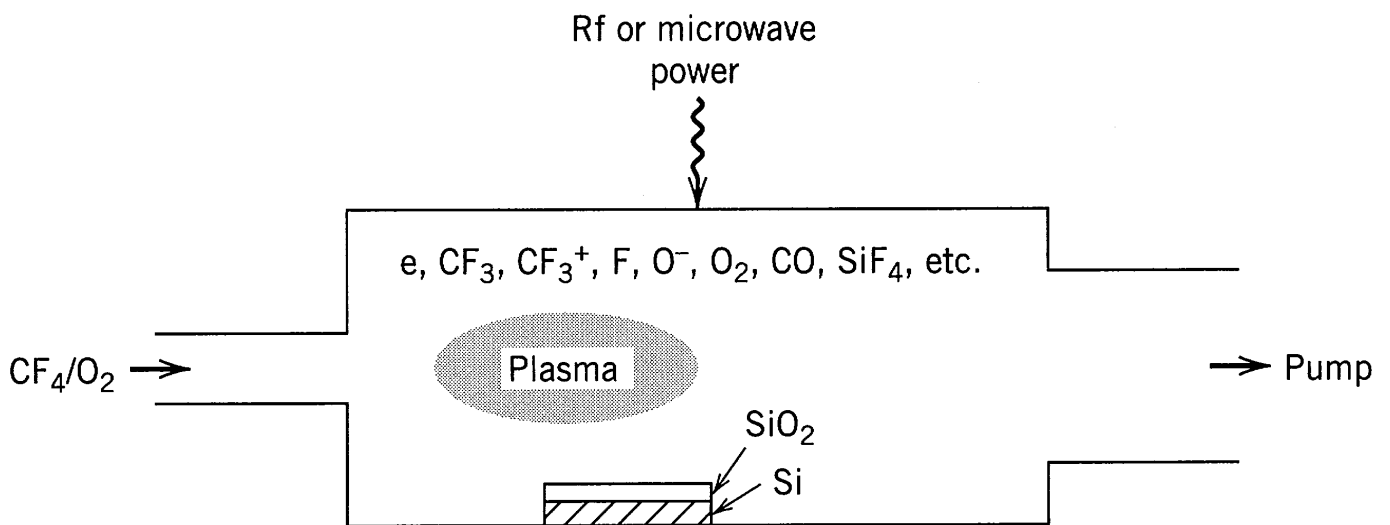


# A MINI-COURSE ON THE PRINCIPLES OF PLASMA DISCHARGES

Michael A. Lieberman



## OUTLINE

- Introduction to Plasma Discharges and Processing
- Summary of Plasma Fundamentals
- Break —
- Summary of Discharge Fundamentals
- Analysis of Discharge Equilibrium
- Inductive RF Discharges

## ORIGIN OF MINI-COURSE

45 hr graduate course at Berkeley  $\implies$

12 hr short course in industry  $\implies$

4 hr mini-course

**INTRODUCTION TO PLASMA DISCHARGES  
AND PROCESSING**

# PLASMAS AND DISCHARGES

- Plasmas:

A collection of freely moving charged particles which is, on the average, electrically neutral

- Discharges:

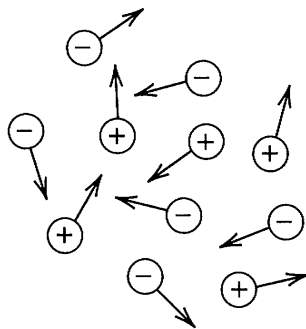
Are driven by voltage or current sources

Charged particle collisions with neutral particles are important

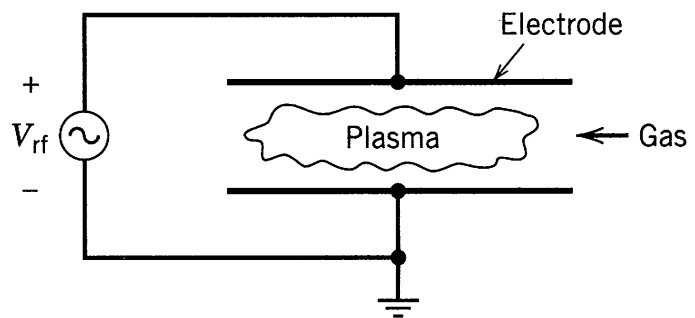
There are boundaries at which surface losses are important

Ionization of neutrals sustains the plasma in the steady state

The electrons are not in thermal equilibrium with the ions



(a)

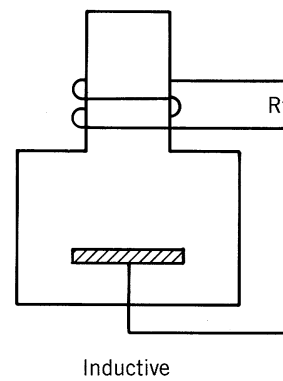
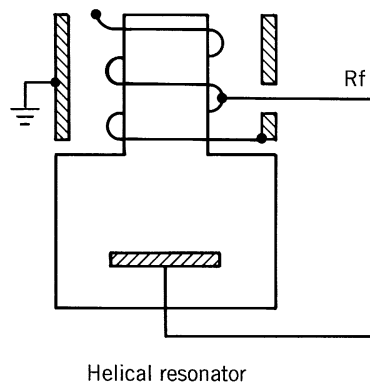
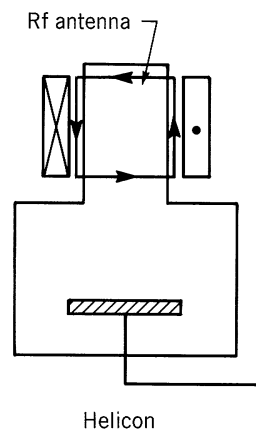
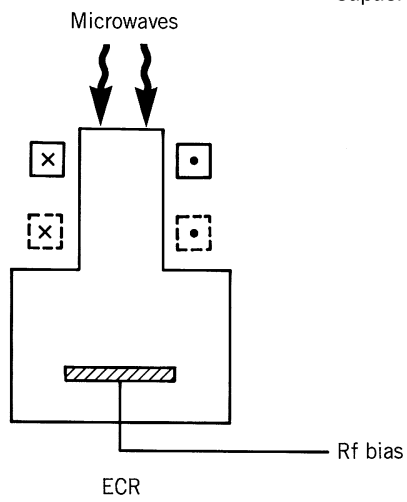
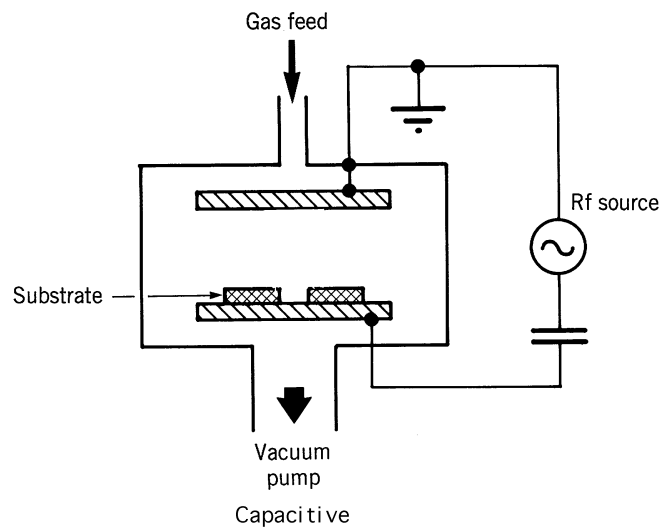


(b)

- Device sizes  $\sim 30$  cm – 1 m

- Driving frequencies from DC to rf (13.56 MHz) to microwaves (2.45 GHz)

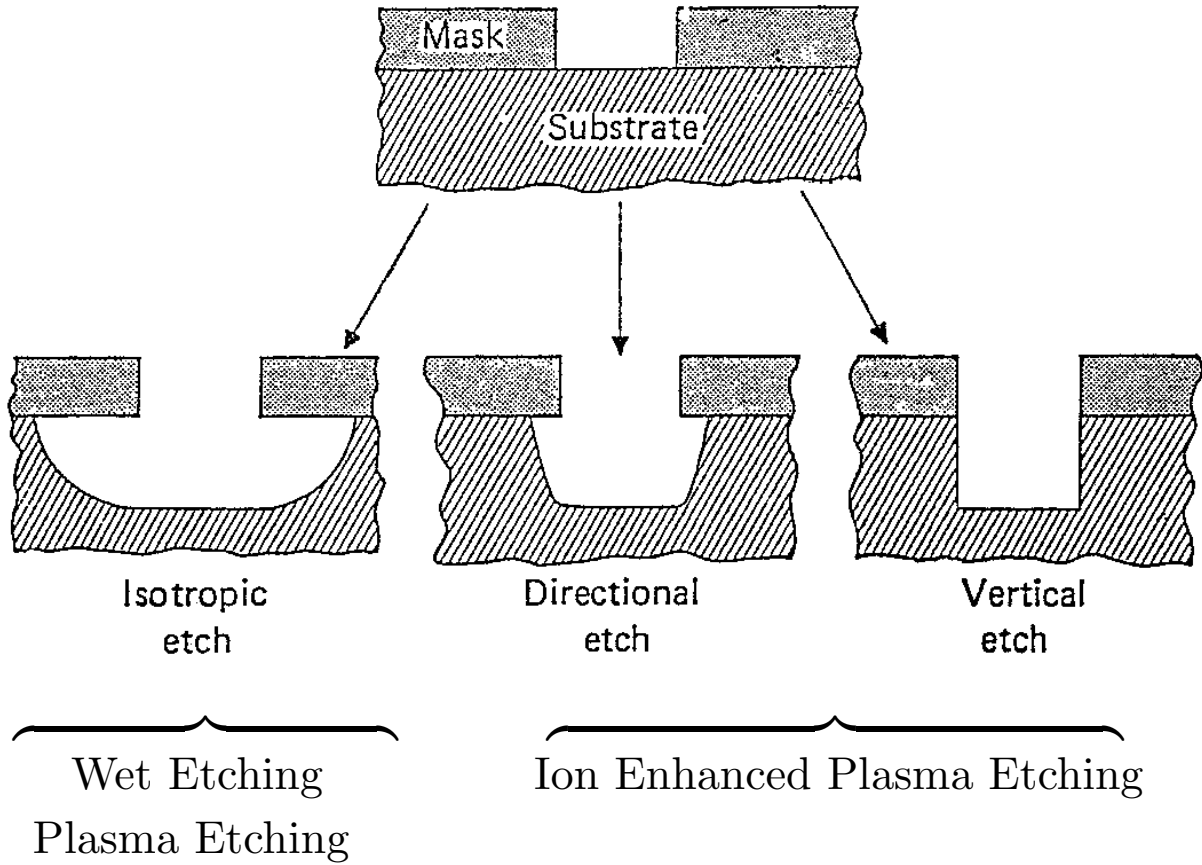
# TYPICAL PROCESSING DISCHARGES



# RANGE OF MICROELECTRONICS APPLICATIONS

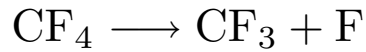
- Etching  
Si, a-Si, oxide, nitride, III-V's
- Ashing  
Photoresist removal
- Deposition (PECVD)  
Oxide, nitride, a-Si
- Oxidation  
Si
- Sputtering  
Al, W, Au, Cu, YBaCuO
- Polymerization  
Various plastics
- Implantation  
H, He, B, P, O, As, Pd

# ANISOTROPIC ETCHING

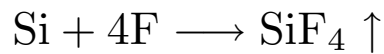


## ISOTROPIC PLASMA ETCHING

1. Start with inert molecular gas  $\text{CF}_4$
2. Make discharge to create reactive species:



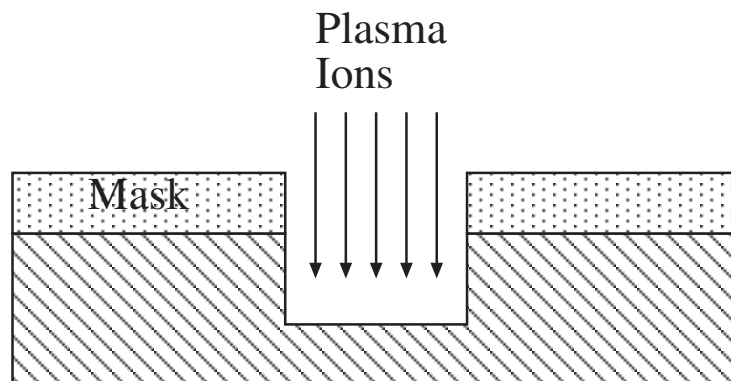
3. Species reacts with material, yielding volatile product:



4. Pump away product
5.  $\text{CF}_4$  does not react with Si;  $\text{SiF}_4$  is volatile

## ANISOTROPIC PLASMA ETCHING

6. Energetic ions bombard trench bottom, but not sidewalls:
  - (a) Increase etching reaction rate at trench bottom
  - (b) Clear passivating films from trench bottom





## UNITS AND CONSTANTS

- SI units: meters (m), kilograms (kg), seconds (s), coulombs (C)

$$e = 1.6 \times 10^{-19} \text{ C, electron charge} = -e$$

- Energy unit is joule (J)

Often use electron-volt

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

- Temperature unit is kelvin (K)

Often use equivalent voltage of the temperature:

