Fusion, space, and solar plasmas as complex systems

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Fusion, space, and solar plasmas as complex systems
Richard Dendy
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Part 4

Quantifying shared information content in complex systems

- Information is a physical quantity: discuss
- What measures are best adapted to nonlinear complex systems?
- Applications to plasmas
Part 4A

Quantifying shared information content in complex systems

-What is the question?
Quantifying physical linkage of two spatiotemporally separated, highly nonlinear, plasma signals: upstream solar wind and ionospheric
terrestrial magnetometer data at high geomagnetic latitude

Solar wind drives magnetotail reconnection: energy release drives ionospheric currents, affecting terrestrial magnetic field
What is information?

Information resides in the number of yes/no (≡ binary 0/1) questions (≡ bits) to which we have the answer. E.g. for \( n = 3 \) questions there are:

\[ 2^3 = 8 \]

possible combinations of yes/no answers, expressible as

8 three-digit (≡ three-bit) binary symbols 101, 110, etc.,

So in general \( n \) bits → an alphabet containing \( M = 2^n \) symbols

Suppose we sample (ask \( n \) questions) the system on \( N \) occasions. The amount of information thereby obtained, \( H \), is the number of questions to which we have answers:

\[ H = N \times n = N \log_2 M \]

If all symbols occur with equal statistical probability \( P = 1/M \), then

\[ H = -N \log_2 P \]

Any digitally sampled measured signal is a time-ordered string of \( N \) \( n \)-bit symbols \( X_1, X_2, \ldots, X_i, \ldots, X_N \) drawn from an alphabet having \( M = 2^n \) symbols.

Different symbols \( X_i \) recur \( N_i \) times, implying different empirical probabilities

\[ P_i = N_i/N \neq 1/M \]
Information and signal measurement

Intuitively, the occurrence in the signal of a statistically rare symbol (small $P_i$, e.g. letter “x”) provides more information $H$ than the occurrence of a frequent one (large $P_i$, e.g. letter “e”).

For the equal probability case, we also know $H = -N \log_2 P$ for $N$ symbols, implying information per symbol $= H/N = - \log_2 P$.

It appears logical to define the information gained from a single occurrence of $X_i$ as $-\log_2 P_i$.

In the signal of length $N$ symbols, $X_i$ occurs $N_i$ times. So the total information provided by the occurrences of $X_i$ is $H_i = -N_i \log_2 P_i$.

The total information in the signal is then

$$H = \sum_i H_i = -\sum_i N_i \log_2 P_i = -\sum_i N P_i \log_2 P_i = -N \sum_i P_i \log_2 P_i$$

Hence the average information per symbol in a real signal is

$$h = H/N = -\sum_i P_i \log_2 P_i$$

This is the Shannon Entropy of the signal: “entropy” because of deep analogies with statistical mechanical entropy and, beyond, to thermodynamic entropy.
Relevance of information theory to complex systems science

Complex systems typically yield highly nonlinear measurements – intermittent, bursty

Hence it may be suboptimal to try to identify correlation and causality via Fourier-derived techniques that rest upon the superposition of linear modes

Information-based analysis is intrinsically nonlinear, being based on sets of probabilities of arbitrary relative magnitude

The strategy is:
- Split each measured signal into a time-ordered string of symbols $X_i$
- Bin the data symbols to establish their probabilities $P_i$
- Calculate how information (meaning $- \sum_i P_i \log_2 P_i$ type quantities) is shared, flows, and decays, both
  - within a given signal
  - between two contemporaneous but separate signals

These techniques are widely used in e.g. genetic sequencing but remain “novel” across a broad range of physics, including complex systems science
Defining linear cross covariance and mutual information

Both provide measures of correlation between two signals $A$ and $B$.

Linear cross covariance

$$ C(A, B) = \frac{E[(A - \overline{A})(B - \overline{B})]}{\sqrt{E[(A - \overline{A})^2]E[(B - \overline{B})^2]}} $$

where $E[...]$ denotes the mathematical expectation value, and $\overline{A} = E[A]$.

