SECOND AUSTRALIAN NATIONAL PHYSICS COMPETITION

THEORY EXAM

Thursday December 2, 2:00 pm, RSPHYSSE, ANU

Time allowed: 3 hours + 10 minutes reading time

Attempt as many problems as you like. The mark value for each problem is written next to the question number. You will only have time to finish a fraction of the questions.

There are two kinds of problems.

(A) **Shorter Quantitative Problems (1-23):** for which a numerical answer is required. NO WORKING FOR THESE PROBLEMS WILL BE MARKED. ANSWERS MUST BE WRITTEN ON THE ANSWER SHEET PROVIDED - IN THE UNITS ASKED FOR TO THE EXACT NUMBER OF SIG FIGS ASKED FOR. For these problems you either get the answer right (and full marks) or nothing.

(B) **Longer Problems (24-30):** which are more open-ended. For these the working will be marked and partial marks will be awarded if it is clear you know what you are doing (but perhaps made a silly mistake). It is important you write clear and legible answers to these problems. In some cases there is a "correct" answer. In other cases the problem is very open-ended, and may require creatitivity and approximation - and there is no right answer.

AFTER MARKING ALL ANSWER SHEETS WILL BE CREMATED

Non-programmable calculators and graph paper may be used.

Useful Information:

1. 1 Angstrom $= 10^{-10}$ metres
2. Schrodinger's Equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + U\psi$$

3. Biot-Savart Law

$$d\mathbf{B} = \frac{\mu_o}{4\pi} I \frac{\mathbf{dl}\times\mathbf{r}}{r^3}$$

4. g in Canberra $= 9.8 ms^{-2}$

Easier Quantitative Problems: No partial marks given

Problem 1. *(1 Mark) A* $2.0\mu F$ *capacitor and a* $5.0\mu F$ *capacitor are connected in series. What is the total capacitance of this system? Answer to 1 significant figure in microFarads.*

Problem 2. *(1 Mark) A motorcyle travels in a straight line in the same direction for 2.5 hours at 80 km/h. What is the displacement of the motorcyle during this time? Answer in metres to 1 sig. fig.*

Problem 3. *(1 Mark) A family sedan of length* 3.0 *metres is travelling along a highway at* 100.0 *km/hour. Due to relativistic effects the length of the car as measured by stationary policewoman is less than* 3.0 *metres. Calculate the apparent change in length in Angstroms to 1 signnificant figure.*

Problem 4. *(1 Mark) A bullet is fired vertically upwards from a revolver and leaves it at speed* $300ms^{-1}$*. How far will the bullet rise, ignoring air resistance? Answer in metres to 1 sig. fig. in.*

Problem 5. *(1 mark)*

When two electrons are separated by a distance r the force between them is 1.0 *Newton. What is the force between them when r is decreased by a factor of* 10*? Answer in Newtons to 1 sig. fig.*

Problem 6. *(2 Marks) A wire of length* 2.0 *metres has a resistance of* 10.0 *Ohms. The wire is then uniformly stretched and it is found that the resistance is* 100.0 *Ohms. What is the new length of the wire? Answer in metres to 1 sig. fig.*

Problem 7. *(2 Marks) A uranium nuclear reactor produces a power output of* $100MW$*. Each fission event releases and energy of* $200MeV$*, and liberates on average* 2.5 *neutrons. How many neutrons per unit time are generated by this reactor? Answer to 1 sig fig in neutrons/sec.*

Problem 8. *(2 Marks) The mass of any charged pion is* $139.6Mev/c^2$*. Calculate the minimum energy a photon must have to undergo the reaction* $\gamma \rightarrow \pi^+ + \pi^-$*. Answer to 1 significant figure in MeV.*

Problem 9. *(2 Marks) 1.0 g of a certain isotope emits* 1.24×10^4 *alpha particles per second. What is the half-life of this isotope? Answer to 1 sig fig in years.*

Problem 10. *(3 Marks) A dueteron has a binding energy of* 2.2 *MeV. What is the minimum kinetic energy that an incoming proton must have to split the deuteron? Answer to 1 sig fig in* MeV.

Problem 11. *(3 Marks) The bob of a simple pendulum of length* $L = 2.0$ *metres is pulled aside from the vertical by an angle of* 90 *degrees. Find the maximum speed of the bob during the motion. Answer to 1 sig. fig. in metres/second.*

Problem 12. *(4 Marks)*

From the period of the motion of the Moon around the Earth and the fact that the radius of the earth is 6.4×10^6 *metres find the centre to centre distance of these two bodies. Answer in metres to 1 sig. fig. .*

Problem 13. *(5 Marks) In the figure below calculate the resistance between points 1 and 2. Give your answer to 1 sig fig in Ohms.*

Problem 14. *(5 Marks) Two identical strings are stretched by* 1.0% *and* 10.0% *respectively. Assuming that the tension in the strings is proportional to the elongation, find the ratio of the fundamental frequencies (i.e. the ratio of the biggest to the smallest). Answer to 1 sig fig.*

Problem 15. *(6 Marks) Light of wavelength* 600 *nm is incident upon two narrow slights with a separation of* 1.0×10^{-3} *metres. The interference pattern appears on a flat screen at a distance of* 2.0 *metres from the plates. Find the number of bright fringes visible in the central* 5 *cm of the pattern. Answer to 1 sig. fig.*

Problem 16. *(6 Marks) A chain of mass* $1.0kg$ *is hung vertically so that its lower end just touches a kitchen scale. The upper end is then released. What is the maximum reading on the scale? Answer to 1 sig fig in kg.*