$$T_e(\text{volts}) = \frac{kT_e(\text{kelvins})}{e}$$

where  $k = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J/K}$

$$1 \text{ V} \iff 11,600 \text{ K}$$

- Pressure unit is pascals (Pa);  $1 \text{ Pa} = 1 \text{ N/m}^2$

Atmospheric pressure  $\approx 10^5 \text{ Pa} \equiv 1 \text{ bar}$

Often use English units for gas pressures

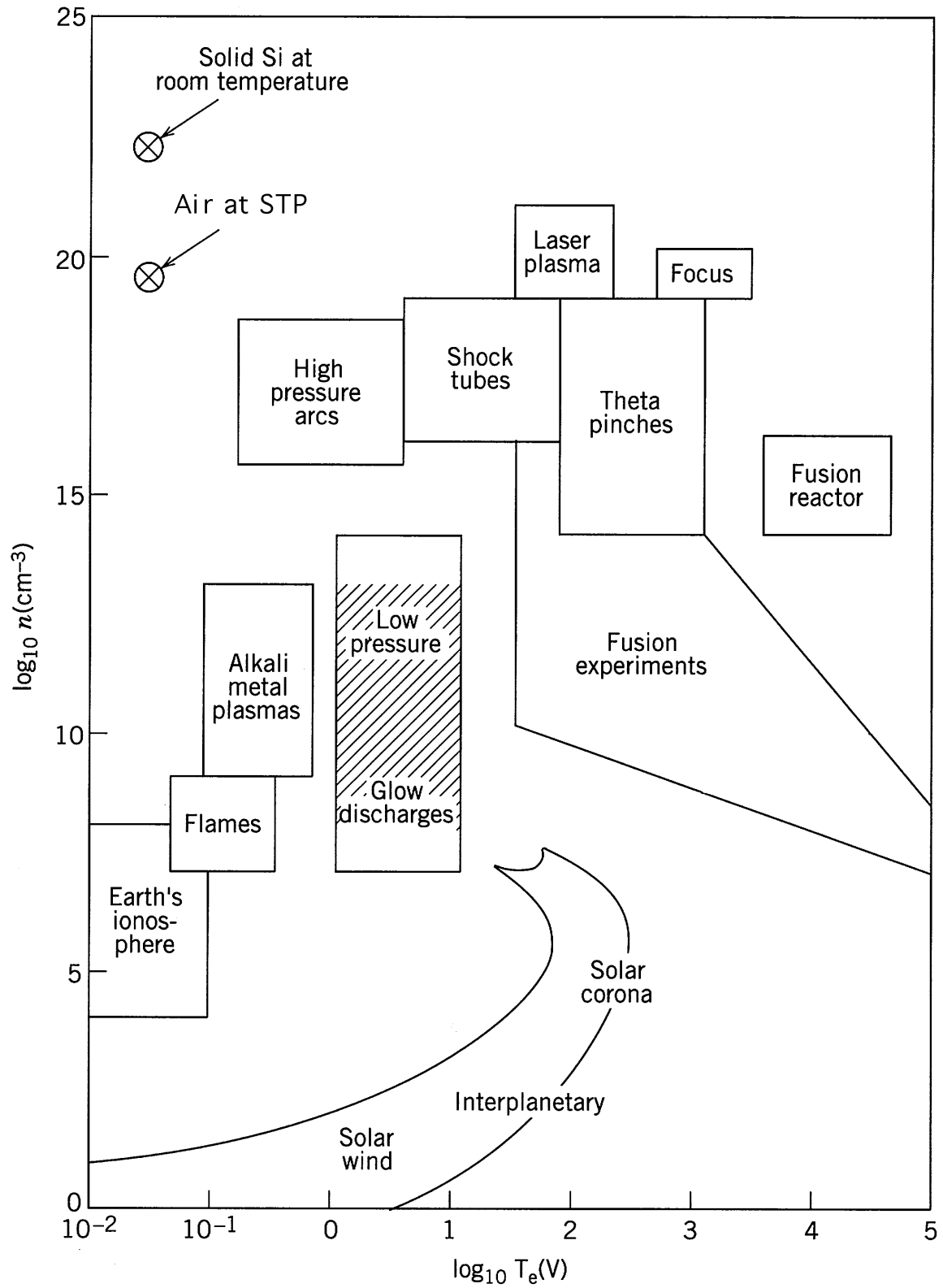
Atmospheric pressure = 760 Torr

$$1 \text{ Pa} \iff 7.5 \text{ mTorr}$$

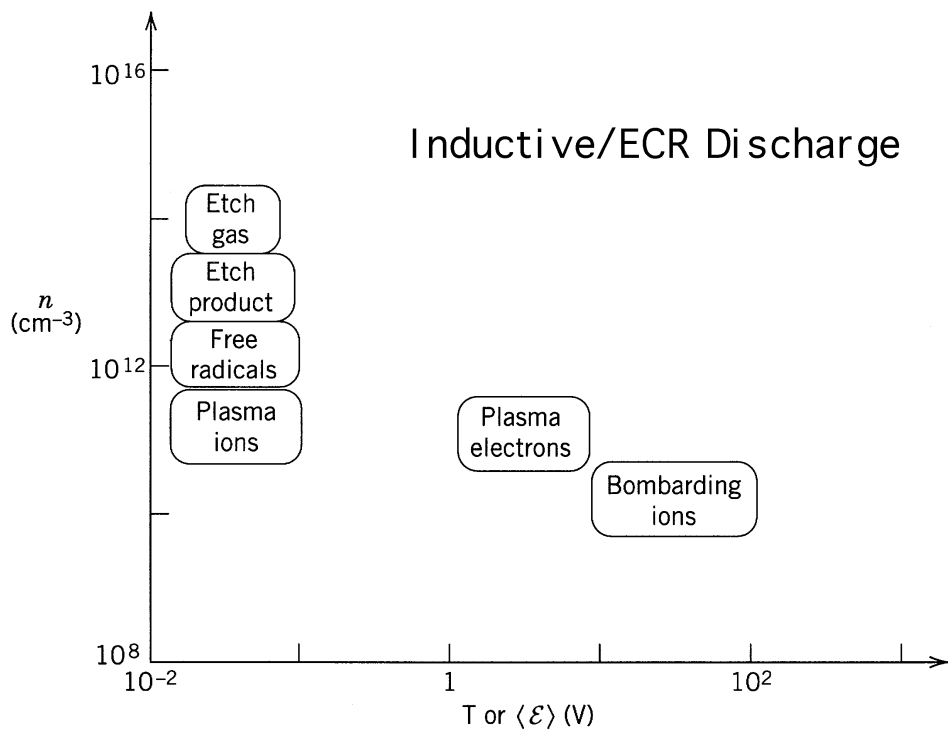
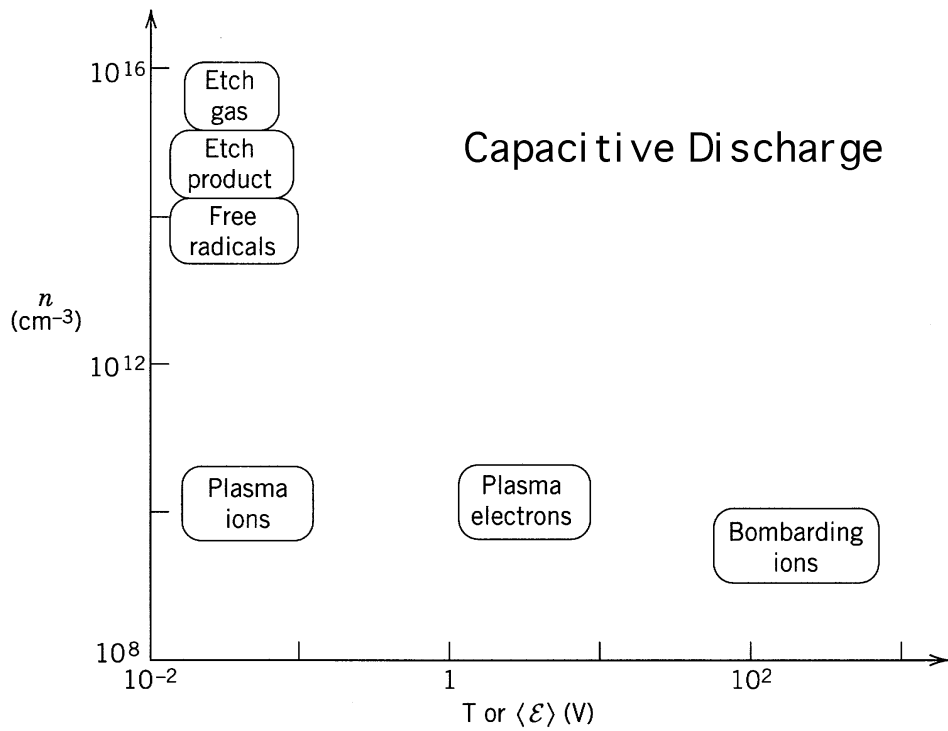
# PHYSICAL CONSTANTS AND CONVERSION FACTORS

Quantity	Symbol	Value
Boltzmann constant	$k$	$1.3807 \times 10^{-23}$ J/K
Elementary charge	$e$	$1.6022 \times 10^{-19}$ C
Electron mass	$m$	$9.1095 \times 10^{-31}$ kg
Proton mass	$M$	$1.6726 \times 10^{-27}$ kg
Proton/electron mass ratio	$M/m$	1836.2
Planck constant	$h$	$6.6262 \times 10^{-34}$ J-s
	$\hbar = h/2\pi$	$1.0546 \times 10^{-34}$ J-s
Speed of light in vacuum	$c_0$	$2.9979 \times 10^8$ m/s
Permittivity of free space	$\epsilon_0$	$8.8542 \times 10^{-12}$ F/m
Permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$ H/m
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/e^2m$	$5.2918 \times 10^{-11}$ m
Atomic cross section	$\pi a_0^2$	$8.7974 \times 10^{-21}$ m <sup>2</sup>
Temperature $T$ associated with $T = 1$ V		11605 K
Energy associated with $\mathcal{E} = 1$ V		$1.6022 \times 10^{-19}$ J
Avogadro number (molecules/mol)	$N_A$	$6.0220 \times 10^{23}$
Gas constant	$R = kN_A$	8.3144 J/K-mol
Atomic mass unit		$1.6606 \times 10^{-27}$ kg
Standard temperature (25 °C)	$T_0$	298.15 K
Standard pressure (760 Torr = 1 atm)	$p^\ominus$	$1.0133 \times 10^5$ Pa
Loschmidt's number (density at STP)	$n^\ominus$	$2.6868 \times 10^{25}$ m <sup>-3</sup>
Pressure of 1 Torr		133.32 Pa
Energy per mole at $T_0$	$RT_0$	2.4789 kJ/mol
calorie (cal)		4.1868 J

# PLASMA DENSITY VERSUS TEMPERATURE

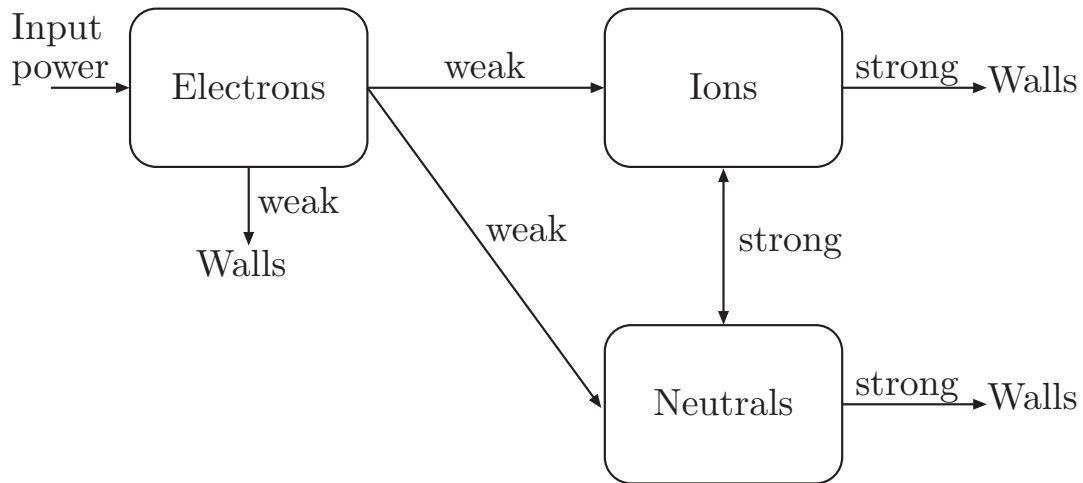


# RELATIVE DENSITIES AND ENERGIES



# NON-EQUILIBRIUM

- Energy coupling between electrons and heavy particles is weak



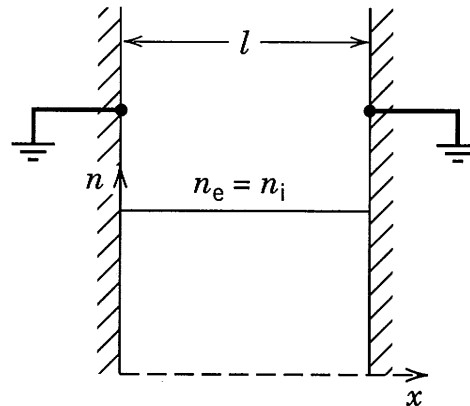
- Electrons are *not* in thermal equilibrium with ions or neutrals

$T_e \gg T_i \quad \text{in plasma bulk}$ $\text{Bombarding } \mathcal{E}_i \gg \mathcal{E}_e \quad \text{at wafer surface}$
--

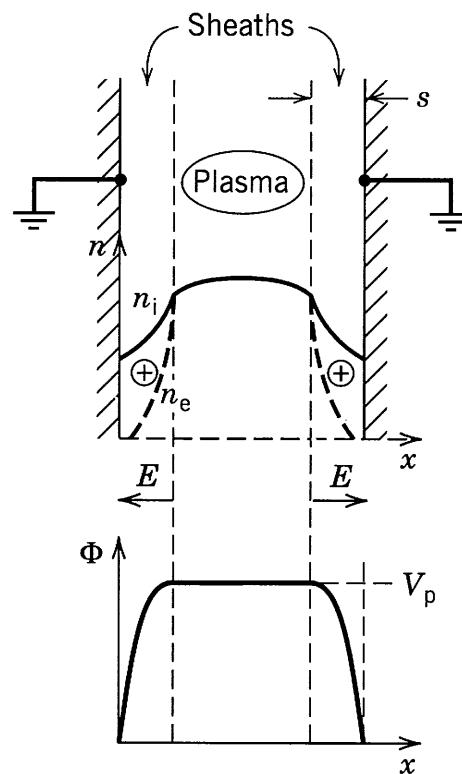
- “High temperature processing at low temperatures”
  1. Wafer can be near room temperature
  2. Electrons produce free radicals  $\implies$  chemistry
  3. Electrons produce electron-ion pairs  $\implies$  ion bombardment

## ELEMENTARY DISCHARGE BEHAVIOR

- Consider uniform density of electrons and ions  $n_e$  and  $n_i$  at time  $t = 0$

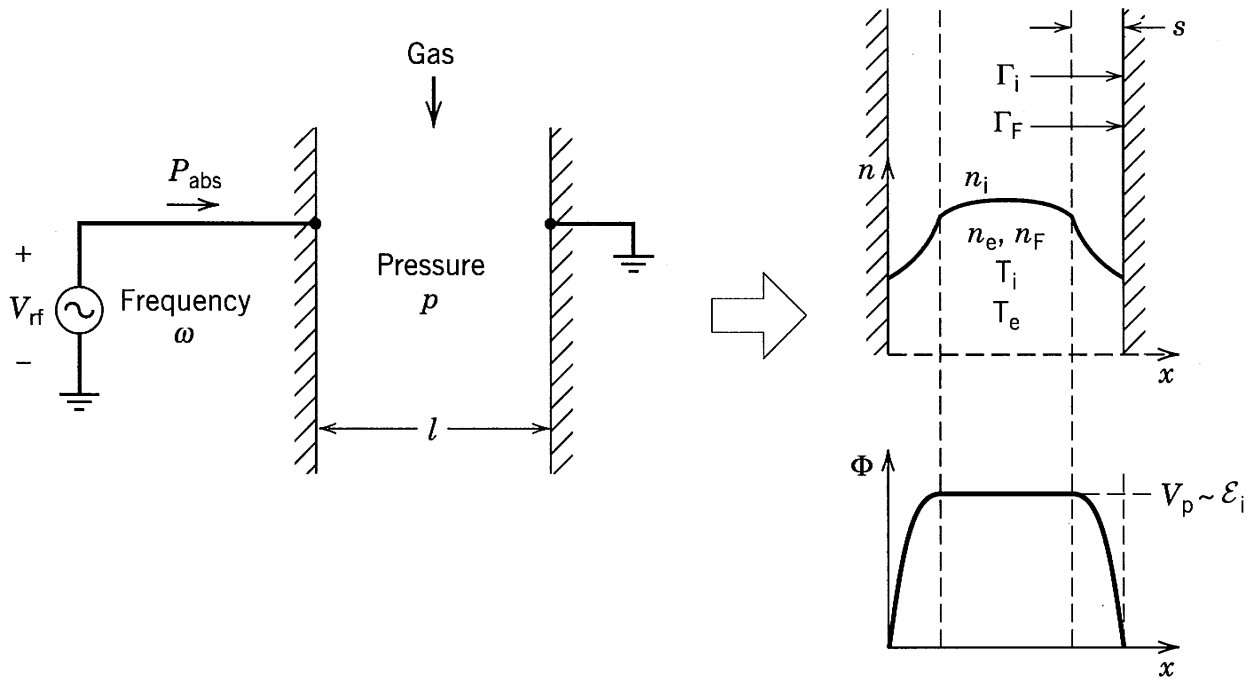


- Warm electrons having low mass quickly drain to the wall, setting up sheaths



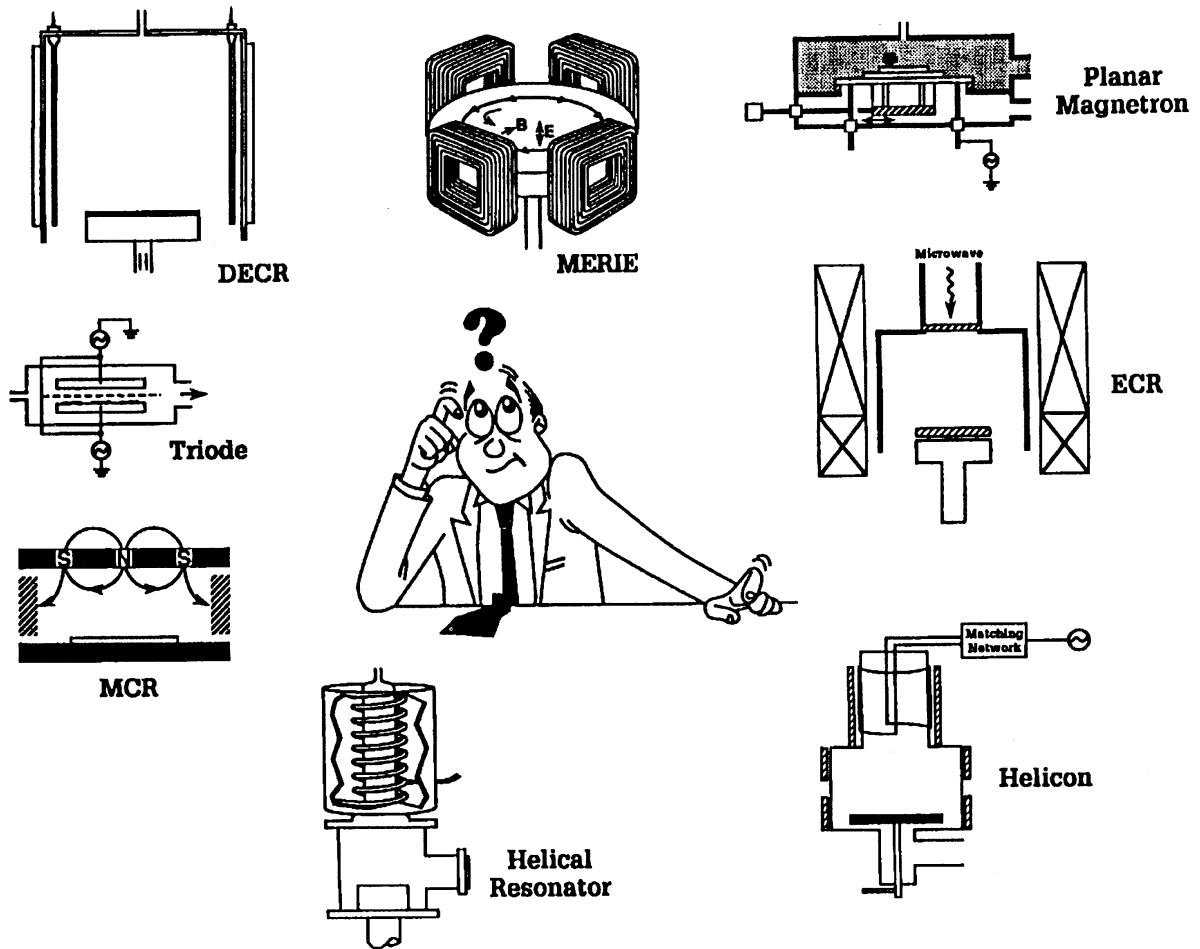
- Ions accelerated to walls; ion bombarding energy  $\mathcal{E}_i = \text{plasma-wall potential } V_p$

