A Study of Alfvén Waves in a Dynamic Magnetic Configuration

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Declaration

This thesis is an account of research undertaken between February 2008 and October 2008 at Plasma Research Laboratory, Research School of Physical Sciences and Engineering, The Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Jesse William Read
October, 2008
First of all I would like to thank my supervisor John Howard for giving me such an exciting project which produced many interesting results, and for his guidance throughout the year. I am also grateful for his many helpful suggestions and discussions on the write up and structure of this thesis, and for his help with making the graphs more presentable. My interest in fusion power has driven me for the past few years and I am very grateful for being given the opportunity to become involved in the plasma fusion research conducted here at The Australian National University. I would also like to thank Boyd Blackwell for many helpful discussions on the physics of Alfvén waves and the results we obtained from these studies, and for his help with editing this thesis.

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Finally I would like to extend my heartfelt thanks to my parents. Without their love and support I would never have made it this far.
Abstract

In this thesis the nature of the Alfvénic modes recently observed in the H-1 Heliac at The Australian National University is explored using two 16-channel photomultiplier arrays located at different toroidal positions. Both arrays were capable of obtaining full profiles of the plasma in a single shot. Using the advanced H-1 power supplies and control system data from these arrays was obtained over a wide range of configurations in a single shot. This is the first time measurements of this Alfvén modes have been performed in a dynamic magnetic configuration. One of the arrays was installed to provide a completely different view of the plasma so that the direction of the poloidal rotation of the modes could be confirmed. The Alfvén modes were found to rotate in the ion diamagnetic direction as expected. An estimate of the plasma resistance was obtained from the observed inductance delay of the magnetic field in the plasma. This is an example of “Alfvén Spectroscopy”, a new field in which Alfvén emission is used to determine plasma parameters.
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Chapter 1

Introduction

Stars (such as our Sun) are powered by thermonuclear fusion reactions. One look at the Sun can demonstrate the enormous energy output of these reactions (especially when one considers the distance between it and the Earth), and we see only a fraction of its power. Imagine then, being able to harness this power in man-made reactors. This has been the dream of plasma physicists since the early 1950s. The difficulty lies in the containment of the fusion reactions. The temperatures required for thermonuclear fusion are too great for any known material to withstand. One solution to this problem is to use magnetic confinement, which prevents the hot fuel (which has been heated enough to become ionised and form a plasma) from coming into contact with any of the structural materials.

Magnetic confinement aims to confine the charged plasma particles in a closed magnetic field, and there are several types of devices designed for this purpose. The most developed design is the tokamak (Figure 1.1). This device uses a toroidal (doughnut shaped) geometry which is basically a solenoid bent round to close on itself. One of the issues with this geometry is that the coils are less dense on the outer radius than they are on the inner radius, which introduces a magnetic field gradient and its associated particle

Figure 1.1: The tokamak (top) and stellarator (bottom) designs [1]. The tokamak uses a toroidal plasma current to create the helical twist in the lines of force. The stellarator uses helical windings around the plasma volume in order to twist the lines of force and thus negates the need for a plasma current.
Introduction

drifts (which lead to particle loss). To counter these drifts the magnetic field is “twisted” by introducing a current through the plasma. This current creates the poloidal field which gives the net field a helical twist [2]. However, this has the added effect of causing the plasma to become susceptible to resonant global instabilities which can grow on the lines of force that close on themselves after a number of turns.

Another design which negates the need for a plasma current is the stellarator (also shown in Figure 1.1). The stellarator uses only external conductors to produce the necessary closed helical magnetic configuration which means that it is not susceptible to the current driven instabilities present in the tokamak design. The helical twist in the confining field is described by the rotational transform, \( \tau = n/m \), where \( n \) and \( m \) are the toroidal and poloidal mode numbers respectively [3]. The field undergoes \( n \) poloidal twists per \( m \) toroidal turns.

An outstanding issue for all magnetic confinement devices is performance degradation in the presence of disturbances such as turbulence, waves and magnetohydrodynamic (MHD) activity. The presence of these disturbances is potentially harmful to the confinement of the plasma particles. This thesis is mainly concerned with Alfvén waves, a type of low frequency MHD activity which is capable of compromising the confinement by ejecting particles from the plasma. Because the plasma particles are trapped on the magnetic field lines these waves are like waves on a taut string, where the plasma particles provide the mass and the magnetic tension provides the restoring force [4]. While theoretical models for the structure and excitation mechanism of these waves exist, they do not adequately explain all of the experimental observations of instabilities that exhibit Alfvénic properties [5]. As confinement is vital for fusion plasmas the excitation of Alfvén instabilities must be avoided and therefore it is important to develop a complete understanding of the structure of Alfvén waves and the mechanisms by which they are excited and can be suppressed.

