist at negative \( \nu \).

Fig. 5 shows a region of existence of dissipative solitons in the case of negative \( \nu \). As we can see, the main difference of this region from the one shown in Fig. 1 is the absence of solitons in the negative dispersion area. The energy of solitons in this positive \( D \) area is relatively lower and does not show any relevant increase at the edges of the region. Instead, at the left-hand side edge of the region we observed a stripe of pulsating solitons. Decreasing further the value of \( D \), no stable localized solution was found.

This example shows once again that the choice of parameters of the master equation is the key for the observation of high-energy solitons. As we have six parameters in Eq. (1), the search for other regions of the parameter space where high-energy solitons can be found is a tedious task. However, these studies can provide us with an efficient guideline for experimental research, so that these efforts are worthy to be made.

Our observations are potentially useful for many applications where the CGLE is used [13]. In particular, the results can be highly relevant for further development of high-power passively mode-locked lasers. Recently, Fernandez et al. [5], may have observed similar high-energy dissipative solitons in their experiment with a Kerr-lens mode-locked Ti:sapphire laser. The direct indication for our statement is their optical spectrum of Fig. 3a in [5], which is very similar to our spectra presented in Figs. 2 and 3, except for the asymmetry which originates from higher-order dispersion. The pulse in the experiment was strongly chirped and of high power as follows from our numerical study. Clearly, further common theoretical and experimental work in this direction will allow the optimization of the parameters and the generation of pulses with exceptionally high energy without additional external amplification.

In conclusion, we studied new dissipative soliton solutions of the cubic–quintic Ginzburg–Landau equation. The most remarkable feature of these new solutions is their energy that can reach almost infinitely high values, while remaining localized. This observation significantly enriches the "world of the Ginzburg–Landau equation" and may have various applications in a wide range of branches of nonlinear science. In particular, new results provide a roadmap for the design of passively mode-locked lasers generating ultra-short, record high-power pulses.

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