# CENTRAL PROBLEM IN DISCHARGE MODELING



- Given  $V_{rf}$  (or  $I_{rf}$  or  $P_{rf}$ ),  $\omega$ , gases, pressure, flow rates, discharge geometry ( $R, l$ , etc), then
- Find plasma densities  $n_e, n_i$ , temperatures  $T_e, T_i$ , ion bombarding energies  $\mathcal{E}_i$ , sheath thicknesses, neutral radical densities, potentials, currents, fluxes, etc
- Learn how to design and optimize plasma reactors for various purposes (etching, deposition, etc)

# CHOOSING PLASMA PROCESSING EQUIPMENT



- How about inductive? (figure published in 1991)

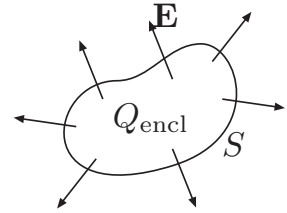


# **SUMMARY OF PLASMA FUNDAMENTALS**

## POISSON'S EQUATION

- An electric field can be generated by charges:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad \text{or} \quad \oiint_S \bar{\mathbf{E}} \cdot d\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$



- For slow time variations (dc, rf, but not microwaves):

$$\mathbf{E} = -\nabla\Phi$$

Combining these yields Poisson's equation:

$$\nabla^2\Phi = -\frac{\rho}{\epsilon_0}$$

- Here  $\mathbf{E}$  = electric field (V/m),  $\rho$  = charge density (C/m<sup>3</sup>),  
 $\Phi$  = potential (V)

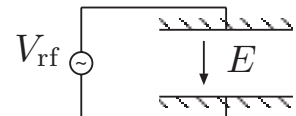
- In 1D:

$$\frac{dE_x}{dx} = \frac{\rho}{\epsilon_0}, \quad E_x = -\frac{d\Phi}{dx}$$

yields

$$\boxed{\frac{d^2\Phi}{dx^2} = -\frac{\rho}{\epsilon_0}}$$

- This field powers a capacitive discharge or the wafer bias power of an inductive or ECR discharge



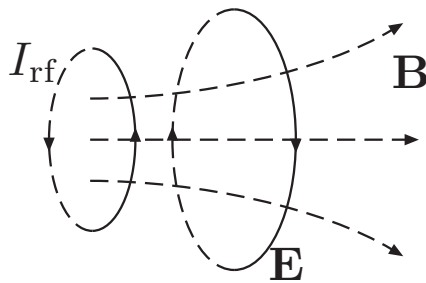
## FARADAY'S LAW

- An electric field can be generated by a time-varying magnetic field:

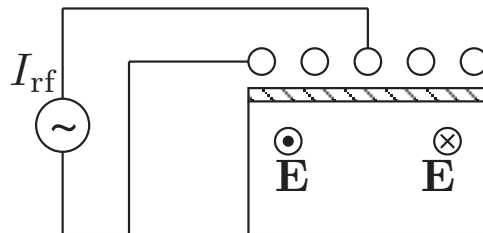
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

or

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \iint_A \mathbf{B} \cdot d\mathbf{A}$$



- Here  $\mathbf{B}$  = magnetic induction vector
- This field powers the coil of an inductive discharge (top power)



## AMPERE'S LAW

- Both conduction currents and displacement currents generate magnetic fields:

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}_T$$

- $\mathbf{J}_c$  = conduction current,  $\epsilon_0 \partial \mathbf{E} / \partial t$  = displacement current,  $\mathbf{J}_T$  = total current,  $\mathbf{H}$  = magnetic field vector,  $\mathbf{B} = \mu_0 \mathbf{H}$  with  $\mu_0 = 4\pi \times 10^{-6}$  H/m
- Note the vector identity:

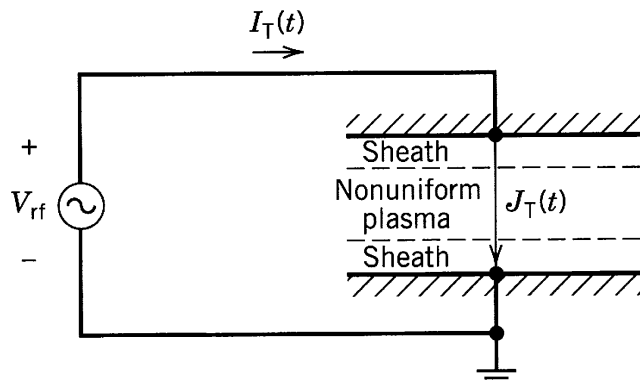
$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 \quad \Rightarrow \quad \nabla \cdot \mathbf{J}_T = 0$$

- In 1D:

$$\frac{\partial J_{Tx}(x, t)}{\partial x} = 0$$

so

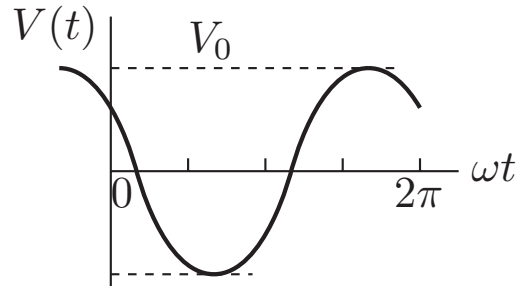
$J_{Tx} = J_{Tx}(t), \text{ independent of } x$



## REVIEW OF PHASORS

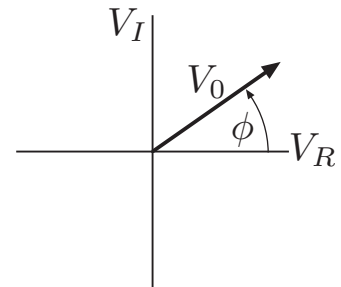
- Physical voltage (or current), a real sinusoidal function of time

$$V(t) = V_0 \cos(\omega t + \phi)$$



- Phasor voltage (or current), a complex number, independent of time

$$\tilde{V} = V_0 e^{j\phi} = V_R + jV_I$$



- Using  $e^{j\phi} = \cos \phi + j \sin \phi$ , we find

$$V_R = V_0 \cos \phi, \quad V_I = V_0 \sin \phi$$

- Note that

$$\boxed{V(t) = \text{Re} \left( \tilde{V} e^{j\omega t} \right)}$$

$$= V_0 \cos(\omega t + \phi)$$

$$= V_R \cos \omega t - V_I \sin \omega t$$

- Hence

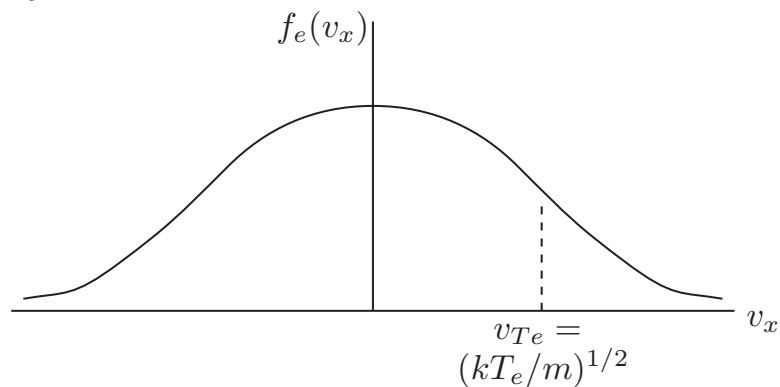
$$V(t) \iff \tilde{V} \quad (\text{given } \omega)$$

## THERMAL EQUILIBRIUM PROPERTIES

- Electrons generally near thermal equilibrium  
Ions generally *not* in thermal equilibrium
- Maxwellian distribution of electrons

$$f_e(v) = n_e \left( \frac{m}{2\pi kT_e} \right)^{3/2} \exp \left( -\frac{mv^2}{2kT_e} \right)$$

where  $v^2 = v_x^2 + v_y^2 + v_z^2$



- Pressure  $p = nkT$

For neutral gas at room temperature (300 K)

$$n_g(\text{cm}^{-3}) \approx 3.3 \times 10^{16} p(\text{Torr})$$

## AVERAGES OVER MAXWELLIAN DISTRIBUTION

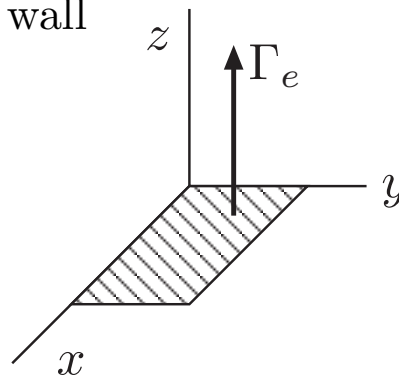
- Average energy

$$\langle \frac{1}{2}mv^2 \rangle = \frac{1}{n_e} \int d^3v \frac{1}{2}mv^2 f_e(v) = \frac{3}{2}kT_e$$

- Average speed

$$\bar{v}_e = \frac{1}{n_e} \int d^3v v f_e(v) = \boxed{\left( \frac{8kT_e}{\pi m} \right)^{1/2}}$$

- Average electron flux lost to a wall



$$\Gamma_e = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_0^{\infty} dv_z v_z f_e(v) = \boxed{\frac{1}{4}n_e \bar{v}_e} \quad [\text{m}^{-2}\text{-s}^{-1}]$$

- Average kinetic energy lost per electron lost to a wall

$$\mathcal{E}_e = 2T_e$$

## FORCES ON PARTICLES

- For a unit volume of electrons (or ions),

$$mn_e \frac{d\mathbf{u}_e}{dt} = qn_e \mathbf{E} - \nabla p_e - mn_e \nu_m \mathbf{u}_e$$

mass  $\times$  acceleration = electric field force +

+ pressure gradient force + friction (gas drag) force

- $m$  = electron mass

$n_e$  = electron density

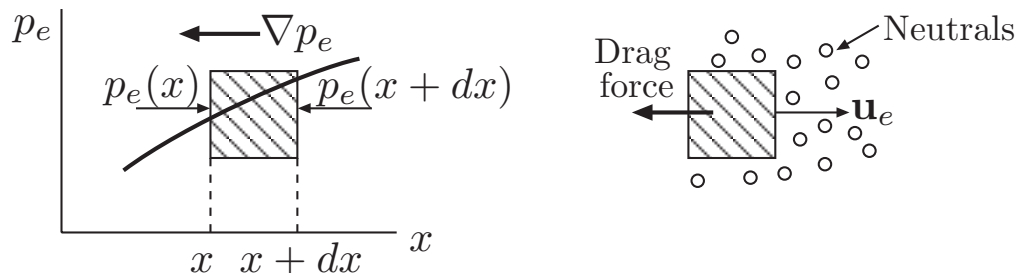
$\mathbf{u}_e$  = electron flow velocity

$q = -e$  for electrons ( $+e$  for ions)

$\mathbf{E}$  = electric field

$p_e = n_e k T_e$  = electron pressure

$\nu_m$  = collision frequency of electrons with neutrals





## BOLTZMANN FACTOR FOR ELECTRONS

- If electric field and pressure gradient forces almost balance:

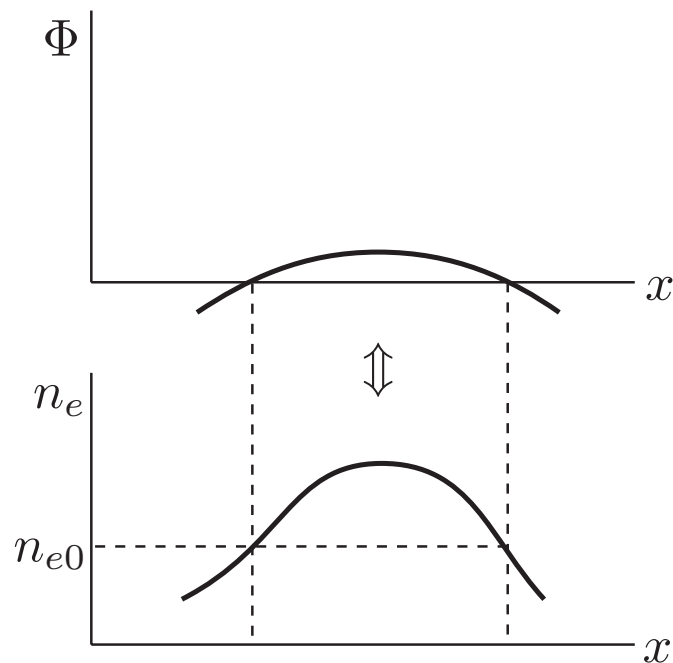
$$0 \approx -en_e \mathbf{E} - \nabla p_e$$

- Let  $\mathbf{E} = -\nabla\Phi$  and  $p_e = n_e kT_e$ :

$$\nabla\Phi = \frac{kT_e}{e} \frac{\nabla n_e}{n_e}$$

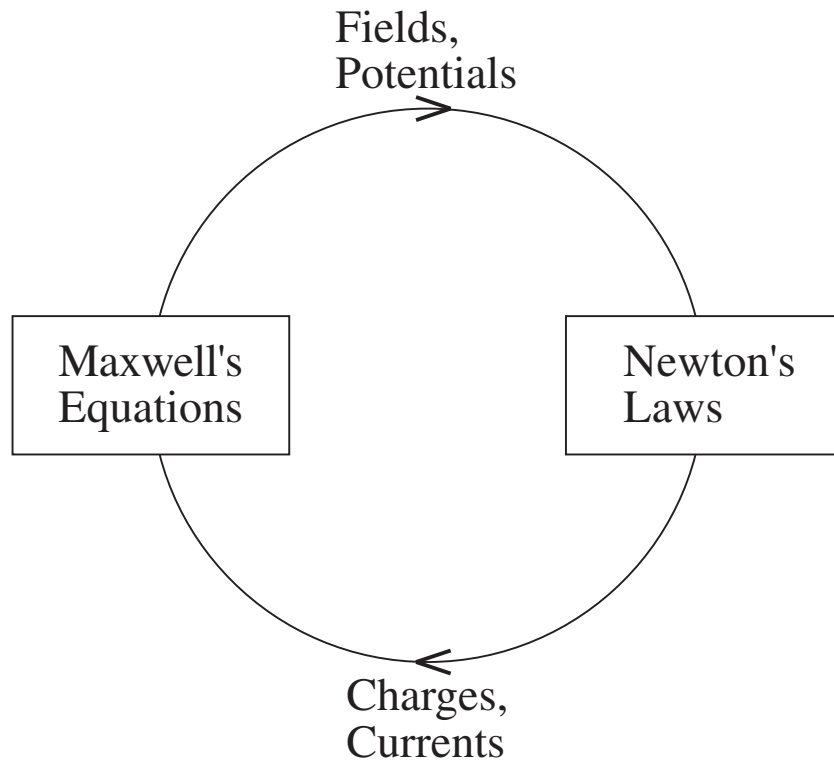
- Put  $kT_e/e = T_e$  (volts) and integrate to obtain:

$$n_e(\mathbf{r}) = n_{e0} e^{\Phi(\mathbf{r})/T_e}$$



# UNDERSTANDING PLASMA BEHAVIOR

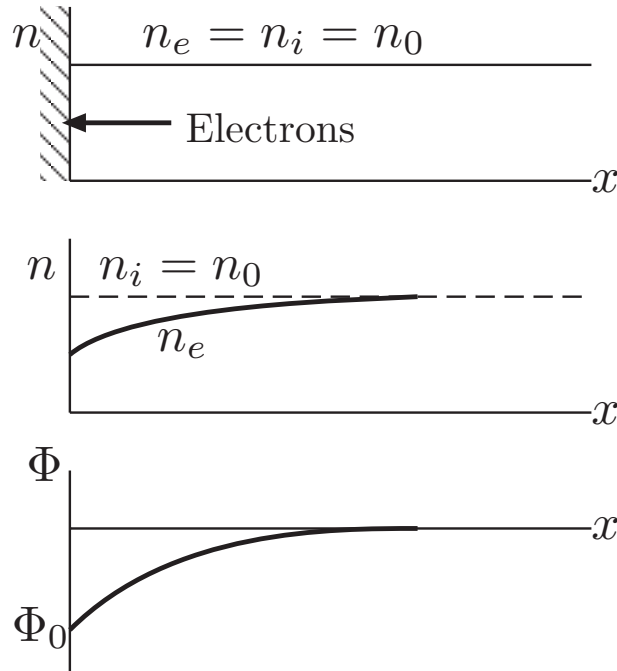
- The field equations and the force equations are coupled



