EXPLORING SPECIAL RELATIVITY
Updated: 18/9/07

Objectives
The aim of this experiment is to increase your understanding of special relativity by observing a virtual world which obeys relativistic physics. It shows how things would appear if you could travel at speeds near that of light, \( c = 3 \times 10^8 \text{ m/s} \). From your observations you can verify, and even deduce, aspects of relativistic physics. Both qualitative and quantitative understanding is sought.

Evaluation
We are evaluating the effectiveness of game like simulations – as used in this lab – for learning physics. To help us, please answer the questions on the evaluation handout, once you have finished the lab. A bonus mark is awarded for doing this.

Skills
In this experiment you will practice: making observations and designing experiments.

Before You Come to this Lab
There are things you need to do before you come to this lab:

1. Study these notes.
2. Do the Prelab Problems.
3. Study the following tutorials on the “Through Einstein’s Eyes” CD or web site: relativistic aberration, Doppler effect, headlight effect [1].
4. Read sections 12 and 13, “Relativistic optics” and “The relativity of simultaneity revisited”, of the PHYS1201 relativity notes. Ignore the references to 4-vectors if you haven’t covered these in lectures yet.
5. Play with the Real Time Relativity program until you feel comfortable with how it works. You can download it onto your own Windows computer – if it has a suitable graphics card – or use it on one of the computers in room 14 next to the 1st year lab. Ask Craig to open it if it’s locked. Try out the things mentioned in the section “Using the Real Time Relativity simulator” below [2].

Basic Principles
Computer simulations are used in science, and other fields such as economics, to conduct investigations that might otherwise be difficult or impossible. In this lab you will use a simulation to explore relativistic physics: the “Real Time Relativity” (RTR) simulator created at ANU by Lachlan McCalman, Antony Searle and Craig Savage [3].
The objects in the simulation are big, because the speed of light is fast. When you start RTR you can see Earth and the Sun. These set the scale of the simulated world, and are respectively about 0.043 and 4.7 light-seconds in diameter. A light-second is the distance light travels in a second: 300,000 km. The square on which most of the objects sit is 35 light-seconds on a side, the colored cubes are 2.5 light-seconds on a side, and the distant bar is 25 light-seconds long.

Three physical effects are particularly apparent in RTR: the Doppler effect, aberration, and the headlight effect.

The Doppler effect is the difference between measurements of the frequency, or colour, of light by relatively moving observers. We consider the usual standard configuration observers S and S′ (see diagram) [4,5]. Let the source of light, either an emitter or a reflector, be at rest in S′. Let this frequency be f′ and the frequency observed in S be f. Let the speed of the observer relative to S′ be v, and the angle of the incoming light relative to the velocity vector be θ, see diagram. Then [4,6]

\[
f = f' \frac{\sqrt{1 - v^2 / c^2}}{1 - (v / c) \cos \theta}.
\]  

(1)

Aberration is the difference in the angle of incoming light rays measured by relatively moving observers. An analogous everyday effect is the change in the angle of rain when you drive through it. Let the angles of the incoming ray be θ and θ′ for the observers S and S′. Then the angles are related by [6]

\[
\tan \left( \frac{\theta'}{2} \right) = \frac{c - v}{c + v} \tan \left( \frac{\theta}{2} \right).
\]  

(2)

The headlight effect is the change in light intensity measured by relatively moving observers. Aberration concentrates the incident light into a narrow cone centred on the direction of motion. In addition, time dilation increases the measured photons per second in the camera frame. Overall there is a brightening in the direction of motion, and a darkening away from the direction of motion. The ratio of the photon fluxes P measured by observers S and S′ scales as the third power of the Doppler shift factor [6]

\[
P' = P \left( \frac{\sqrt{1 - v^2 / c^2}}{1 - (v / c) \cos \theta} \right)^3.
\]  

(3)
**Prelab Study**
See “Before You Come to this Lab” at the start of this document.

