

Velocity of heat dissipative solitons in optical fibers

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In the fiber fuse, a pulse of high temperature travels toward the input end of the fiber, where high-power laser light is launched into the fiber. At any point along the fiber, the soliton can be ignited. The fiber core is damaged in the process so that light cannot propagate beyond the hot spot. This phenomenon is an example of a dissipative soliton that can exist only in the presence of an external energy supply and internal loss. We analyze this phenomenon, derive an expression for the velocity of the soliton, and determine its width as functions of the physical parameters of the laser and the fiber material. © 2008 Optical Society of America
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The fascinating phenomenon of the fiber fuse, which can operate at modest pump powers [1–4], is an interesting example of a dissipative soliton [5]. In this effect, a bright surge of high temperature (“flame”) moves along an optical fiber, fed by an optical pump. The process occurs because damage of the glass dramatically increases the light absorption in the fiber. This has a threshold and occurs only when the local temperature is >1000 K.

The most obvious naturally occurring analogy is a grass fire, that is advancing on a linear front [6]. In the central part, which is on fire, the temperature is above the ignition point, and it initiates grass in front to start burning. At the same time, the burnt grass has no further fuel, and it cools down. The grass-fire line advances at a velocity that depends on the amount of burnable material and the external (ambient) temperature. The maximum temperature reached also depends on these factors. Like any dissipative soliton, it needs a continuous input of matter/energy to sustain it.

In a purely 1D system with no lateral heat flow, it is straightforward to calculate the velocity [1]. When side heat loss is included, however, to our knowledge only numerical solutions have so far been reported [1,7]. These numerical results provide useful qualitative estimates, but to our knowledge analytic expressions for the fuse velocity have not been reported. Here, we provide such expressions for fuse velocity and width. This is of particular relevance to high-power fiber lasers and amplifiers, where the possibility of catastrophic fiber destruction is a major concern and where a detailed knowledge of the conditions leading to fuse formation is clearly of great importance.

As was shown in [7], the dynamics of the fiber fuse phenomenon is governed by the following set of three equations. The first is the 1D heat transfer equation with source and relaxation terms:

$$T_t = D \frac{\partial^2 T}{\partial x^2} + \chi \alpha I - k(T - T_0), \quad (1)$$

where χ is a geometric factor and D is the thermal conductivity. $T(x, t)$ is the local temperature of the fiber, and $\alpha(x, t)$ is the absorption, both averaged over the fiber cross section, while $I(x, t)$ is the laser power in the fiber, integrated over the fiber cross section. The last term in this equation represents the relaxation (with rate k) of the temperature to the ambient temperature, T_0 .

The second equation gives the rate of change of the absorption coefficient as a function of the temperature, $T(x, t)$. This equation follows from the experimental data given in Fig. 1 of [3]. The absorption stays close to zero at temperatures below the threshold and grows roughly linearly at higher temperatures:

$$\alpha_t = F(T) = \begin{cases} 0, & \text{if } T < T_1 \\ m(T - T_1), & \text{if } T > T_1 \end{cases}. \quad (2)$$

Here, m is a factor [$\approx 10^{-3} \text{m}^{-1} \text{s}^{-1} (\text{K})^{-1}$] that depends on the specific mechanism of glass damage and the transverse dimensions of the fiber. The critical temperature T_1 at which the damage starts is related to the formation of color centers in the glass at high temperature.

Finally, the third equation defines the laser power, which is absorbed along the fiber according to

$$I_x = -\alpha I. \quad (3)$$

This set of three equations gives a simple 1D description of the phenomenon. Mathematically, it is a complete model that states the problem on a phenomenological basis and is related to some others in combustion theory [8], apart from the fiber damage and relaxation mechanisms.

It was shown, numerically, that this set of equations allows for steady flame propagation along the fiber. Experiments show both steady-state and pulsating propagation of the flame [2,4]. The latter corresponds to a pulsating dissipative soliton [9]. Here, we consider only steady motion. Then the set can be reduced to a single ordinary differential equation (ODE) [7]:

$$D\phi''' - v\phi'' - k\phi' - \frac{J}{v}(e^{-\phi} - 1) = \frac{D\mu_1 + k\xi}{v}\tau_1, \quad (4)$$

where the function $\phi = \phi(\xi)$ and its first and second derivatives are initially zero. Here, $\phi(\xi) = \int_0^\xi \alpha dx$, $J = m\chi I$, $\tau_1 = m(T_1 - T_0)$ is the normalized threshold temperature, $\xi = x + vt$, v is the velocity of the traveling waves, and μ_1 is given by

$$\mu_1 = \frac{v}{2D} + \sqrt{\frac{v^2}{4D^2} + \frac{k}{D}}. \quad (5)$$

The derivation of Eq. (4) and related explanations are in [7]. The normalized temperature is $\tau = m(T - T_0)$, and the continuity of its slope at $\xi = 0$ provides the condition $\phi'''(0) = \tau_1\mu_1/v$. The thermal dissipative soliton (flame) travels backward [i.e., toward the laser source] at a (positive) velocity v . It is a function of D, J, k . This problem involves finding the distributions of temperature $\tau(\xi)$ and absorption $\alpha(\xi)$ and the velocity dependence on the three parameters D, J, k . As Eq. (2) is given by a stepwise function, then ξ needs to be divided into three regions where the temperature τ is defined as below or above the threshold. Naturally, the absorption is constant in the tails of the soliton where the temperature is below the threshold and increases only in the central part. When deriving Eq. (4), the variable ξ has been chosen in such a way that the first boundary is $\xi = 0$.