## DEBYE LENGTH $\lambda_{De}$

- The characteristic length scale of a plasma
- Low voltage sheaths  $\sim$  few Debye lengths thick
- Let's consider how a sheath forms near a wall:

Electrons leave plasma before ions and charge wall negative



Assume electrons in thermal equilibrium and stationary ions

## DEBYE LENGTH $\lambda_{De}$ (CONT'D)

- Newton's laws

$$n_e(x) = n_0 e^{\Phi/T_e}, \quad n_i = n_0$$

- Use in Poisson's equation

$$\frac{d^2\Phi}{dx^2} = -\frac{en_0}{\epsilon_0} \left(1 - e^{\Phi/T_e}\right)$$

- Linearize  $e^{\Phi/T_e} \approx 1 + \Phi/T_e$

$$\frac{d^2\Phi}{dx^2} = \frac{en_0}{\epsilon_0 T_e} \Phi$$

- Solution is

$$\Phi(x) = \Phi_0 e^{-x/\lambda_{De}}, \quad \lambda_{De} = \left(\frac{\epsilon_0 T_e}{en_0}\right)^{1/2}$$

- In practical units

$$\lambda_{De}(\text{cm}) = 740 \sqrt{T_e/n_0}, \quad T_e \text{ in volts, } n_0 \text{ in cm}^{-3}$$

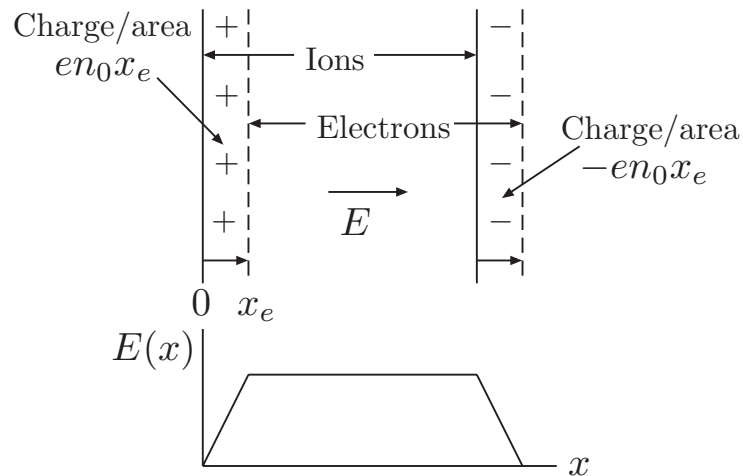
- Example

At  $T_e = 1$  V and  $n_0 = 10^{10}$  cm $^{-3}$ ,  $\lambda_{De} = 7.4 \times 10^{-3}$  cm

$\implies$  Sheath is  $\sim 0.15$  mm thick (Very thin!)

## ELECTRON PLASMA FREQUENCY $\omega_{pe}$

- The fundamental timescale for a plasma
- Consider a plasma slab (no walls). Displace all electrons to the right a small distance  $x_{e0}$ , and release them:



- Maxwell's equations (parallel plate capacitor)

$$E = \frac{en_0x_e(t)}{\epsilon_0}$$

- Newton's laws (electron motion)

$$m \frac{d^2x_e(t)}{dt^2} = -\frac{e^2n_0}{\epsilon_0}x_e(t)$$

- Solution is electron plasma oscillations

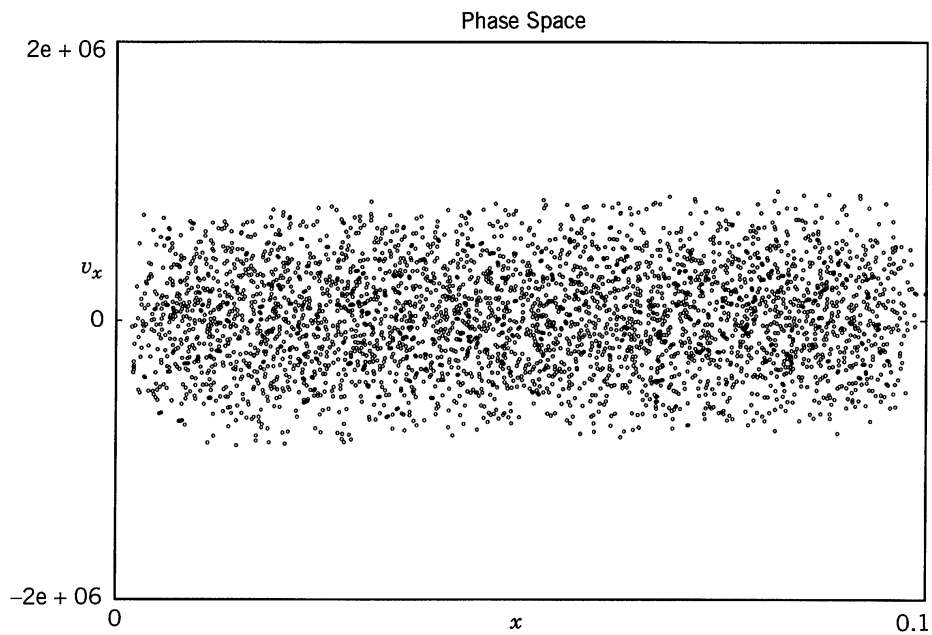
$$x_e(t) = x_{e0} \cos \omega_{pe}t, \quad \boxed{\omega_{pe} = \left( \frac{e^2n_0}{\epsilon_0 m} \right)^{1/2}}$$

- Practical formula is  $f_{pe}(\text{Hz}) = 9000\sqrt{n_0}$ ,  $n_0$  in  $\text{cm}^{-3}$   
 $\implies$  microwave frequencies ( $\gtrsim 1$  GHz) for typical plasmas

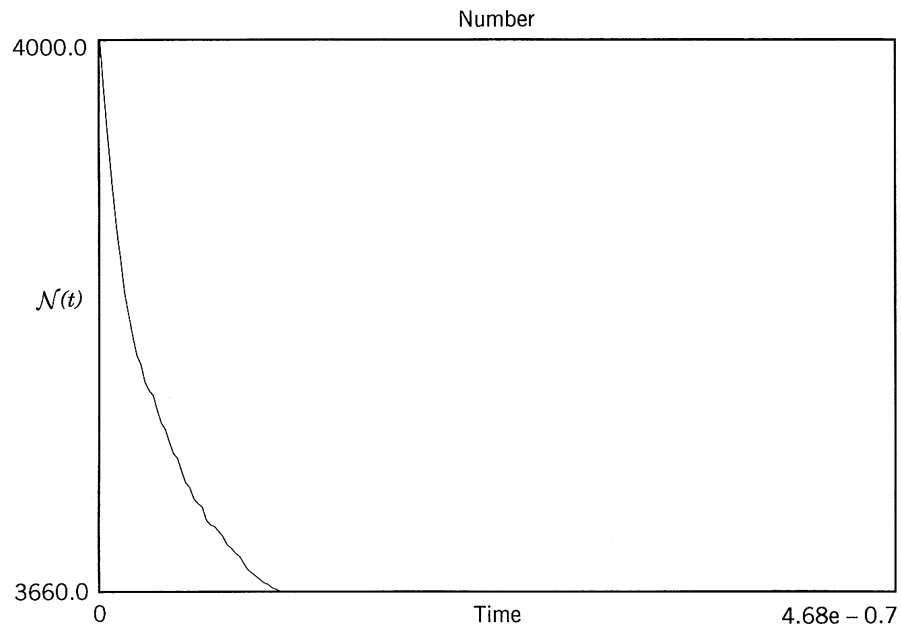
# 1D SIMULATION OF SHEATH FORMATION

$$(T_e = 1 \text{ V}, n_e = n_i = 10^{13} \text{ m}^{-3})$$

- Electron  $v_x$ - $x$  phase space at  $t = 0.77 \mu\text{s}$

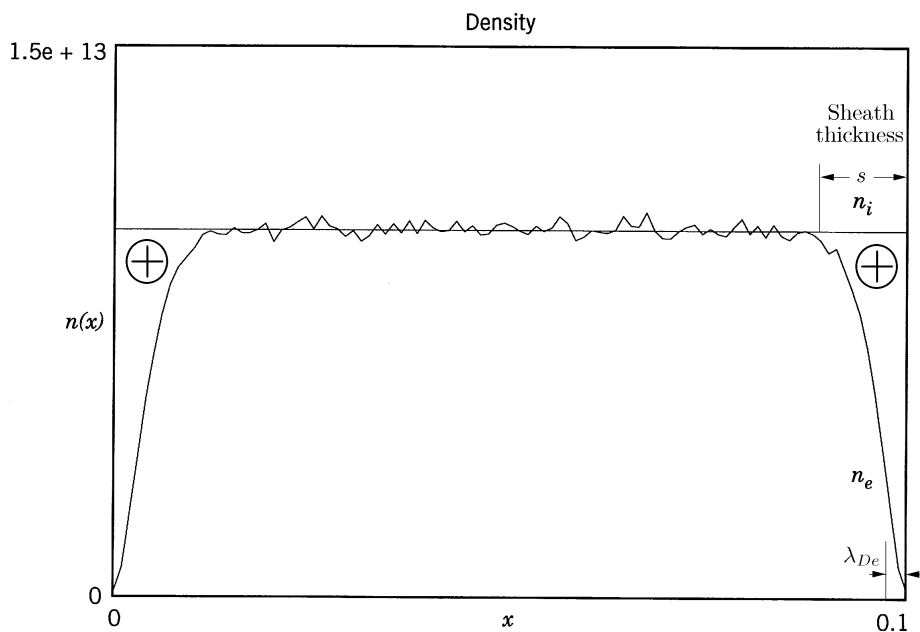


- Electron number  $\mathcal{N}$  versus  $t$

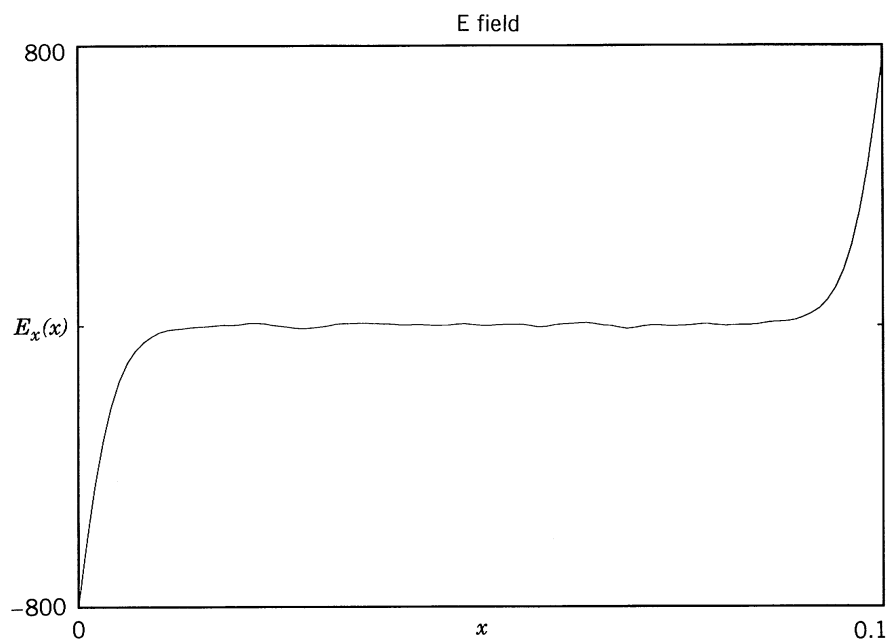


# 1D SIMULATION OF SHEATH FORMATION (CONT'D)

- Electron density  $n_e(x)$  at  $t = 0.77 \mu s$

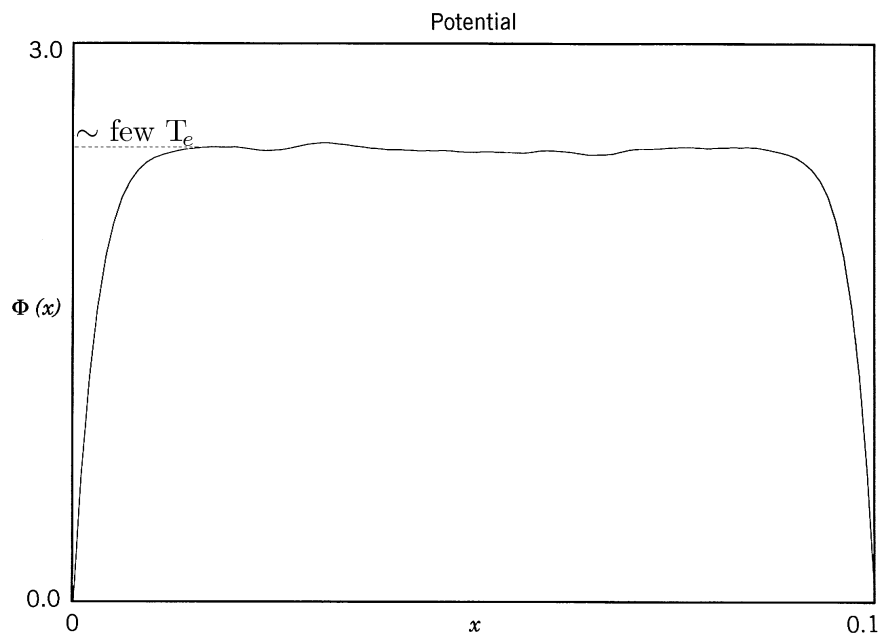


- Electric field  $E(x)$  at  $t = 0.77 \mu s$

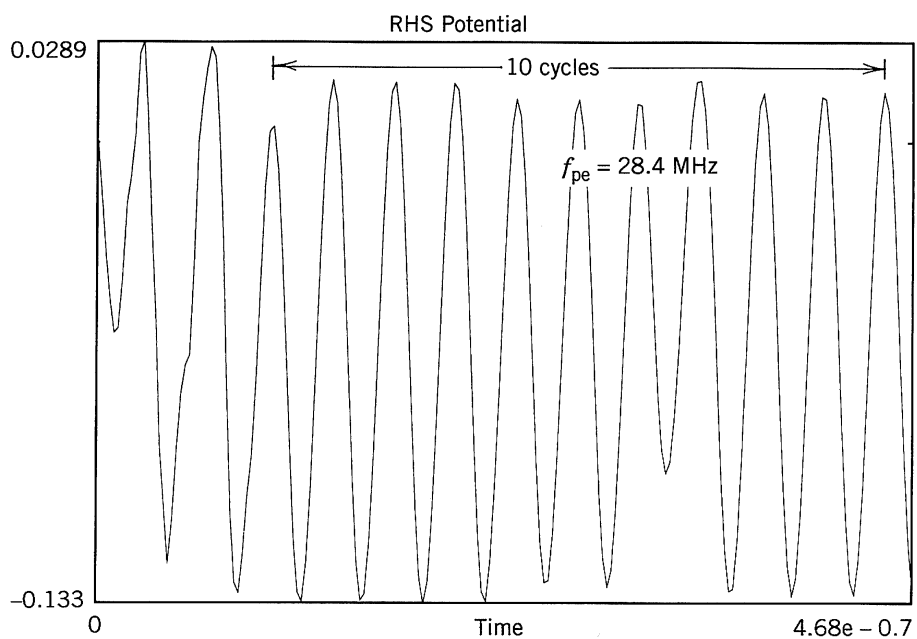


# 1D SIMULATION OF SHEATH FORMATION (CONT'D)

- Potential  $\Phi(x)$  at  $t = 0.77 \mu\text{s}$



- Right hand potential  $\Phi(x = l)$  versus  $t$





## PLASMA DIELECTRIC CONSTANT $\epsilon_p$

- RF discharges are driven at a frequency  $\omega$

$$E(t) = \text{Re} (\tilde{E} e^{j\omega t}), \quad \text{etc}$$

- Define  $\epsilon_p$  from the total current in Maxwell's equations

$$\nabla \times \tilde{H} = \underbrace{\tilde{J}_c + j\omega\epsilon_0\tilde{E}}_{\text{Total current } \tilde{J}} \equiv j\omega\epsilon_p\tilde{E}$$

