2001 EXAM

1. The Equipartition Theorem.

(a) State carefully (using complete sentences) the equipartition theorem for a system with N quadratic terms in the Hamiltonian.

(b) A system has Hamiltonian $U = \frac{3}{4}x^2 + \frac{4}{5}y^2 + z^2$ where x, y, z are the usual 3D Cartesian coordinates. Calculate the average energy of this system when placed in a heat bath at temperature T.

(c) A very small mass m is suspended from a spring with a very small spring constant k in a gravitational field g. The mass is constrained to move in the vertical direction. At zero temperature the mass reaches some equilibrium position z_{eq}. The temperature is now increased to room temperature T. Calculate the square deviation of the the position of the mass from equilibrium i.e. $\langle (x - x_{eq})^2 \rangle$. (Hint: Use the equipartition theorem).

2. Capillarity

(a) Explain what is meant by the contact angle and give an argument which yields Young's law for a liquid drop resting on a flat solid surface in equilibrium with the vapour of the drop. Use complete sentences.

(b) Explain in physical terms (using complete sentences), why the pressure inside a champagne bubble must be greater than the pressure in the surrounding champagne.

(c) A swimming pool with a flat bottom is filled to a depth h_0. Show that the pool water is stable against a perturbation in the height of $h = h_0 \sin(kx)$ where x lies along the horizontal and k is a constant. Neglect gravity.

3. Partition Functions

(a) A particle is constrained to move in one dimension between $x = x_0$ and $x = x_1$ and has no external potential acting on it. Calculate the partition function for this system at temperature T, and the average position of the particle $\langle x \rangle$.

(b) The particle (in (a)) is now subjected to an extra force $f_x = -A/x$ where $A > 0$ is a constant. Again calculate the partition function and the average position.

(c) Explain physically, using complete sentences what you expect for the answer to (b) in the limits $A \to 0$ and $A \to \infty$. Show that your answer to b agrees with your expectations.