Nonlinear mutual information

$$ I(A, B) = \sum_{i,j}^m P(a_i, b_j) \log_2 \left( \frac{P(a_i, b_j)}{P(a_i)P(b_j)} \right) $$

where signals $A$ and $B$ have been partitioned into exhaustive discrete alphabets $\{a_i\}$, $\{b_j\}$, with

- each symbol having empirical probability $P(a_i)$, $P(b_j)$,
- $P(a_i, b_j)$ is the joint probability of $a_i$ and $b_j$
Mutual information in the Lorenz system attractor

The strange attractor of the famous Lorenz system of three coupled nonlinear equations is canonical.

For the distribution of values taken by each variable \(x, y, z\), mutual information can be calculated as a function of time separation.
Linear cross covariance and mutual information for a simple sine wave: here $A = \sin \omega t$ and $B = \sin \omega (t – \text{lag})$
Part 4B

Quantifying shared information content in complex systems

Flocking
A word from our feathered friends

Emergent self organisation, or what?

Quelea

Starling
Quantifying clumpiness and flocking in complex systems, including plasmas

The Vicsek model* for flocking birds, fish,...
- Each flying bird (swimming fish...) takes account of the velocity orientation of its near neighbours, and does its best (subject to noise) to align with them.
- Speed is constant, velocity orientation and position change.

For each bird, at each successive time step:
- Update position using current velocity.
- Identify the other birds within radius R, take their average velocity orientation, and add noise.

\[
\begin{align*}
    x_{n+1} &= x_n + \vec{v} \delta t \\
    \theta_{n+1} &= \langle \theta_n \rangle_R + \delta \theta_n
\end{align*}
\]

Noise range is \(-\eta < \delta \theta < \eta\)

Critical phase transition at noise $\eta = \eta_c$ in the Vicsek model

At low noise level $\eta$, a small number of flocks form and move together in roughly straight line.

At high noise level $\eta$, disordered Brownian motion.

Structure on all scales when $\eta \approx \eta_c$. 
Quantifying the phase change in the Vicsek model

Classical physics measures are
- “Order parameter” $\phi$, in this case mean velocity

$$\phi = \frac{1}{N^{\alpha_0}} \sum_{i=1}^{N} \sum_{\nu=1}^{\nu_{\max}} \psi_i$$

- “Susceptibility” $\chi$, in this case velocity dispersion

$$\chi = \sigma^2(\phi) = \frac{1}{N} \left( \langle \phi^2 \rangle - \langle \phi \rangle^2 \right)$$

Information theory measure is derived from the probability distribution of the birds’ positions and velocities $\{x_n, \theta_n\} \equiv A \equiv \{a_1, a_2, a_3, \ldots\}$: the “signal” comprising the “alphabet” (i.e., pre-assigned set of strings) $a_i$, each of which is found to occur with measured probability $p(a_i)$

From these probabilities we can construct the Shannon entropy

$$H(A) = -\sum_{i=1}^{n} P(a_i) \log_2 (P(a_i))$$

Given two such signals, we can measure their information theoretic correlation in terms of their normalised mutual information

$$NMI (A, B) = \frac{H(A) + H(B)}{H(A, B)} - 1$$
Strategy for calculating mutual information for Vicsek system

1. Take a snapshot

2. Discretise data – positions $x$ and velocity orientations $\theta$ work best

3. Calculate entropies
Measuring mutual information in the Vicsek system

Move from actual distributions to $P(x)$, $P(\theta)$, and $P(x, \theta)$
Classical and information theory results for Vicsek

Plot measured mutual information (blue) and susceptibility (red) versus noise $\eta$

Error bars near the peak identifying the phase transition are
- at their smallest for mutual information
- at their largest for susceptibility
In this respect the intrinsically nonlinear measure is “better”

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Part 4C

Quantifying shared information content in complex systems

Recurrence plot technique
Data strings as vectors

- The complete data series $S = \{s_1, s_2, s_3, \ldots, s_n\}$ has $n$ elements.

- A string of $d$ sequential data elements, starting from the $k$th element, can be expressed as the $d$-dimensional vector $p_k = \{s_k, s_{k+1}, s_{k+2}, \ldots, s_{k+d-1}\}$: time delay embedding.

- The total number of such vectors, $N = n - d - 1$.