- Conduction current  $\tilde{J}_c = -en_e\tilde{u}_e$  is mainly due to electrons
- Newton's law (electric field and neutral drag) is

$$j\omega m\tilde{u}_e = -e\tilde{E} - m\nu_m\tilde{u}_e$$

- Solve for  $\tilde{u}_e$  and evaluate  $\tilde{J}_c$  to obtain

$$\epsilon_p = \epsilon_0 \left[ 1 - \frac{\omega_{pe}^2}{\omega(\omega - j\nu_m)} \right]$$

- For  $\omega \gg \nu_m$ ,  $\epsilon_p$  is mainly real (nearly lossless dielectric)  
For  $\nu_m \gg \omega$ ,  $\epsilon_p$  is mainly imaginary (very lossy dielectric)

## RF FIELDS IN LOW PRESSURE DISCHARGES

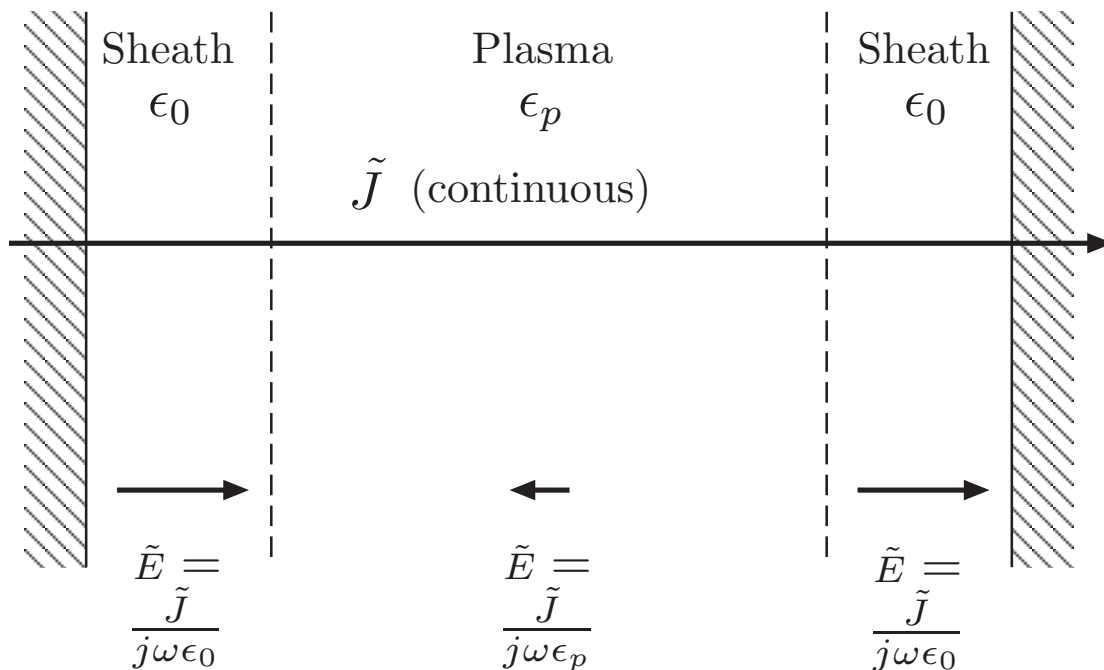
- Consider mainly lossless plasma ( $\omega \gg \nu_m$ )

$$\epsilon_p = \epsilon_0 \left( 1 - \frac{\omega_{pe}^2}{\omega^2} \right)$$

- For almost all RF discharges,  $\omega_{pe} \gg \omega$

$\implies \epsilon_p$  is negative

- Typical case:  $\epsilon_p = -1000 \epsilon_0$



- Electric field in plasma is  $1000 \times$  smaller than in sheaths!
- Although field in plasma is small, it sustains the plasma!

## PLASMA CONDUCTIVITY $\sigma_p$

- Useful to introduce the plasma conductivity  $\tilde{J}_c \equiv \sigma_p \tilde{E}$
- RF plasma conductivity

$$\sigma_p = \frac{e^2 n_e}{m(\nu_m + j\omega)}$$

- DC plasma conductivity ( $\omega \ll \nu_m$ )

$$\sigma_{dc} = \frac{e^2 n_e}{m\nu_m}$$

- The plasma dielectric constant and conductivity are related by:

$$j\omega\epsilon_p = \sigma_p + j\omega\epsilon_0$$

- Due to  $\sigma_p$ , rf current flowing through the plasma heats electrons (just like a resistor)

## OHMIC HEATING POWER

- Time average power absorbed/volume

$$p_d = \langle \mathbf{J}(t) \cdot \mathbf{E}(t) \rangle = \frac{1}{2} \operatorname{Re}(\tilde{J} \cdot \tilde{E}^*) \quad [\text{W/m}^3]$$

- Put  $\tilde{J} = (\sigma_p + j\omega\epsilon_0)\tilde{E}$  to find  $p_d$  in terms of  $\tilde{E}$

$$p_d = \frac{1}{2} |\tilde{E}|^2 \sigma_{\text{dc}} \frac{\nu_m^2}{\omega^2 + \nu_m^2}$$

- Put  $\tilde{E} = \tilde{J}/(\sigma_p + j\omega\epsilon_0)$  to find  $p_d$  in terms of  $\tilde{J}$ .

For almost all rf discharges ( $\omega_{pe} \gg \omega$ )

$$\boxed{p_d = \frac{1}{2} |\tilde{J}|^2 \frac{1}{\sigma_{\text{dc}}}}$$

## **SUMMARY OF DISCHARGE FUNDAMENTALS**

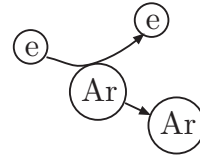
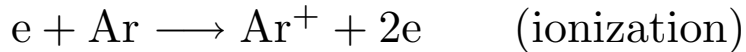
## ELECTRON COLLISIONS WITH ARGON

- Maxwellian electrons collide with Ar atoms (density  $n_g$ )

$$\frac{dn_e}{dt} = \nu n_e = K n_g n_e$$

$\nu$  = collision frequency [ $s^{-1}$ ],  $K(T_e)$  = rate coefficient [ $m^3/s$ ]

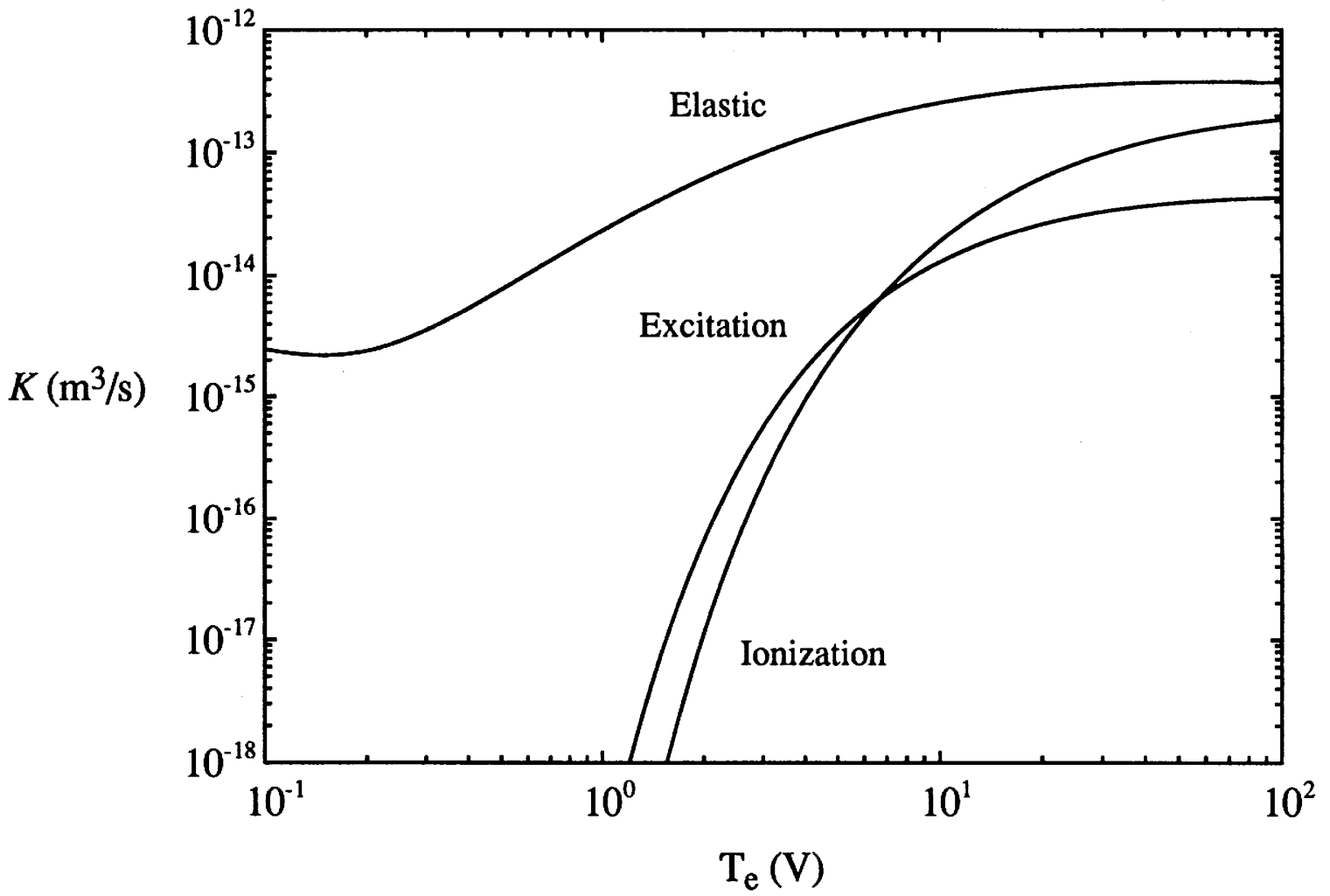
- Electron-Ar collision processes



- Rate coefficient  $K(T_e)$  is average of cross section  $\sigma$  [ $m^2$ ] for process, over Maxwellian distribution

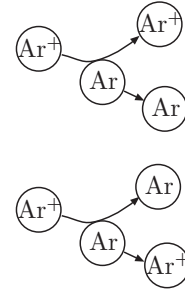
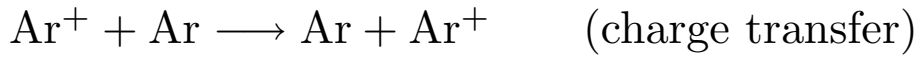
$$K(T_e) = \langle \sigma v \rangle_{\text{Maxwellian}}$$

# ELECTRON-ARGON RATE COEFFICIENTS



## ION COLLISIONS WITH ARGON

- Argon ions collide with Ar atoms



- Total cross section for room temperature ions  $\sigma_i \approx 10^{-14} \text{ cm}^2$
- Ion-neutral mean free path

$$\lambda_i = \frac{1}{n_g \sigma_i}$$

- Practical formula

$$\lambda_i(\text{cm}) = \frac{1}{330 p}, \quad p \text{ in Torr}$$

- Rate coefficient for ion-neutral collisions

$$K_i = \frac{\bar{v}_i}{\lambda_i}$$

with  $\bar{v}_i = (8kT_i/\pi M)^{1/2}$



## THREE ENERGY LOSS PROCESSES

1. Collisional energy  $\mathcal{E}_c$  lost per electron-ion pair created

$$K_{\text{iz}}\mathcal{E}_c = K_{\text{iz}}\mathcal{E}_{\text{iz}} + K_{\text{ex}}\mathcal{E}_{\text{ex}} + K_{\text{el}}(2m/M)(3T_e/2)$$

$$\implies \mathcal{E}_c(T_e) \quad (\text{voltage units})$$

$\mathcal{E}_{\text{iz}}$ ,  $\mathcal{E}_{\text{ex}}$ , and  $(3m/M)T_e$  are energies lost by an electron due to an ionization, excitation, and elastic scattering collision

2. Electron kinetic energy lost to walls

$$\mathcal{E}_e = 2 T_e$$

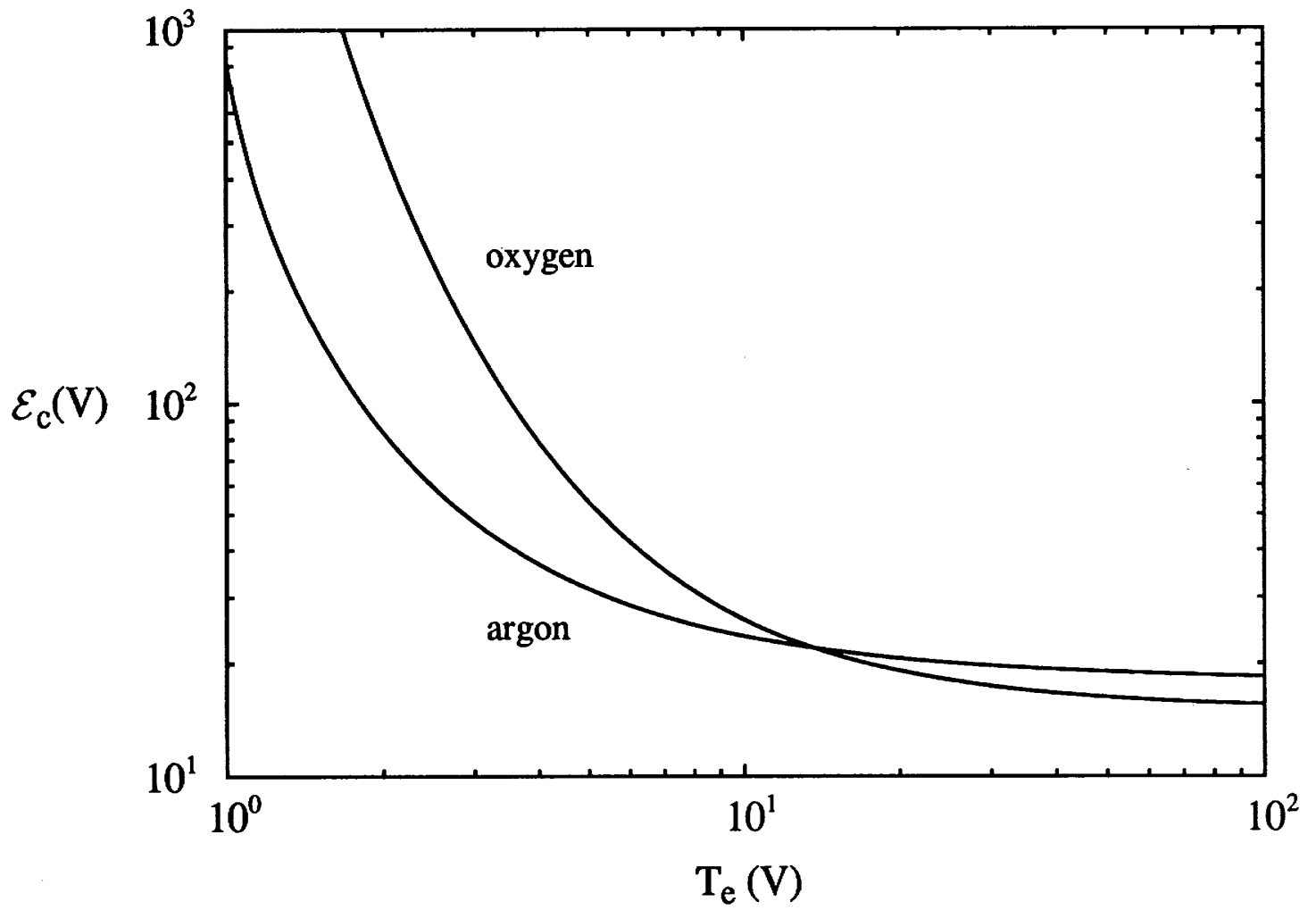
3. Ion kinetic energy lost to walls is mainly due to the dc potential  $\bar{V}_s$  across the sheath

$$\mathcal{E}_i \approx \bar{V}_s$$

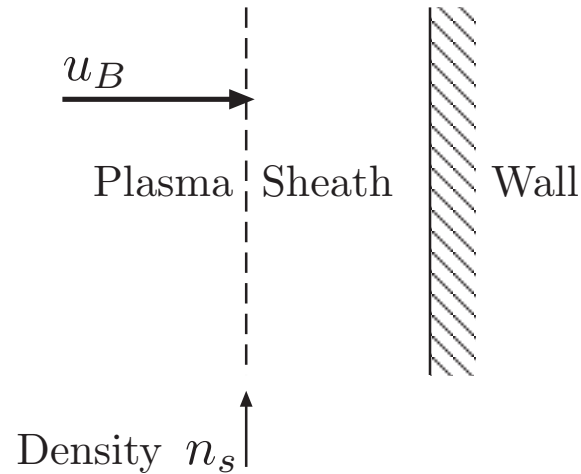
- Total energy lost per electron-ion pair lost to walls

$$\boxed{\mathcal{E}_T = \mathcal{E}_c + \mathcal{E}_e + \mathcal{E}_i}$$

# COLLISIONAL ENERGY LOSSES



## BOHM (ION LOSS) VELOCITY $u_B$



- Due to formation of a “presheath”, ions arrive at the plasma-sheath edge with directed energy  $kT_e/2$

$$\frac{1}{2}Mu_i^2 = \frac{kT_e}{2}$$

- At the plasma-sheath edge (density  $n_s$ ), electron-ion pairs are lost at the Bohm velocity

$$u_i = u_B = \left( \frac{kT_e}{M} \right)^{1/2}$$

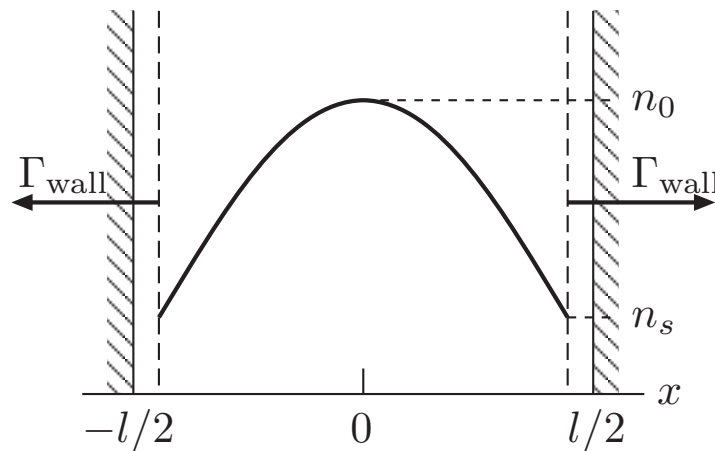
## AMBIPOLAR DIFFUSION AT HIGH PRESSURES