Thus, in region 1, ($\xi < 0$, see Fig. 1, Eq. (4) can be significantly simplified, and we have the exponential solution $\tau = \tau_1 \exp(\mu_1 \xi)$, with $\mu_1 > 0$. Region 2 ranges from $\xi = 0$ to the point $\xi = \xi_f$, where the temperature drops below the threshold and the absorption growth stops. In this region, we have to solve the complete Eq. (4). In region 3, ($\xi > \xi_f$), the term $e^{-\phi(\xi)}$ is very

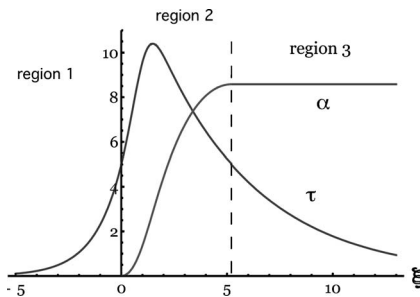


Fig. 1. Distribution of temperature τ and absorption α for $D=3$, $J=35$. We define ξ_f (here 5.2) as the width of the central region, i.e., region 2. At this point, the absorption reaches its final value, α_∞ , which in this case is about 8.582.

small, so we effectively have the same equation as in region 1. Now the solution is $\tau = \tau_1 \exp(\mu_2 \xi)$ with $\mu_2 < 0$:

$$\mu_2 = \frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \frac{k}{D}}. \quad (6)$$

Thus, we now find the solution in region 2 ($0 < \xi < \xi_f$), and we then match boundary conditions at $\xi = 0$ and $\xi = \xi_f$.

In region 2, for ξ close to ξ_f , we can write an approximation:

$$\phi(\xi) \approx \alpha_\infty \xi + \frac{\mu_2 \tau_1}{6v} (\xi - \xi_f)^3, \quad (7)$$

so that $\tau - \tau_1 = v\phi''(\xi) = \mu_2 \tau_1 (\xi - \xi_f)$, with the slope of the temperature profile $\tau'(\xi) = \mu_2 \tau_1$ at the point $\xi = \xi_f$. By using the first equation given, and finding the zero-order terms at the boundary between regions 2 and 3, we find a relation between the parameters, *viz.*,

$$J + D\tau_1(\mu_2 - \mu_1) - \alpha_\infty k v - k\tau_1 \xi_f = 0. \quad (8)$$

We know μ_1 and μ_2 , so we can now write down the velocity:

$$v = \frac{\alpha_\infty k (J - k\tau_1 \xi_f) - \tau_1 \sqrt{c_1}}{\alpha_\infty^2 k^2 - \tau_1^2}, \quad (9)$$

where

$$\begin{aligned} c_1 &= (J - k\tau_1 \xi_f)^2 + 4Dk(\alpha_\infty k - \tau_1)(\alpha_\infty k + \tau_1) \\ &= (J - k\tau_1 \xi_f)^2 + 4Dk(\alpha_\infty^2 k^2 - \tau_1^2). \end{aligned}$$

The expression (9) is the central result of our work. It gives the velocity in terms of the parameters of the system, i.e., conductivity, pump intensity, and parameters of the mechanism of fiber damage, such as the temperature threshold. Sample curves of velocity versus D and velocity versus pump power, given in Figs. 2 and 3 respectively, are very close to those found using direct numerical simulations of the partial differential equation (PDE) in [7]. Having an analytical expression for the velocity is more convenient, of course, as it allows direct comparisons to be made with experimental results. For the parameters of the problem given in physical units $D[\text{m}^2/\text{Ks}]$, $\alpha[\text{m}^{-1}]$, $k[\text{s}^{-1}]$, the velocity v is given in $[\text{ms}^{-1}]$.

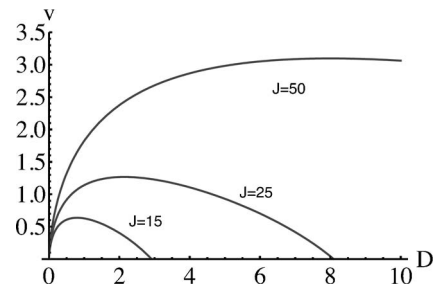
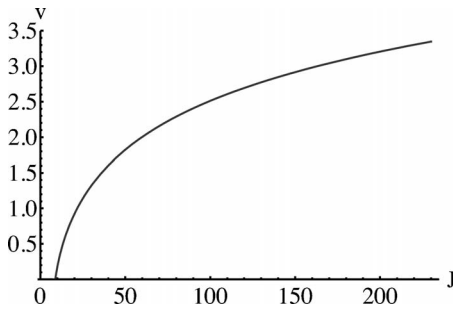


Fig. 2. Velocity as a function of D for various J . Here, the top curve is for $J=50$, the middle one is for $J=25$, and the lowest one is for $J=15$. (Here $k=1/2$.)