- Example: for embedding space dimension $d = 3$
  
  $p_1 = \{s_1, s_2, s_3\}$, $p_2 = \{s_2, s_3, s_4\}$, ..., $p_k = \{s_k, s_{k+1}, s_{k+2}\}$, ..., $p_N = \{s_{n-2}, s_{n-1}, s_n\}$

- Quantifying similarity between data sequences then reduces to quantifying distances between pairs of vectors $p_j, p_k$ in the $d$-dimensional embedding space.
Recurrence plot construction

- Distance between vectors quantifies similarity between segments. Maximum norm

\[ ||a - b|| = \max_{j=1,d} \{|a_j - b_j|\} \]

- How close implies similar? Choose a threshold distance \( \varepsilon \)

- Thresholded recurrence plot \( T_{ij} \) has elements

\[ T_{ij} = \theta(\varepsilon - ||p_i - p_j||) = \begin{cases} 1 & \text{if } ||p_i - p_j|| < \varepsilon \\ 0 & \text{if } ||p_i - p_j|| > \varepsilon \end{cases} \]

- Visual realisation of recurrence plot:

\[ N \times N \text{ array having} = \begin{cases} \text{black point} & \iff T_{ij} = 1 \\ \text{white space} & \iff T_{ij} = 0 \end{cases} \]
Calculating elements of a recurrence plot: example

Suppose $S = \{1, 3, 7, 4, 2, 9, 5, 3, 7, 4, 8, 1, \ldots\}$

$d = 2$
\[ p_1 = (1, 3) \quad p_7 = (5, 3) \Rightarrow T_{17} = 0 \]
\[ p_2 = (3, 7) \quad p_8 = (3, 7) \Rightarrow T_{28} = 1 \]
\[ p_3 = (7, 4) \quad p_9 = (7, 4) \Rightarrow T_{39} = 1 \]
\[ p_4 = (4, 2) \quad p_{10} = (4, 1) \Rightarrow T_{410} = 0 \]

$d = 3$
\[ p_1 = (1, 3, 7) \quad p_7 = (5, 3, 7) \Rightarrow T_{17} = 0 \]
\[ p_2 = (3, 7, 4) \quad p_8 = (3, 7, 4) \Rightarrow T_{28} = 1 \]
\[ p_3 = (7, 4, 2) \quad p_9 = (7, 4, 8) \Rightarrow T_{39} = 0 \]
Examples of recurrence plots

Threshold $\varepsilon$ chosen to give 20% coverage with black dots in both examples:

Three hundred points of white noise on the unit interval

Three hundred iterations of the logistic map $x_{t+1} = \mu x_t (1 - x_t)$ in the chaotic regime with $\mu = 4$
Unthresholded recurrence plot

- Instead of on/off threshold at $||p_i - p_j|| = \varepsilon$, retain distance information:
  $U_{ij} = ||p_i - p_j||$

- Requires greyscale or colour for visualisation. Plasma physics example*:

Geomagnetic AE timeseries 1995

Visualisation of mutual information

Colour scale proportional to $R_{ij} \equiv |a_i - a_j|$

- Shared structure due to periods of both low and high activity
Recurrence plots

• Visualise and quantify similarities between data sets

• Good for data that are
  – not time stationary
  – contaminated by noise
  – brief

• Quantitative measures applied to recurrence plots describe
  – information and mutual information content
  – correlation, causation, predictability
Quantifying recurrence plots

- The fraction of a thresholded recurrence plot $T_{ij}$ that is covered in black dots equals the fraction of pairs of vectors that are closer together than $\varepsilon$:

\[
C_d(\varepsilon) \equiv \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} T_{ij} = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \theta(\varepsilon - ||p_i - p_j||)
\]

- $C_d(\varepsilon)$ is known as the correlation sum or recurrence rate.

- In terms of the unthresholded recurrence plot $U_{ij}$,

\[
C_d(\varepsilon) \equiv \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \theta(\varepsilon - U_{ij})
\]

- $T_{ij}$ shows points that contribute to a particular value of $C_d(\varepsilon)$

- $U_{ij}$ is a visualisation of the functional dependence of $C_d(\varepsilon)$ on $\varepsilon$
Recurrence rate $C_d(\varepsilon)$ and information content

- If the probability of an event occurring is $p$, Shannon suggested that the information gained on observing the event is $-\log_2 p$ bits.