Problem 17. *(6 Marks) Two spheres of mass* $1.0\ kg$ *and* $2.0\ kg$ *are placed so as to slide on a smooth horizontal rod, and are connected together by a spring of stiffness* $k = 2.4 \times 10^1$ *N/m. When set into motion what is the oscillation frequency? Ans to 1 sig fig in rad/s.*

Problem 18. *(6 Marks) A ball of mass* $m = 0.5kg$ *is hung vertically from an elastic cord with spring constant* $k = 50Nm^{-1}$*. The ball is pulled down from its equilibrium position and set in motion. Find the maximum distance the ball can be pulled down for the motion to remain harmonic. Ans in metres to 1 sig fig.*

Problem 19. *(7 Marks) A circular cylinder with base area* $1.0m^2$ *has small hole of area* $1.0 \times 10^{-4}m^2$ *cut in its base. The cylinder is filled with water to a depth of* $1.5m$*. Neglecting the viscosity of water how long will it take to empty the cylinder? Ans to 1 sig fig in seconds.*

Problem 20. *(8 Marks) In a primitive telescope there are two spherical glass balls. The larger one of radius* $5.0cm$ *is the objective and the smaller one of has radius* $1.0\ cm$*. What is the distance between the centre of the spheres? The refractive index of glass is* $n = 1.50$*. Answer to 1 sig fig in cm.*

Problem 21. *(10 Marks) In the infinite resistance ladder shown below* $R = 10.0Ohms$*. Find the total resistance between the points* 1 *and* 2*. Answer to 1 sig fig in Ohms.*

Problem 22. *(10 Marks) A non-conducting sphere with radius* $R = 50mm$ *carries a uniform surface charge density* $\sigma = 10.0 \times 10^{-6}Cm^{-2}$*. It rotates about an axis through its centre at* $\omega = 70rad/s$*. Find the magnitude of the magnetic induction (B) at the centre of the sphere. Answer to 1 sig fig in Tesla.*

Problem 23. *(12 Marks) You are given a square membrane of area* $2.0m^2$ *in which the perpendicular vibrations propagate at speed* $v = 1.0 \times 10^2 ms^{-1}$. *Between circular freqencies* ω *and* $\omega + d\omega$ *there are* dN *natural vibration modes. Find* $dN/d\omega$ *for* $\omega = 1.0 \times 10^5 rads^{-1}$. *Answer to 1 sig fig in s/rad.*

More Difficult Problems: Partial Marks Given

Problem 24. *(10 Marks) Suppose the night is very very dark. What is roughly the maximum distance you can see on ordinary battery-powered torch with the naked eye?*

Problem 25. *(10 Marks) Estimate the wind speed needed to blow over a bus.*

Problem 26. *(15 marks) Two parallel narrow slits aligned in the* x *direction are cut in a screen located in the plane* $z = -L$. *They are covered with different pieces of polarizing material and illuminated by plane parallel light described by the electric field vector:*

$$\mathbf{E} = 2^{-1/2}(\hat{\mathbf{x}} + \hat{\mathbf{y}})E_0 \cos 2\pi(ct - z)/\lambda$$

The first slit at $y = -D/2$ *passes light linearly polarized parallel to the* y *axis, while the other, at* $y = D/2$ *passes light linearly polarized parallel to the* x *axis. The midpoint of each slit is at* $x = 0$. *A detector is placed along the line* $x = z = 0$ *and it can move in the* y *direction. The detector is sensitive only to light polarized parallel to the vector* $\hat{\mathbf{x}} - \hat{\mathbf{y}}$. *Assume that the range of* y *over which the detector moves allows the use of small-angle approximations. What is the variation with* y *of the light intensity seen by the detector?*

Problem 27. *(30 Marks) A smooth wire is bent into a circle of radius* a *and rotates with uniform angular velocity* ω *about an axis through the centre of the wire. The circle is vertical and gravity is uniform. A particle of mass* m *slides on the wire. Use the lowest point of the wire as the origin and define an arc-length coordinate* s *for the position of the particle, i.e.* $-\pi a < s < \pi a$. *Write down the equation of motion for the particle i.e. an equation of the form* $d^2s/dt^2 = ..$, *but do not try and solve it. Hence or otherwise find the positions of equilibrium of the particle and discuss the nature of each of these as a function of the dimensionless parameter* $\mu \equiv \omega^2 a/g$. *Find also the period of the motion (for small oscillations) about any stable positions of equilibrium. (Cambridge Mathematical Tripos Examination , January 2, 1879, 9am to 12 am).*

Problem 28. *(30 Marks) An electron is trapped in a cubical box of side* a. *The walls of the box are impermeable to the electron. 4 of the walls are insulators and the other 2, which are*

opposite each other are perfect conductors. A potential V is applied across the conducting plates and creates a uniform field in the box. Neglecting image effects find approximately the ground state energy of this system by quantum mechanics. Find this in two limits (a) When the potential drop is small, and (b) when it is very large.

Problem 29. *(30 Marks) Suppose we have a parallel plate capacitor, with air as the dielectric and where the spacing between the plates, d, is very small. Every boy and girl knows that the capacitance between the plates per unit area is ϵ_0/d. Now suppose the lower plate is not flat but is subjected to one-dimensional sinusoidal ripples of amplitude A and wavelength λ. Here $A \ll \lambda$ and $A \ll d$, and $\lambda \ll d$ and λ is much less than the dimensions of the plates. Find approximately how this affects the capacitance per unit area.*

Problem 30. *(30 Marks) A solid homogeneous hemisphere of radius a rests in neutral equilbrium (i.e. between stable and unstable) on top of a fixed sphere of radius b. The curved surfaces are in contact. Find a relation between a and b. (Cambridge Mathematical Tripos Exams, 1869). (See Routh, Analytical Statics, Cambridge 1896, in the section "Rocking Stones.")*