<table>
<thead>
<tr>
<th>Prelab Problems</th>
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<tbody>
<tr>
<td>1. An object is 5 light-seconds away. How long does it take light from the object to reach you?</td>
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<tr>
<td>2. You are at rest relative to a clock 1 light-minute away. You set your watch to agree with the time you see on the clock. Is your watch reading fast or slow? By how much?</td>
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<tr>
<td>3. (a) Express a light-second per second in ms$^{-1}$. (b) Express a light-second per second per second in ms$^{-2}$, and in units of “gees”, the acceleration due to gravity, $g = 9.8$ ms$^{-2}$. Use $c = 3 \times 10^8$ ms$^{-1}$.</td>
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<tr>
<td>4. Does the Doppler effect increase or decrease the frequency when the ray is coming from: (a) in front, $\theta &lt; 90$ degrees? (b) behind, $\theta &gt; 90$ degrees?</td>
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<tr>
<td>5. Show that a ray incoming at $\theta = 90$ degrees has the Doppler shift $f = f' \sqrt{1 - v^2 / c^2}$. This is called “time dilation”. The source of light in $S'$ can be thought of as a clock producing $f'$ oscillations per second. In the $S$ frame a smaller number $f$ oscillations per second is observed. Hence each oscillation takes longer, and the $S'$ time is said to be “dilated”.</td>
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<tr>
<td>6. Consider two incoming rays with slightly different angles. Does the angular difference increase or decrease for rays coming from: (a) in front, $\theta = 0$ degrees. (b) behind, $\theta = 180$ degrees. See [6] for help.</td>
</tr>
</tbody>
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**Using the Real Time Relativity simulator**
These instructions apply to version 0.8 of RTR. See the RTR Manual provided in the lab, and on the website for more detailed instructions [7].

To start RTR double click the RTR icon on the desktop. To exit it press Escape.

The mouse controls where you are looking, but not the direction your rocket is moving. Try looking around a bit with the mouse. You'll notice that the red cross always points in the same direction—this tells you which way your ship is facing. Placing the red cross in the centre (F key) means you are looking in the direction of travel. It will be useful in the lab to be able to look 90 degrees to the direction of motion: L & R keys.
The **keyboard** controls the rocket. The **up** and **down arrow keys** control pitch (tilting the nose up and down). The **A & D keys** control yaw (rotating side to side). Try moving around a little with the keys. You'll notice that the red cross remains fixed in the centre of the screen.

**To accelerate and decelerate, use the W and S keys.** To stop, use the **1 key**. To instantly accelerate to 0.866c, use the **2 key**. To instantly accelerate to -0.866c, use the **3 key**. These speeds correspond to a length contraction, and time dilation, factor of 2. The **4 & 5 keys** accelerate to a contraction factor of 4.

If you get lost you can reset to the starting configuration by using the **O (Oh) key**.

When first getting familiar with RTR it is useful to turn off all the relativistic effects by pressing the **M key**. Pressing it again turns relativity back on.

By default, the **Doppler effect** and **headlight effect** are turned off in the simulation, but can be toggled on and off with the **K key** respectively. Indicators at the top left of the screen tell you whether they are on or off.

To Pause press the **F2 key**. To capture the RTR window press the two-key combination **Alt-PrintScreen**. This puts the image on the clipboard. It can then be pasted into an application such as Paint, and annotated if you like, and **printed**. Paint may be opened while RTR is paused. It is found under “Accessories” in the “All Programs” menu. I suggest you reduce images in size (stretch) and paste a number of images on the same page before printing.

**The experiment**
Press the **O (Oh) key** to go to the starting position looking along the columns. Turn on the headlight and Doppler effects (K & H keys) - note the indicators at top left of the screen. Describe in your log book what you see as you continuously accelerate from rest (W key) to 0.999c. Note that the virtual world has “identified boundaries” – if you fly too far in one direction you come out on the opposite side of the world. Print out a screen showing some interesting visual effects and include it in your lab book. Be sure to refer to it in your description. Explain what you observe in terms of the relativistic optics effects described in the introduction.