- The mean amount of information gained from single observation is then the Shannon entropy of the system

$$H = -\sum_i p_i \log_2 p_i$$

- Generalise to the order-$q$ Renyi entropies

$$H_q = \frac{1}{1-q} \ln \left( \sum_i p_i^q \right)$$

$H_1 = H_{\text{Shannon}}$

- Second Renyi entropy for the black/white probabilities at given $\varepsilon$

$$H_2(\varepsilon) \approx -\log_2 C_d(\varepsilon)$$
Mutual Shannon information between two datasets

• If series $A$ and $B$ are independent, a simultaneous measurement of $A$ and $B$ yields an amount $H(A, B)$ of information (joint entropy) identical to that gained by measuring $A$ and $B$ separately:

$$H(A, B) = H(A) + H(B)$$

• If series $A$ and $B$ are not independent, measuring the state of $A$ yields some information about the state of $B$:

$$H(A, B) \leq H(A) + H(B)$$

• Shared information (mutual information)

$$I_{AB} = H(A) + H(B) - H(A, B)$$

• Here

$$H(A) = -\sum_{i}^{m} p_{i}^{a} \log_{2} p_{i}^{a}, etc$$
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Part 4C

Quantifying shared information content in space plasmas

Specifically, the solar wind
Quantifying physical linkage of two spatiotemporally separated, highly nonlinear, plasma signals: upstream solar wind and ionospheric

Solar wind from WIND satellite at sunward libration point

Terrestrial magnetometer data at high geomagnetic latitude

Solar wind drives magnetotail reconnection: energy release drives ionospheric currents affecting terrestrial magnetic field
How much information do the solar wind and magnetometer data share in common?


- Distinguish between hypotheses concerning solar wind propagation
  - Project ST data series in time according to different hypotheses for $v_{sw}$ and $n$:
    \[ \Delta t = (P_w - P_E) \cdot n/v \cdot n \]

- Time lag introduced by magnetospheric plasma processes
  - Additional $\Delta t'$ to accommodate this

- Compute mutual information between SW(t) and AE (t + $\Delta t + \Delta t'$), and maximise
Mutual information between AE and SW

For four different SW propagation hypotheses, as a function of additional time lag $\Delta t'$

Physics output:
- Best hypothesis for propagation
- Shared information quantified
- Best time lag in magnetosphere
Simultaneous two-spacecraft measurements in the solar wind

During 2005-2006 the WIND and ACE spacecraft were at the Sun-Earth libration point L₁ in the distant upstream solar wind – a near-ideal (remote boundaries, broad range of scales) turbulent plasma.

This enables measurements of spatial correlations in the measured, highly nonlinear, plasma and magnetic field properties of the solar wind, over a range 30 to 100 Earth radii (Re).
Simultaneous solar wind measurements from WIND and ACE

Typical observations of $B_x$ [nT] from WIND (blue) and ACE (red)

Four day trace

Embedded one-day trace

Strongly nonlinear signals exhibiting correlation across a range of timescales
Mutual information measures the spatial decay of correlation

These measurements enable us to distinguish between the different possible MHD characteristics (shear vs. compressional Alfvénic) of the turbulent structures.

Wicks et al, Astrophys J (accepted) 2008
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Part 5

Scaling and plasma turbulence

- Turbulence and complexity
- Measuring spatial and temporal scaling properties of turbulence
- Applications to fusion and space plasmas
Capturing statistical self similarity through structure functions

The generalised structure function $S(p, l)$ of order $p$ on scale $l$, for a signal $z(x)$, is:

$$S(p, l) = \langle \{ z(x + l) - z(x) \}^p \rangle$$

Here $\langle \rangle$ denotes an ensemble average – for example, an integral over $x$.

Simple self similarity is reflected empirically by scaling of the form

$$S(p, l) \sim l^{\zeta(p)}$$

Basic fluid turbulence models yield scaling that is linear in $p$, i.e. $\zeta(p) = ap$, so that

$$S(p, l) \sim l^{ap}$$

where the value of $a$ may be inferred from theory, e.g.