- Plasma bulk is quasi-neutral ( $n_e \approx n_i = n$ ) and the electron and ion loss fluxes are equal ( $\Gamma_e \approx \Gamma_i \approx \Gamma$ )
- Fick's law

$$\Gamma = -D_a \nabla n$$

with ambipolar diffusion coefficient  $D_a = kT_e/M\nu_i$

- Density profile is sinusoidal



- Loss flux to the wall is

$$\Gamma_{\text{wall}} = h_l n_0 u_B$$

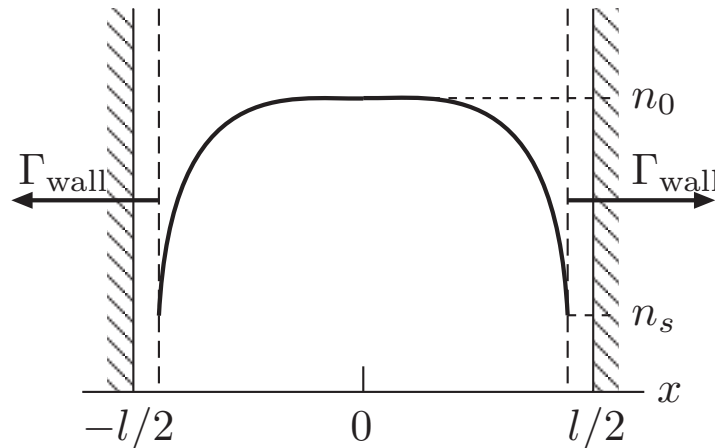
where the edge-to-center density ratio is

$$h_l \equiv \frac{n_s}{n_0} = \frac{\pi}{l} \frac{u_B}{\nu_i}$$

- Applies for pressures  $> 100$  mTorr in argon

## AMBIPOLAR DIFFUSION AT LOW PRESSURES

- The diffusion coefficient is not constant
- Density profile is relatively flat in the center and falls sharply near the sheath edge



- For a cylindrical plasma of length  $l$  and radius  $R$ , loss fluxes to axial and radial walls are

$$\Gamma_{\text{axial}} = h_l n_0 u_B, \quad \Gamma_{\text{radial}} = h_R n_0 u_B$$

where the edge-to-center density ratios are

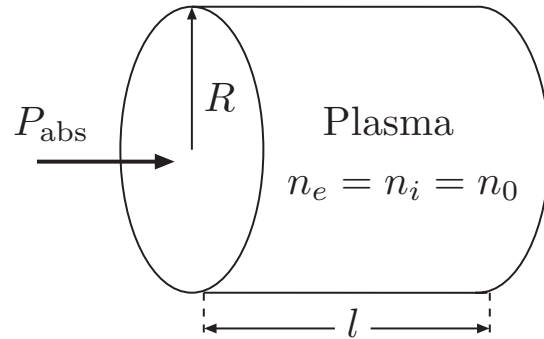
$$h_l \approx \frac{0.86}{(3 + l/2\lambda_i)^{1/2}}, \quad h_R \approx \frac{0.8}{(4 + R/\lambda_i)^{1/2}}$$

- Applies for pressures  $< 100$  mTorr in argon

# ANALYSIS OF DISCHARGE EQUILIBRIUM

## PARTICLE BALANCE AND $T_e$

- Assume uniform cylindrical plasma absorbing power  $P_{\text{abs}}$



- Particle balance

Production due to ionization = loss to the walls

$$K_{\text{iz}} n_g n_0 \pi R^2 l = (2\pi R^2 h_l n_0 + 2\pi R l h_R n_0) u_B$$

- Solve to obtain

$$\frac{K_{\text{iz}}(T_e)}{u_B(T_e)} = \frac{1}{n_g d_{\text{eff}}}$$

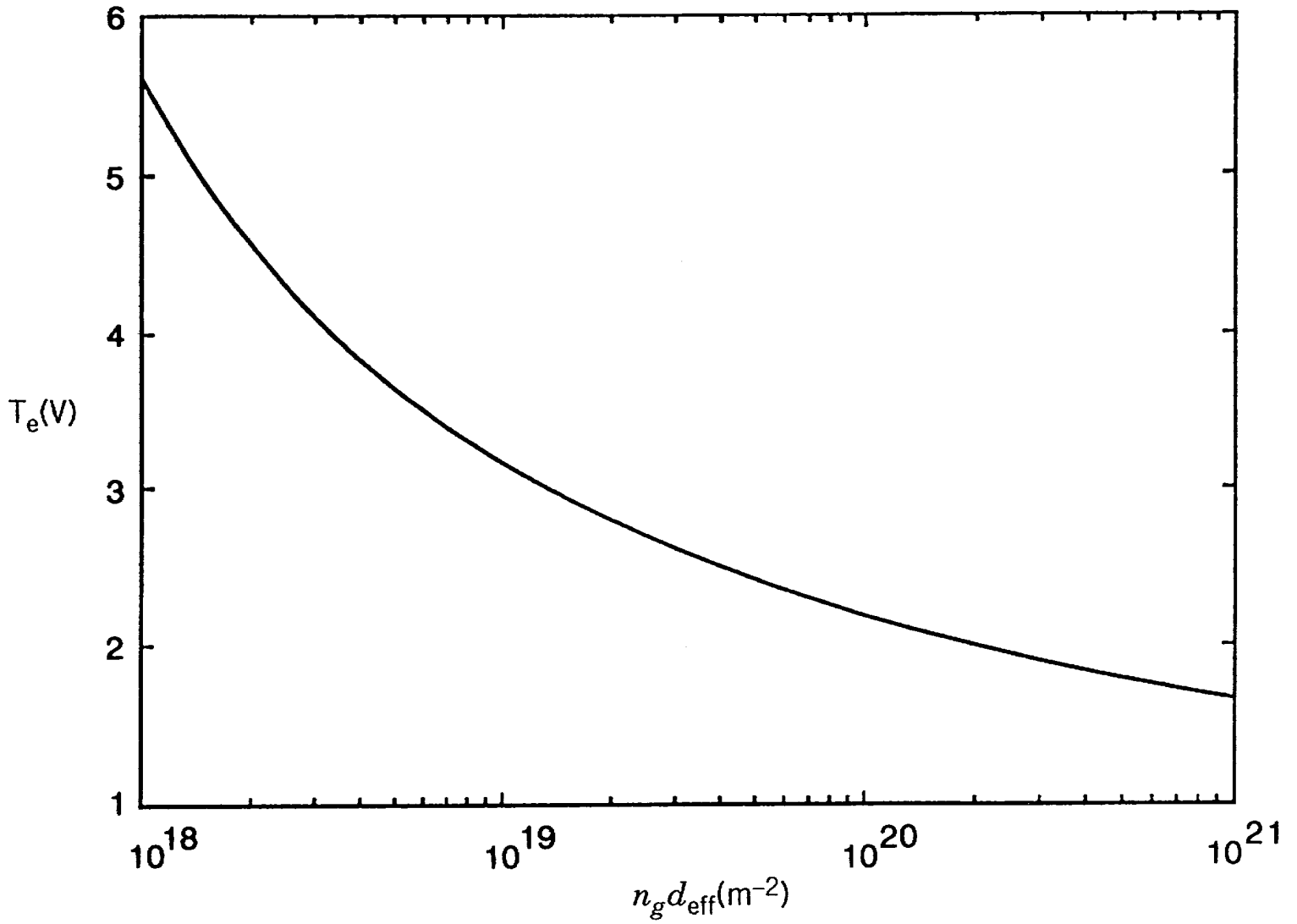
where

$$d_{\text{eff}} = \frac{1}{2} \frac{Rl}{Rh_l + lh_R}$$

is an effective plasma size

- Given  $n_g$  and  $d_{\text{eff}} \implies$  electron temperature  $T_e$
- $T_e$  varies over a narrow range of 2–5 volts

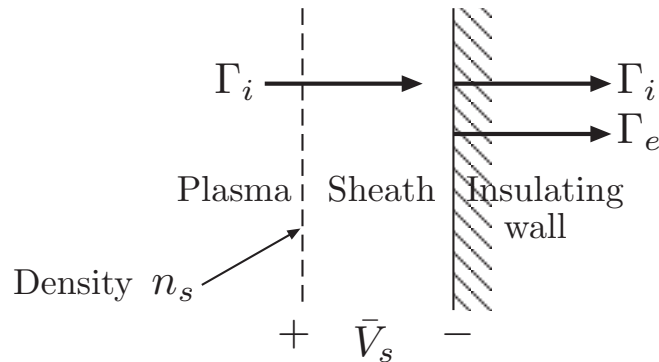
# ELECTRON TEMPERATURE IN ARGON DISCHARGE





## ION ENERGY FOR LOW VOLTAGE SHEATHS

- $\mathcal{E}_i$  = energy entering sheath + energy gained traversing sheath
- Ion energy entering sheath =  $T_e/2$  (voltage units)
- Sheath voltage determined from particle conservation in the sheath



$$\Gamma_i = n_s u_B, \quad \Gamma_e = \frac{1}{4} n_s \bar{v}_e e^{-\bar{V}_s/T_e}$$

with  $\bar{v}_e = (8eT_e/\pi m)^{1/2}$

- The ion and electron fluxes must balance

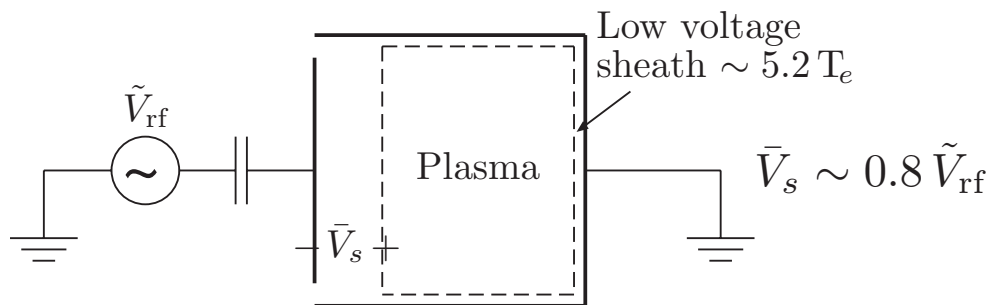
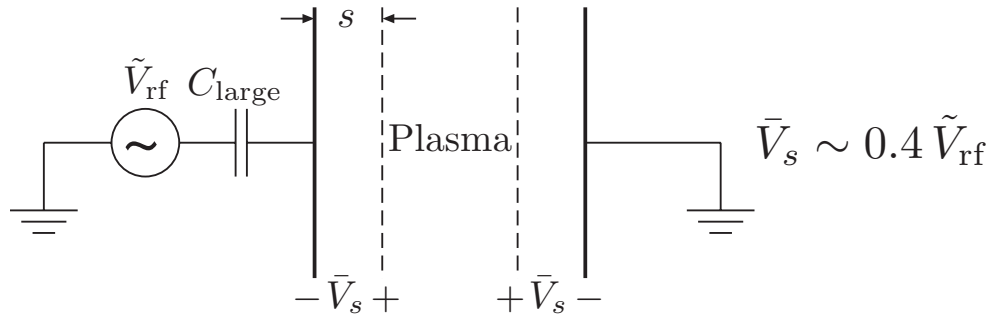
$$\bar{V}_s = \frac{T_e}{2} \ln \left( \frac{M}{2\pi m} \right)$$

or  $\bar{V}_s \approx 4.7 T_e$  for argon

- Accounting for the initial ion energy,  $\mathcal{E}_i \approx 5.2 T_e$

## ION ENERGY FOR HIGH VOLTAGE SHEATHS

- Large ion bombarding energies can be gained near rf-driven electrodes embedded in the plasma



- The sheath thickness  $s$  is given by the Child Law

$$\bar{J}_i = en_s u_B = \frac{4}{9} \epsilon_0 \left( \frac{2e}{M} \right)^{1/2} \frac{\bar{V}_s^{3/2}}{s^2}$$

- Estimating ion energy is not simple as it depends on the type of discharge and the application of bias voltages

## POWER BALANCE AND $n_0$

- Assume low voltage sheaths at all surfaces

$$\mathcal{E}_T(T_e) = \underbrace{\mathcal{E}_c(T_e)}_{\text{Collisional}} + \underbrace{2T_e}_{\text{Electron}} + \underbrace{5.2T_e}_{\text{Ion}}$$

- Power balance

Power in = power out

$$P_{\text{abs}} = (h_l n_0 2\pi R^2 + h_R n_0 2\pi Rl) u_B e \mathcal{E}_T$$

- Solve to obtain

$$n_0 = \frac{P_{\text{abs}}}{A_{\text{eff}} u_B e \mathcal{E}_T}$$

where

$$A_{\text{eff}} = 2\pi R^2 h_l + 2\pi Rl h_R$$

is an effective area for particle loss

- Density  $n_0$  is proportional to the absorbed power  $P_{\text{abs}}$
- Density  $n_0$  depends on pressure  $p$  through  $h_l$ ,  $h_R$ , and  $T_e$

## PARTICLE AND POWER BALANCE

- Particle balance  $\implies$  electron temperature  $T_e$   
(independent of plasma density)
  
- Power balance  $\implies$  plasma density  $n_0$   
(once electron temperature  $T_e$  is known)

## EXAMPLE 1

- Let  $R = 0.15$  m,  $l = 0.3$  m,  $n_g = 3.3 \times 10^{19}$  m<sup>-3</sup> ( $p = 1$  mTorr at 300 K), and  $P_{\text{abs}} = 800$  W
- Assume low voltage sheaths at all surfaces
- Find  $\lambda_i = 0.03$  m. Then  $h_l \approx h_R \approx 0.3$  and  $d_{\text{eff}} \approx 0.17$  m
- From the  $T_e$  versus  $n_g d_{\text{eff}}$  figure,  $T_e \approx 3.5$  V
- From the  $\mathcal{E}_c$  versus  $T_e$  figure,  $\mathcal{E}_c \approx 42$  V. Adding  $\mathcal{E}_e = 2T_e \approx 7$  V and  $\mathcal{E}_i \approx 5.2T_e \approx 18$  V yields  $\mathcal{E}_T = 67$  V
- Find  $u_B \approx 2.9 \times 10^3$  m/s and find  $A_{\text{eff}} \approx 0.13$  m<sup>2</sup>
- Power balance yields  $n_0 \approx 2.0 \times 10^{17}$  m<sup>-3</sup>
- Ion current density  $J_{il} = eh_l n_0 u_B \approx 2.9$  mA/cm<sup>2</sup>
- Ion bombarding energy  $\mathcal{E}_i \approx 18$  V

## EXAMPLE 2

- Apply a strong dc magnetic field along the cylinder axis  
 $\implies$  particle loss to radial wall is inhibited
  - For no radial loss,  $d_{\text{eff}} = l/2h_l \approx 0.5$  m
  - From the  $T_e$  versus  $n_g d_{\text{eff}}$  figure,  $T_e \approx 3.3$  V
  - From the  $\mathcal{E}_c$  versus  $T_e$  figure,  $\mathcal{E}_c \approx 46$  V. Adding  $\mathcal{E}_e = 2T_e \approx 6.6$  V and  $\mathcal{E}_i \approx 5.2T_e \approx 17$  V yields  $\mathcal{E}_T = 70$  V
  - Find  $u_B \approx 2.8 \times 10^3$  m/s and find  $A_{\text{eff}} = 2\pi R^2 h_l \approx 0.043$  m<sup>2</sup>
  - Power balance yields  $n_0 \approx 5.8 \times 10^{17}$  m<sup>-3</sup>
  - Ion current density  $J_{il} = eh_l n_0 u_B \approx 7.8$  mA/cm<sup>2</sup>
  - Ion bombarding energy  $\mathcal{E}_i \approx 17$  V
- $\implies$  Significant increase in plasma density  $n_0$