 Fig. 3. Velocity as a function of J for $D=1$.

Now, Eq. (9) is rather involved. We can make a few simplifications in limiting cases. The absorption at plus infinity, α_∞ , has the form J^g/D^r . So, for small D , $v \approx D^r$. This means we need $r > 0$, since $v=0$ when $D=0$. We take $k=1/2$ and $\tau_1=5$, since this allows comparisons with earlier work. From numerical runs solving the ODE, we find that $\alpha_\infty = J^{4/5}/D^{2/3}$. Thus, $v \approx D^{2/3}$ for small D .

A more accurate estimate can be obtained using $\xi_f \approx J/7 - 5D/J$, which is found from our numerical simulations. Then, we obtain, for velocity at small D :

$$v = \frac{D^{2/3}(-9J^2 + 10D^{2/3}\sqrt{c_2^5 J} - 175D)}{700D^{4/3}\sqrt[5]{J} - 7J^{9/5}}, \quad (10)$$

where

$$c_2 = \frac{30625D^2}{J^2} - 6650D + 81J^2 + \frac{98J^{8/5}}{\sqrt[3]{D}}.$$

When D is high, the pulse spreads out, and the maximum temperature decreases below the critical value for optical damage. Then there is no absorption, and the thermal soliton cannot propagate. Hence, there is no fiber fusion above some critical value of D . One can increase input intensity J to achieve fusion in the high- D regime, as seen from the two upper curves in Fig. 2.

To find the temperature profile, we have to solve Eq. (4) in region 2. The slope of the temperature curve at $\xi=0$, i.e., τ' is $\tau_1\mu_1$. Thus the shape of the main function must be of the form

$$\phi = \frac{\tau_1\mu_1}{6v}\xi^3 f(\xi),$$

where the unknown function $f(\xi)$ has the initial value $f(0)=1$. Clearly, for small positive ξ , we have $\alpha(\xi) = \tau_1\mu_1\xi^2/(2v)$. It is possible to make a neat approximation that determines the quantities needed. This is done by approximating the solution by the function $f(\xi) = \exp[(b-a)\xi]\text{sech}(b\xi)$, where a and b satisfy $b > a > 0$. Near ξ_f , we can use the asymptotic form of the solution, hence,

$$\phi \approx \frac{\tau_1\mu_1}{3v}\xi^3 \exp[-a\xi]. \quad (11)$$

Then, matching the second-order derivative, we have

$$\tau - \tau_1 = v\phi''(\xi) \approx \frac{\tau_1\mu_1(6 - 6a\xi + a^2\xi^2)}{3\exp(a\xi)}, \quad (12)$$

and this has its first zero when $\xi = \xi_f$, so the first parameter is $a = (3 - \sqrt{3})/\xi_f$. Expanding Eq. (4) to first order then allows us to obtain the second parameter, $b = a + (k + \mu_1 v)/(4D\mu_1)$.

The functions $\alpha(\xi)$ and $\tau(\xi)$ in region 2 can be obtained by differentiating $\phi(\xi)$. The slope of the temperature curve at the start of region 2, $\tau'(0) = \tau_1\mu_1$, is larger in magnitude than its slope at the end of region 2, $\tau'(\xi_f) = \tau_1\mu_2$. Thus, as would be expected, the maximum temperature occurs in the first half of region 2, i.e., in the range $\xi < \xi_f/2$. The maximum temperature increases with J when D is fixed, and decreases with D when J is fixed.

The expression for absorption in region 2 is

$$\alpha(\xi) = \frac{\tau_1\mu_1\xi^2[3 - a\xi + b\xi - b\xi \tanh(b\xi)]}{6v \cosh(b\xi)\exp[(a-b)\xi]}, \quad (13)$$

so that the maximum absorption on the right-hand side of region 2 is

$$\alpha_\infty = \alpha(\xi_f) = \frac{\tau_1\mu_1}{\sqrt{3}v}\xi_f^2 \exp(\sqrt{3}-3). \quad (14)$$

Thus, we have obtained good approximations for temperature and absorption profiles and they agree with the numerical results of [7]. Their stability has been confirmed by direct simulations of PDEs (1)–(3) in [7].

In conclusion, we have found analytic expressions for the velocity of a heat dissipative soliton (fiber fuse), its temperature and absorption profiles, and its width. We have discussed the maximum temperature that occurs for various parameters. The analytic results are in good agreement with the numerical results found in previous work.

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