- $a = 1/3$ in Kolmogorov’s fluid turbulence approach of 1941
- $a = 1/4$ in the Iroshnikov-Kraichnan theory of Alfvénic MHD turbulence

In general, $\zeta(p)$, which arises from the cascade processes that constitute the turbulence:

- can be a nonlinear function of $p$, to capture intermittency, and
- may include terms reflecting dissipation.
Structure function scaling beyond the ideal

Related question: what is “a turbulence theory”?

Even for linear fluid scaling, different cascade processes may operate over different ranges of separations $l$, hence

$$ S(p, l) = < \{ z(x + l) − z(x) \}^p > \sim l^{ap} \text{ for } l < L_1, $$

$$ S(p, l) \sim l^{bp} \text{ for } l > L_1 $$

Finite system size $L_{max}$ affects large-$l$ properties of $S(p, l)$

Dissipation on small scales affects small-$l$ properties of $S(p, l)$

Real turbulent signals also typically reflect **intermittency**: meaning large events which, although infrequent, are so large that they cannot be “neglected” or “ordered out” – a consequence of the rise and fall of the most strongly dissipating structures. Self similarity is then reflected empirically by scaling of the form

$$ S(p, l) \sim l^{ζ(p)} $$

where $ζ(p)$ depends **nonlinearly** on $p$.

A model that predicts the functional form of $ζ(p)$ from first principles, e.g. by linking it to the dimension of dissipating structures, is a **turbulence theory**.
Addressing dissipative effects: extended self similarity

Given a plot of log $S(p, l)$ versus log $l$ which is curved because of the consequences of dissipation, finite system size, intermittency, etc.,

Can the invariant statistical properties of a single underlying turbulent process, if one operates, nevertheless shine through?

Benzi et al. (Phys. Rev. E 48, R29 1993) hypothesise that:
1. There is an unknown “generalised lengthscale” $G(l)$ instead of $l$, such that
   
   $S(p, l) \sim [G(l)]^{\zeta(p)}$ instead of $S(p, l) \sim l^{\zeta(p)}$

2. This generalised scaling penetrates into the dissipation range.

Hence formally $S(q, l) \sim [G(l)]^{\zeta(q)}$, and taking ratios of structure functions at two different orders, $p$ and $q$

   $S(p, l) \sim [S(q, l)]^{\zeta(p)/\zeta(q)}$

If, empirically, a plot of log $S(p + 2, l)$ versus log $S(p, l)$ yields a straight line, its gradient is $\zeta(p + 2)/\zeta(p)$, and this reflects extended self similarity (“ESS”).

If the plot remains curved, this reflects deep nonlinear dependence of $\zeta(p)$ on $p$, due e.g. to intermittency
Example of application of extended self similarity

In 2000 Biskamp & Müller reported the first $512^3$ 3D numerical simulation of MHD turbulence:

\[ k^{5/3} E(k) \text{ versus } k \]

\[ \zeta(p) \text{ versus } p \]

\[ S(5) \text{ versus } S(3) \]
Turbulence scaling applied to plasma turbulence

There is a quest to understand the extent (if any) of “universality” – i.e. shared, scalable phenomenology – in plasma turbulence in the three main toroidal magnetic confinement approaches:

Large aspect ratio tokamak:

Stellarator

Spherical tokamak
Comparing plasma turbulence in stellarator and spherical tokamak

The Mega Amp Spherical Tokamak, MAST, Euratom/UKAEA Culham

The Large Helical Device, LHD at NIFS, Tajimi, Japan

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<th>Mega-Amp Spherical Tokamak (MAST)</th>
<th>Large Heical Device (LHD)</th>
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Edge turbulence measurements from MAST and LHD
Treatment of plasma turbulent fluctuation measurements

The inserted probes measure ion saturation current \( I_{\text{sat}} \sim n_e T_e^{1/2} \). We know that we are addressing non-Maxwellian fluctuations with measured statistics:

- MAST
- LHD (16)
- LHD (17)
- LHD (18)