## ELECTRON HEATING MECHANISMS

- Discharges can be distinguished by electron heating mechanisms
  - (a) Ohmic (collisional) heating (capacitive, inductive discharges)
  - (b) Stochastic (collisionless) heating (capacitive, inductive discharges)
  - (c) Resonant wave-particle interaction heating (Electron cyclotron resonance and helicon discharges)
- Achieving adequate electron heating is a central issue
- Although the heated electrons provide the ionization required to sustain the discharge, the electrons tend to short out the applied heating fields within the bulk plasma

## **INDUCTIVE DISCHARGES**

## **DESCRIPTION AND MODEL**

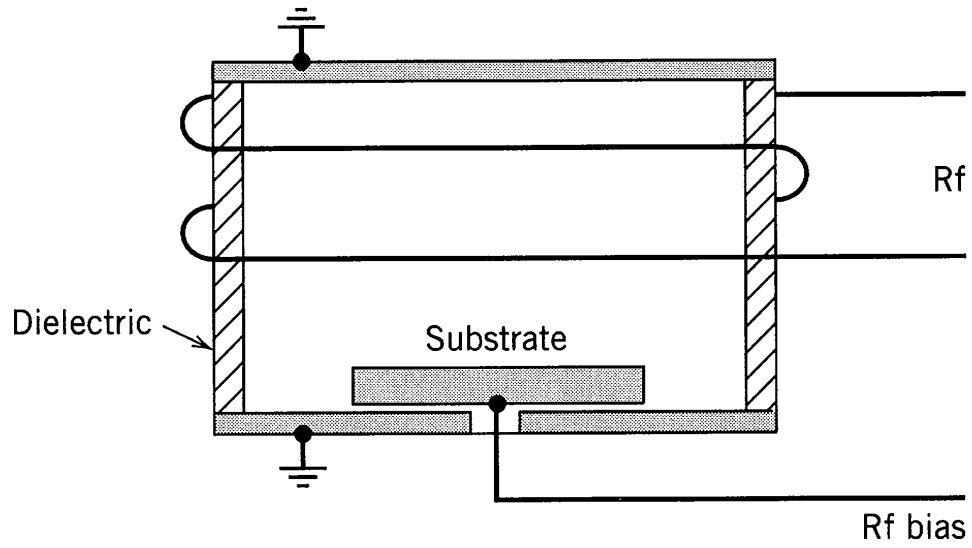


## MOTIVATION

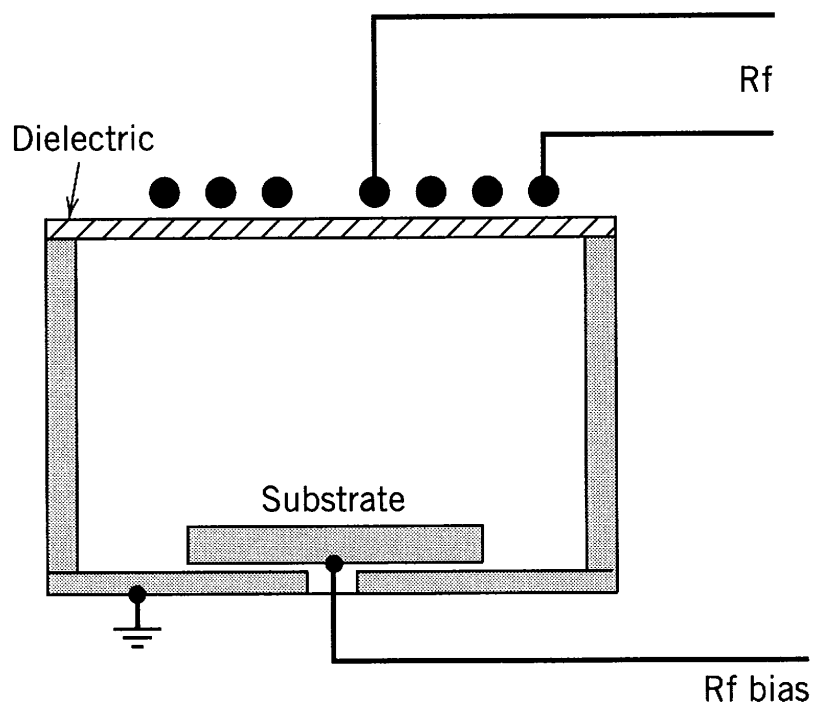
- Independent control of plasma density and ion energy
- Simplicity of concept
- RF rather than microwave powered
- No source magnetic fields

# CYLINDRICAL AND PLANAR CONFIGURATIONS

- Cylindrical coil

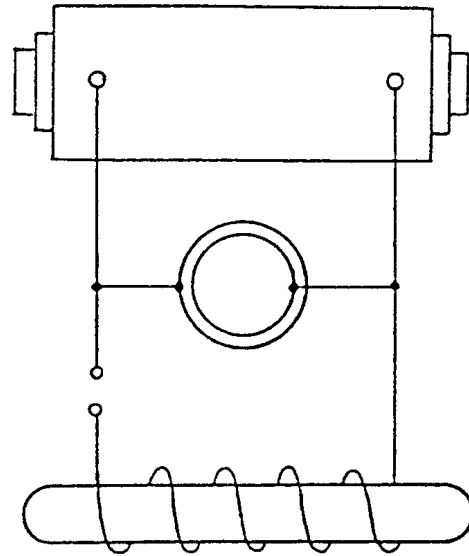


- Planar coil

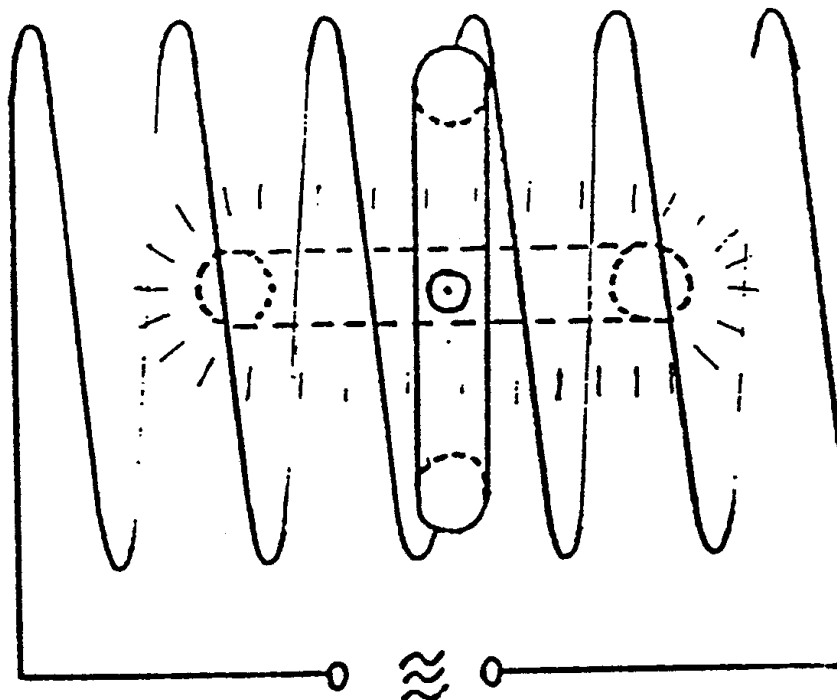


## EARLY HISTORY

- First inductive discharge by Hittorf (1884)

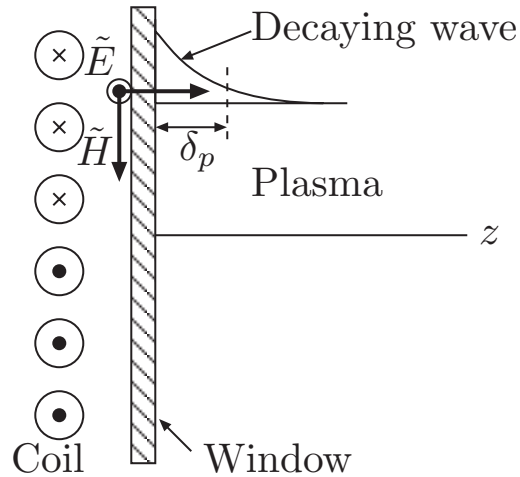


- Arrangement to test discharge mechanism by Lehmann (1892)



## HIGH DENSITY REGIME

- Inductive coil launches electromagnetic wave into plasma



- Wave decays exponentially into plasma

$$\tilde{E} = \tilde{E}_0 e^{-z/\delta_p}, \quad \delta_p = \frac{c}{\omega} \frac{1}{\text{Im}(\kappa_p^{1/2})}$$

where  $\kappa_p =$  plasma dielectric constant

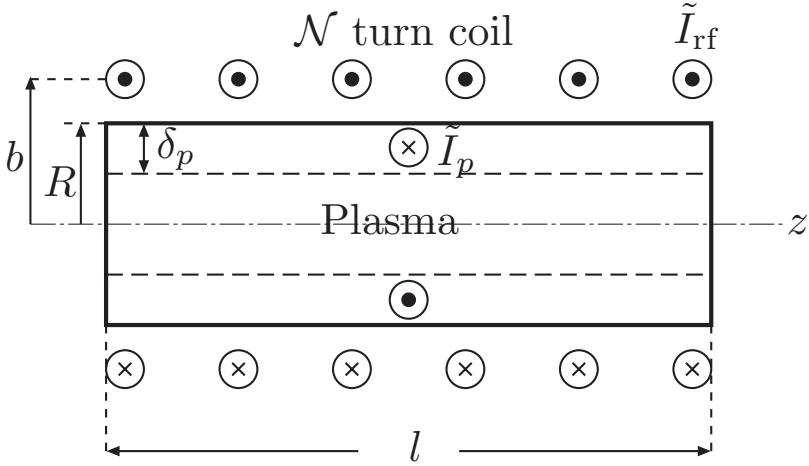
$$\kappa_p = 1 - \frac{\omega_{pe}^2}{\omega(\omega - j\nu_m)}$$

For typical high density, low pressure ( $\nu_m \ll \omega$ ) discharge

$$\delta_p \approx \frac{c}{\omega_{pe}} = \left( \frac{m}{e^2 \mu_0 n_e} \right)^{1/2} \sim 1-2 \text{ cm}$$

# TRANSFORMER MODEL

- For simplicity consider long cylindrical discharge



- Current  $\tilde{I}_{rf}$  in  $\mathcal{N}$  turn coil induces current  $\tilde{I}_p$  in 1-turn plasma skin

$\implies$  A transformer

## PLASMA RESISTANCE AND INDUCTANCE

- Plasma resistance  $R_p$

$$R_p = \frac{1}{\sigma_{dc}} \frac{\text{circumference of plasma loop}}{\text{cross sectional area of loop}}$$

where

$$\sigma_{dc} = \frac{e^2 n_{es}}{m \nu_m}$$

$$\Rightarrow R_p = \frac{2\pi R}{\sigma_{dc} l \delta_p}$$

- Plasma inductance  $L_p$

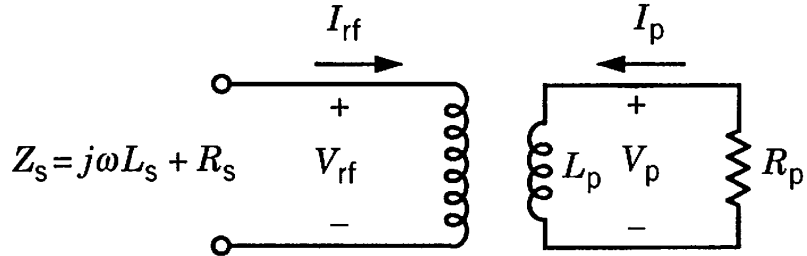
$$L_p = \frac{\text{magnetic flux produced by plasma current}}{\text{plasma current}}$$

- Using magnetic flux =  $\pi R^2 \mu_0 \tilde{I}_p / l$

$$\Rightarrow L_p = \frac{\mu_0 \pi R^2}{l}$$

## COUPLING OF PLASMA AND COIL

- Model the source as a transformer



$$\tilde{V}_{\text{rf}} = j\omega L_{11} \tilde{I}_{\text{rf}} + j\omega L_{12} \tilde{I}_p$$

$$\tilde{V}_p = j\omega L_{21} \tilde{I}_{\text{rf}} + j\omega L_{22} \tilde{I}_p$$

- Transformer inductances

$$L_{11} = \frac{\text{magnetic flux linking coil}}{\text{coil current}} = \frac{\mu_0 \pi b^2 \mathcal{N}^2}{l}$$

$$L_{12} = L_{21} = \frac{\text{magnetic flux linking plasma}}{\text{coil current}} = \frac{\mu_0 \pi R^2 \mathcal{N}}{l}$$

$$L_{22} = L_p = \frac{\mu_0 \pi R^2}{l}$$

- Put  $\tilde{V}_p = -\tilde{I}_p R_p$  in transformer equations and solve for impedance

$Z_s = \tilde{V}_{\text{rf}} / \tilde{I}_{\text{rf}}$  seen at coil terminals

$$\boxed{Z_s = j\omega L_{11} + \frac{\omega^2 L_{12}^2}{R_p + j\omega L_p}}$$

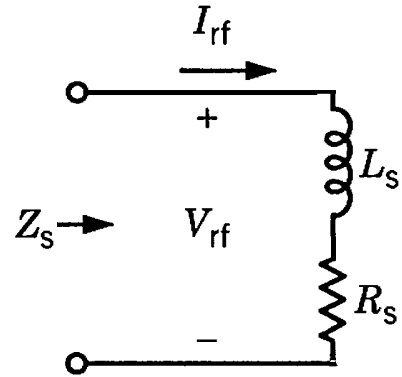
## SOURCE CURRENT AND VOLTAGE

- Equivalent circuit at coil terminals

$$Z_s = R_s + j\omega L_s$$

$$R_s = \mathcal{N}^2 \frac{2\pi R}{\sigma_{dc} l \delta_p}$$

$$L_s = \frac{\mu_0 \pi R^2 \mathcal{N}^2}{l} \left( \frac{b^2}{R^2} - 1 \right)$$