**All screenshots in your logbook should have full captions describing what they are and the speed at which they were taken.**
Holding the W key produces constant proper acceleration of 0.2 light-seconds per second$^2$, or $6.1 \times 10^6$ g. This is the acceleration you would feel if you were actually in the rocket (Would you survive?). However, since you cannot exceed the speed of light relative to the world frame, your world frame acceleration decreases as you speed up. When the acceleration is along the direction of motion, proper acceleration $a_p$ and world frame acceleration $a_w$ are related by [8]

$$a_p = a_w \left(1 - \frac{v^2}{c^2}\right)^{-3/2}.$$

Verify the particular Doppler formula discussed in prelab problem 5:

$$f = f' \sqrt{1 - \frac{v^2}{c^2}},$$

where $f$ is the frequency (colour) observed when moving with speed $v$ relative to an object whose rest frame colour corresponds to frequency $f'$. The verification should be quantitative, but approximate, and be for a range of speeds. The L and R keys should be useful for this: they point the camera 90 degrees to the left or right of your direction of motion. The F key points the camera forward and B backwards. When studying the Doppler effect you should turn the headlight effect off (H key). Otherwise it swamps the Doppler effect. Think of it as “computer correction” of your camera image.

**Extra Information:**

Red light frequency: $4.3 \times 10^{14}$ s$^{-1}$.
Green light frequency: $5.7 \times 10^{14}$ s$^{-1}$.
Blue light frequency: $6.8 \times 10^{14}$ s$^{-1}$.

A complication is that the observed colour may be shifted from the object’s rest ultraviolet “colours”. Hence RTR makes an assumption about what these colours are. Different assumptions give rise to different observed colours. RTR provides two models for how the ultraviolet colours are related to the visible ones: described in detail in the User-Guide. The major difference is that in model 1 objects become dark in the ultraviolet more rapidly than in model 0. This can make the Doppler effect easier to see. **Hence I suggest you base your observations on model 1** – in any case note in your logbook which model you used. Pressing the K key cycles through the two models – the model in use is indicated as either “Doppler 0” or “Doppler 1” at the top left of the screen.

The final, and major, part of this lab is to work through the observations and analysis discussed in section 13 of the PHYS1201 Relativity notes. **You must have studied section 13 before you do this lab.** Make your own observations, record them in your logbook, and analyse them to demonstrate time dilation, length contraction, and the relativity of simultaneity.
To do the required calculations you need to understand the coordinate system used in RTR. The origin \( x = y = z = 0 \) is the RTR starting point, in front of the columns (O key). The negative \( x \) direction is down the columns, the positive \( y \) direction is down, and the positive \( z \) direction is to the left. The units are light-seconds. The position of the camera is shown at the top left of the screen.

The starting position is reached using the \( I \) key. This places you at rest looking to your left at the clocks, which read seconds. The clocks all have the same \( y = 0 \) & \( z = -17 \) light-seconds coordinates. Their \( x \) coordinates are respectively \( x = -10, -5, 0, 5, 10 \) light-seconds. So the clocks are 5 light-seconds apart. Take a screenshot and include it in your logbook: call this screenshot 1. Verify that the times read by the clocks in your screenshot are explained by the light propagation delay.

Next press the \( C \) key to place you moving at \( v = 0.5c \) parallel to the line of clocks and looking to your left. The clocks are just becoming visible. When you are opposite the middle clock take a screenshot: call this screenshot 2. Immediately press the \( I \) key to bring you to rest relative to the clocks. Now use the mouse to look back at the clocks. Take a screenshot of the clocks: call this screenshot 3. Put these two screenshots in your logbook – remember to give them full captions.

Your three screenshots should look something like those in section 13 of the notes.

By comparing screenshots 1 and 2 quantitatively verify length contraction – use a ruler! By comparing screenshots 2 and 3 quantitatively verify that the relativity of simultaneity can be explained by light propagation delay in the clock rest frame. Include complete arguments and calculations in your logbook.

Provide a discussion of the sources of error in your measurements, and how you might minimise them. For example: is it better to be closer to or further away from the clocks? Should the clocks be closer together or further apart?
Extension work (optional)
If you have time repeat your verifications of length contraction, time dilation, and the relativity of simultaneity for other speeds.

Achievements
You know how things would look in a relativistic world! You have developed your observational and experimental design skills.

References