Treat each \( I_{\text{sat}} \) measurement as a stochastic increment, i.e. a step on a random walk, on the shortest possible timescale \( \tau_{\text{min}} \) which is defined by the probe sampling rate. Fluctuations on longer timescales \( \tau \) are constructed by summing and detrending:

\[
\delta I_{\text{sat}}(t, \tau) = \sum_{t' = t}^{t + \tau - \tau_{\text{min}}} (I_{\text{sat}}(t') - \langle I_{\text{sat}} \rangle_t) / \sigma
\]

The associated structure functions are constructed from the absolute moments

\[
S_m(\tau) \equiv \langle |\delta X(t, \tau)|^m \rangle \propto \tau^{\zeta(m)}
\]
Generalised structure functions for MAST and LHD turbulence

Recall that if we plot the logarithm of $S(p, l) = \langle \{z(x + l) - z(x)\}^p >$ against $l$ for different $p$, basic turbulent scaling dependence of the form $S(p, l) \sim l^a$ will show as linear traces.

For both MAST and LHD, we obtain excellent scaling of this form, with two well defined scaling regions:

![Graphs for MAST and LHD showing scaling regions]

**References**

Scaling properties of turbulence in MAST and LHD

We have thus identified a regime corresponding to fluid turbulence having $S(p, l) \sim l^{ap}$, or equivalently for $I_{sat}$ probe measurements

$$S_{mn}(\tau) \equiv \langle |\delta X(t, \tau)|^m \rangle \propto \tau^{\zeta(m)}$$

with $\zeta(m) = \alpha m$ where $\alpha$ can be measured from the plots:

MAST

LHD

This provides rigorous quantification of plasma turbulence properties for comparison
- between different confinement systems
- between measurement and numerical simulations
Solar wind plasma: a classic turbulence laboratory

- High magnetic Reynolds number, i.e. ratio of convective to dissipative terms
- Wide range of length scales, hence a well defined inertial range turbulent cascade

The Ulysses spacecraft has a unique out-of-ecliptic heliocentric orbit, achieved by gravity assistance from a Jupiter swing-by

Ulysses has spent many months above the sun’s polar coronal holes, taking measurements of magnetic field components in the quiet fast solar wind
Solar wind magnetic field power spectrum measured by Ulysses

Logarithmic plots of spectral power [$nT^2/Hz$] versus frequency [Hz] for the components of $B$

The low frequency $1/f$ range is probably due to the coronal driver of the solar wind. The high frequency $1/f^{5/3}$ inertial range is fairly well defined.
Scaling properties of the solar wind: structure functions versus sampling time

Logarithmic plots of $S(p = 3, \tau)$ versus $\tau$

There is significant curvature, hence not $S(p, \tau) \sim \tau^{ap}$

Evidence for a spectral break near $\log_{10}\tau = 3$

Systematic trends with increasing time, corresponding to greater heliocentric radial distance
Extended self similarity in the solar wind

Logarithmic plots* of $S(p = 2, \tau)$ versus $S(p = 3, \tau)$

Evidence for $S(p = 3, \tau) \sim [S(p = 2, \tau)]^{\zeta(3)/\zeta(2)}$, where $\zeta(2)/\zeta(3) = 0.75$

Conclusions: 1 of 2

• Complex systems science enables rigorous, few-parameter, model-independent characterisation of strongly nonlinear plasma datasets and behaviour

• Quantitative comparison is now possible between
  – Different plasma physics models for a given nonlinear dataset
  – Nonlinear datasets from similar phenomena in different plasma systems

• Applications span fusion, space, solar, and astrophysical plasmas

• Complex systems models help to identify the key physics

• Necessary for a robust predictive capability
Conclusions: 2 of 2

- Plasmas provide a challenging proving ground for the techniques of complex systems science, in part because the datasets are far from “ideal”

- Complex systems techniques yield unique, and otherwise inaccessible, information about plasmas

- There are few *a priori* grounds for expecting either natural or laboratory plasmas to respond well to these techniques, nevertheless they do

- The physical constituency for complex systems science is substantially increased by its applicability to plasma science, since plasmas
  - account for most of the baryonic matter in the universe, and
  - generate most of the photons
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