- Power balance  $\implies \tilde{I}_{rf}$

$$P_{\text{abs}} = \frac{1}{2} \tilde{I}_{rf}^2 R_s$$

- From source impedance  $\implies V_{rf}$

$$\tilde{V}_{rf} = \tilde{I}_{rf} Z_s$$

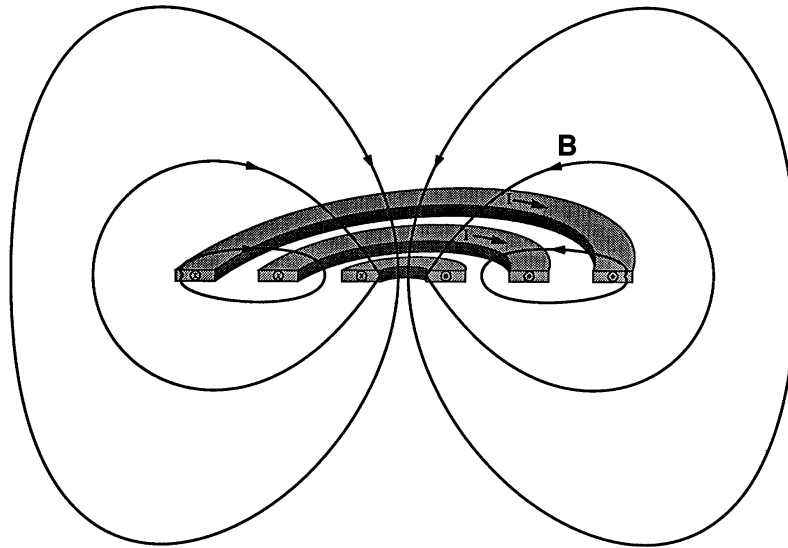


## EXAMPLE

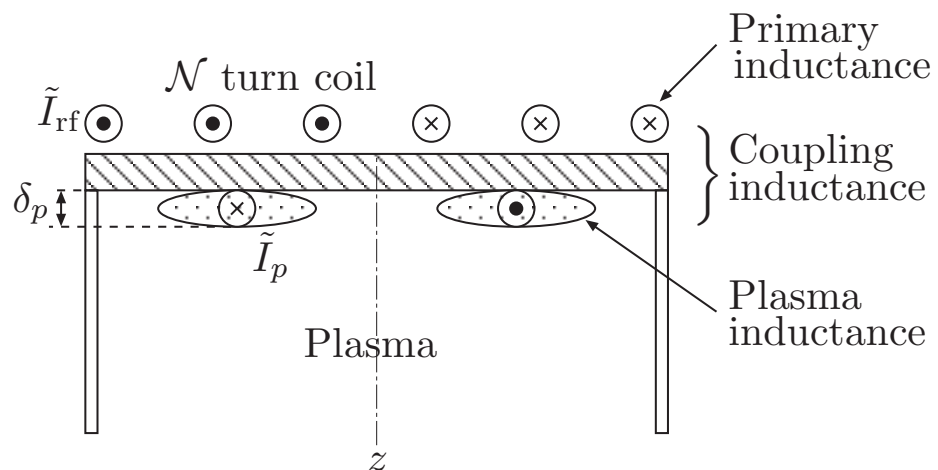
- Assume plasma radius  $R = 10$  cm, coil radius  $b = 15$  cm, length  $l = 20$  cm,  $\mathcal{N} = 3$  turns, gas density  $n_g = 1.7 \times 10^{14}$  cm<sup>-3</sup> (5 mTorr argon at 300 K),  $\omega = 85 \times 10^6$  s<sup>-1</sup> (13.56 MHz), absorbed power  $P_{\text{abs}} = 600$  W, and low voltage sheaths
- At 5 mTorr,  $\lambda_i \approx 0.6$  cm,  $h_l \approx h_R \approx 0.19$ , and  $d_{\text{eff}} \approx 17.9$  cm
- Particle balance ( $T_e$  versus  $n_g d_{\text{eff}}$  figure) yields  $T_e \approx 2.6$  V
- Collisional energy losses ( $\mathcal{E}_c$  versus  $T_e$  figure) are  $\mathcal{E}_c \approx 58$  V  
Adding  $\mathcal{E}_e + \mathcal{E}_i = 7.2 T_e$  yields total energy losses  $\mathcal{E}_T \approx 77$  V
- $u_B \approx 2.5 \times 10^5$  cm/s and  $A_{\text{eff}} \approx 350$  cm<sup>2</sup>
- Power balance yields  $n_e \approx 5.6 \times 10^{11}$  cm<sup>-3</sup> and  $n_{se} \approx 1.0 \times 10^{11}$  cm<sup>-3</sup>
- Use  $n_{se}$  to find skin depth  $\delta_p \approx 1.7$  cm; estimate  $\nu_m = K_{\text{el}} n_g$  ( $K_{\text{el}}$  versus  $T_e$  figure) to find  $\nu_m \approx 1.4 \times 10^7$  s<sup>-1</sup>
- Use  $\nu_m$  and  $n_{se}$  to find  $\sigma_{\text{dc}} \approx 113$   $\Omega^{-1}\text{-m}^{-1}$
- Evaluate impedance elements  $R_s \approx 14.7$   $\Omega$  and  $L_s \approx 2.2$   $\mu\text{H}$ ;  
 $|Z_s| \approx \omega L_s \approx 190$   $\Omega$
- Power balance yields  $\tilde{I}_{\text{rf}} \approx 9.0\text{A}$ ; from impedance  $\tilde{V}_{\text{rf}} \approx 1720$  V

# PLANAR COIL DISCHARGE

- Magnetic field produced by planar coil



- RF power is deposited in ring-shaped plasma volume



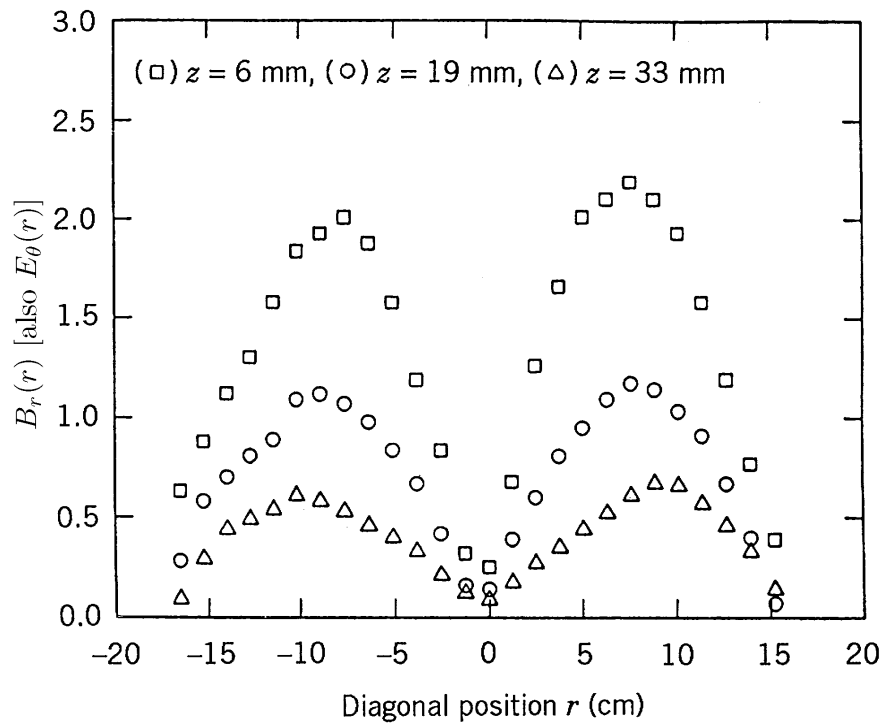
- As for a cylindrical discharge, there is a primary ( $L_{11}$ ), coupling ( $L_{12} = L_{21}$ ) and secondary ( $L_p = L_{22}$ ) inductance

## PLANAR COIL FIELDS

- A ring-shaped plasma forms because

$$\text{Induced electric field} = \begin{cases} 0, & \text{on axis} \\ \text{max}, & \text{at } r \approx \frac{1}{2}R_{\text{wall}} \\ 0, & \text{at } r = R_{\text{wall}} \end{cases}$$

- Measured radial variation of  $B_r$  (and  $E_\theta$ ) at three distances below the window (5 mTorr argon, 500 W)



## **INDUCTIVE DISCHARGES**

### **POWER BALANCE**

## RESISTANCE AT HIGH AND LOW DENSITIES

- Plasma resistance seen by the coil

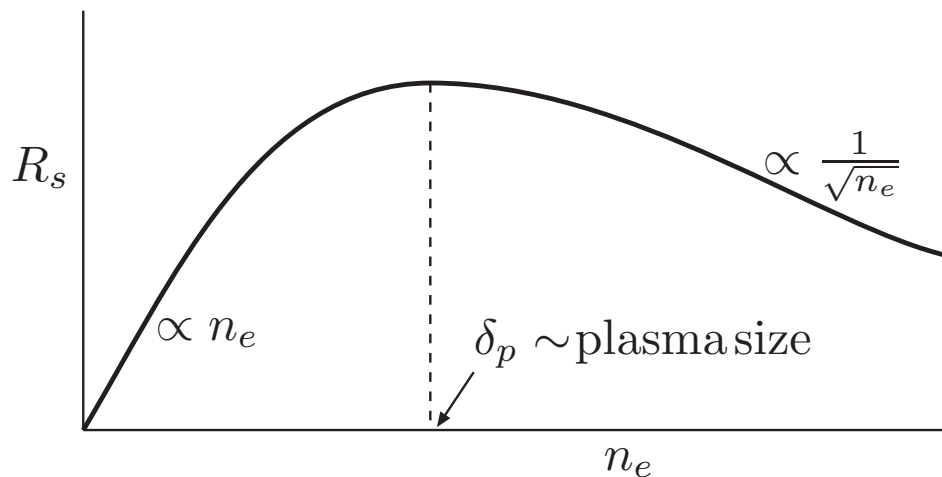
$$R_s = R_p \frac{\omega^2 L_{12}^2}{R_p^2 + \omega^2 L_p^2}$$

- High density (normal inductive operation)

$$R_s \approx R_p \propto \frac{1}{\sigma_{dc} \delta_p} \propto \frac{1}{\sqrt{n_e}}$$

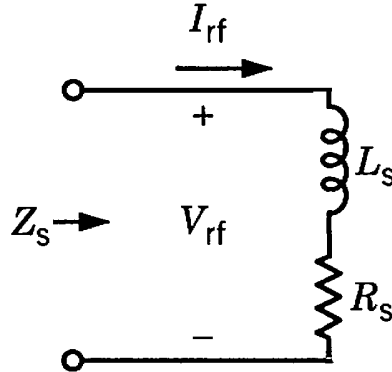
- Low density (skin depth > plasma size)

$R_s \propto$  number of electrons in the heating volume  $\propto n_e$

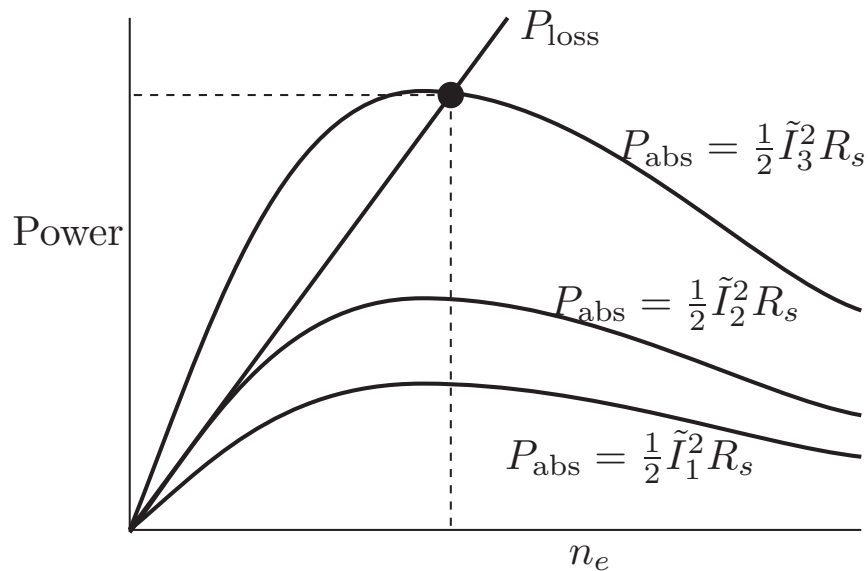


## POWER BALANCE WITHOUT MATCHING

- Drive discharge with rf current



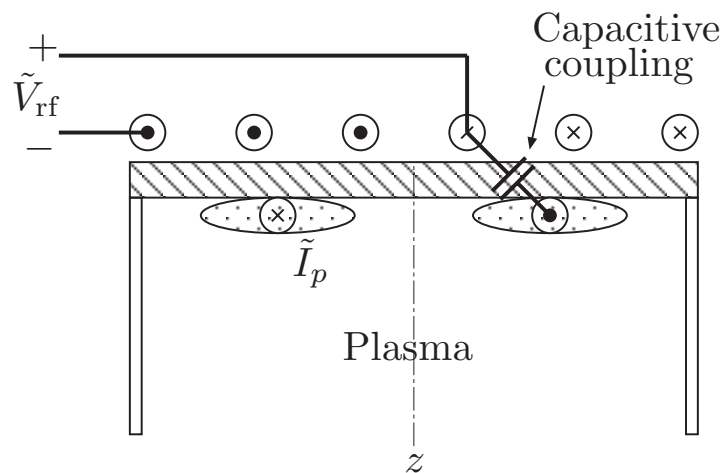
- Power absorbed by discharge is  $P_{\text{abs}} = \frac{1}{2} |\tilde{I}_{\text{rf}}|^2 R_s(n_e)$   
Power lost by discharge  $P_{\text{loss}} \propto n_e$
- Intersection gives operating point; let  $\tilde{I}_1 < \tilde{I}_2 < \tilde{I}_3$



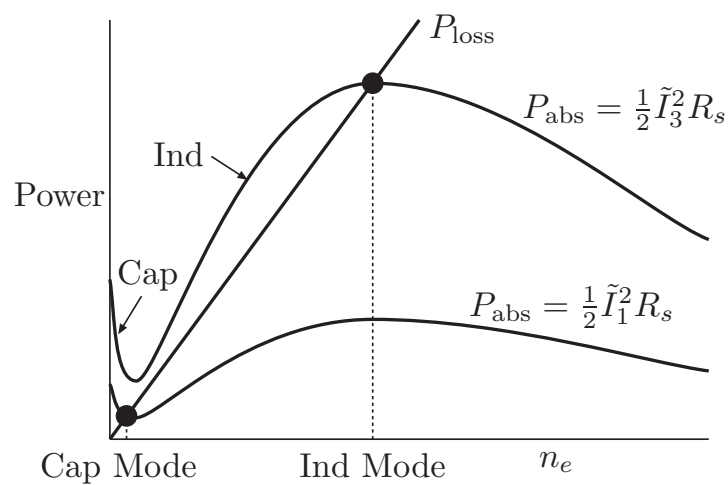
- Inductive operation impossible for  $\tilde{I}_{\text{rf}} \leq \tilde{I}_2$

## CAPACITIVE COUPLING OF COIL TO PLASMA

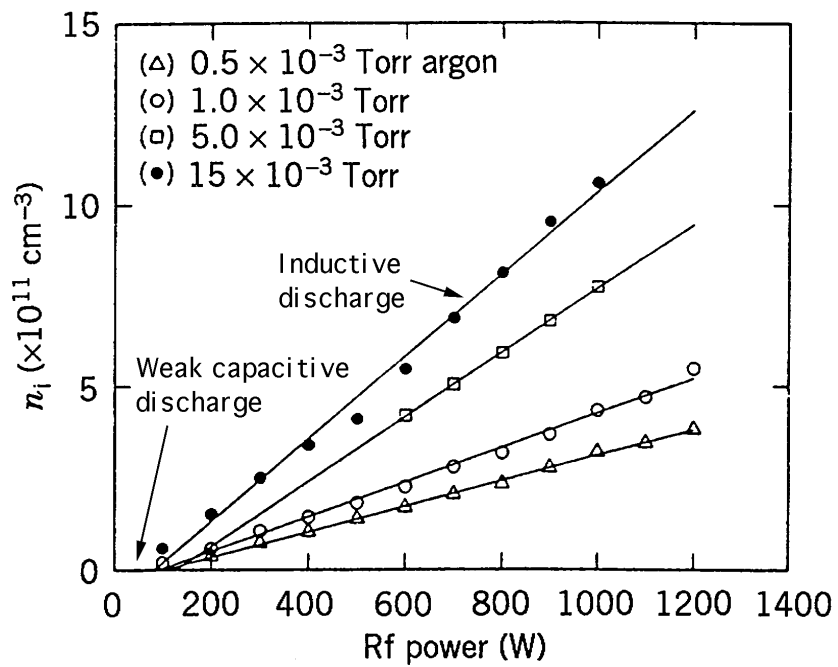
- For  $\tilde{I}_{\text{rf}}$  below the minimum current  $\tilde{I}_2$ , there is only a weak capacitive coupling of the coil to the plasma



- A small capacitive power is absorbed  
 $\implies$  low density capacitive discharge



# MEASUREMENTS OF ARGON ION DENSITY



- Above 100 W, discharge is inductive and  $n_e \propto P_{\text{abs}}$
- Below 100 W, a weak capacitive discharge is present

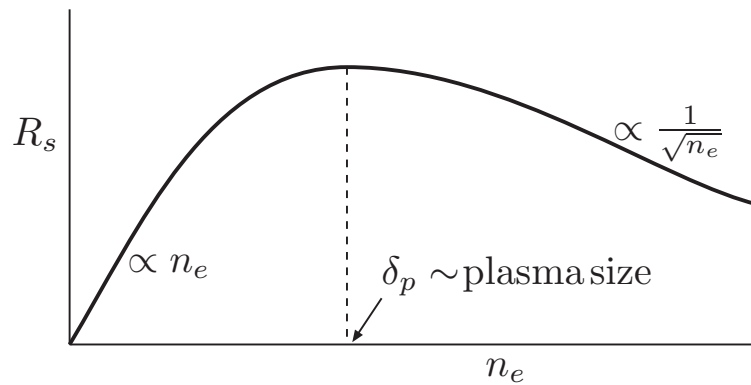


## SOURCE EFFICIENCY

- The source coil has some winding resistance  $R_{\text{coil}}$
- $R_{\text{coil}}$  is in series with the plasma resistance  $R_s$
- Power transfer efficiency is

$$\eta = \frac{R_s}{R_s + R_{\text{coil}}}$$

- High efficiency  $\implies$  maximum  $R_s$



- Power transfer efficiency decreases at low and high densities
- Poor power transfer at low or high densities is analogous to poor power transfer in an ordinary transformer with an open or shorted secondary winding

## CONCLUSIONS

- Plasma discharges are widely used for materials processing and are indispensable for microelectronics fabrication
- The coupling of the equations for the fields and the charged particles is the key to plasma analysis
- Neutral particles play a key role in ionization, energy loss, and diffusion processes in discharges
- The particle and energy balance relations are the key to the analysis of discharge equilibrium
- The particle balance determines the electron temperature; the energy balance determines the plasma density
- A transformer model along with the particle and energy balance relations are the key to the analysis of inductive discharges