1.1 Alfvén activity in the H-1 Heliac

The studies discussed in this thesis were conducted at the H-1NF National Fusion Facility at The Australian National University. The H-1 Heliac (Figure 1.2), like any stellarator, uses only external currents to shape its magnetic fields. However, H-1 uses the geometry

![Figure 1.2: The H-1 Heliac [6]. This device uses the geometry of its toroidal field coils along with a ring conductor to shape its confining magnetic field. The helical winding wrapped around the ring conductor allows the rotational transform to be finely tuned. Only 18 of 36 coils are shown for clarity. The major radius of this device is 1m while the average minor radius is 0.1-0.2m.](image)
Alfvén activity in the H-1 Heliac

1.1 Alfvén activity in the H-1 Heliac

of its toroidal field coils combined with a central ring conductor to give the field its helical twist as opposed to the helical windings used by the “classical” stellarator design. It is this feature that gives H-1 the title of “Heliac”. The H-1 Heliac also has a helical winding wrapped around its central ring conductor (the blue line in Figure 1.2). By adjusting the ratio of the current in the winding to the current in the toroidal field coils ($\kappa_h$) it is possible to finely control the rotational transform of the magnetic lines of force. This grants access to a range of magnetic configurations which makes the H-1 Heliac an ideal design for studying the fundamental physics of magnetically confined plasmas.

The H-1 Heliac is an experimental device, designed to confine low power plasmas. Alfvén instabilities are not expected in low power plasmas due to the expected absence of the energetic particles that excite them. However, instabilities resembling Alfvén eigen-modes have recently been discovered in the H-1 Heliac [7]. Figure 1.3 shows the magnetic and electron density measurements of these fluctuations. The dependence of the frequency of the instabilities on the magnetic configuration control parameter is consistent with resonant MHD activity. The strong correlation between the magnetic and electron density fluctuations and their relative magnitudes is consistent with Alfvénic behaviour.

Figure 1.3: Magnetic fluctuations in the H-1 Heliac measured with the Mirnov (magnetic) pickup coils (left), and electron density fluctuations in H-1 measured using the scanning interferometer (right, these diagnostic techniques are discussed in the next chapter) [5]. The dependence on the magnetic configuration control parameter ($\kappa_h$) is consistent with resonant magnetohydrodynamic (MHD) type activity. The strong correlation between the two is consistent with the identification of Alfvén instabilities.

Although some of the observed modes have been identified as shear Alfvén waves, other modes are awaiting positive identification [7]. The source of these instabilities in H-1 is not yet known, although there are several possibilities being considered (section 1.1.3). The presence of these instabilities in H-1 provides a unique and unforeseen opportunity to study their structure and how they vary with changes in the rotational transform (configuration). The experimental observations can then be compared to the theoretical 3-D MHD models of Alfvén instabilities.

The transverse Alfvén wave (also known as the shear, torsional, or slow Alfvén wave) is a hydromagnetic wave which propagates along the magnetic field lines of the confining magnetic geometry (its wavenumber $k$ is directed parallel to the magnetic field lines). They are transverse waves with a velocity, $v_A$ (the Alfvén velocity), given by:

$$v_A = \frac{B_0}{\sqrt{\rho_0 \mu_0}}$$

(1.1)
where $\rho$ is the mass density $n_0 M$, $\mu_0$ is the permeability of free space, and $B_0$ is the strength of the magnetic field lines along which the wave propagates [8]. These waves are analogous to transverse waves in a taut string, where the “tension” is given by $B_0^2/\mu_0$ (the magnetic tension) and the corollary of the density of the string is given by the mass density of the plasma particles.

To take into account the geometry in which the plasma is confined, the periodic boundary conditions that close the torus need to be applied [5]. This means that $k_\parallel$, the wavenumber parallel to the magnetic field (in straight magnetic field coordinates), is now proportional to $|n - \iota m|$ (where $n$ and $m$ are now the mode numbers of the wave, these can differ from the mode numbers of the field lines along which it propagates). For the torsional Alfvén wave the wave frequency is given approximately by $\omega = k_\parallel v_A$ and so it now depends upon the rotational transform [9]. Figure 1.4 shows the observed frequency of the instabilities in H-1 compared to $k_\parallel v_A$ for different rotational transforms [7]. As can be seen, the instabilities in H-1 exhibit Alfvénic properties.

Straight field line coordinates are the deformation of the two “angular” coordinates in the toroidal coordinate system into orthogonal coordinates in which the twisted magnetic field lines appear as straight lines [10]. In this system $\iota$ becomes the pitch angle between the field lines and the vertical axis (Figure 1.5).

**Figure 1.4:** $k_\parallel v_A$ (line) compared with the observed “frequencies” of the instabilities in H-1 (crosses). Both are normalised by $\sqrt{n_e}$. The vertical axis gives the normalised frequency and the horizontal axis is the rotational transform [5]

**Figure 1.5:** An illustration of straight field line coordinates [10]. As can be seen the coordinates close on themselves after a complete turn (in both the poloidal, $\theta$, and toroidal, $\phi$, directions).
1.1 Alfvén activity in the H-1 Heliac

Estimates of the mode numbers for the Alfvénic mode observed in H-1 have been made based on the results from the Mirnov coils (which are discussed in section 2.1) and the density interferometer (discussed in section 2.2) [5]. These results indicated dominant $m = 3$ and $m = 4$ modes with $n = 4$ and $n = 5$ respectively [5]. These estimates still need to be confirmed. The modes were also observed to rotate poloidally in the diamagnetic drift direction [5]. The rotation and propagation of the mode structures is discussed in the next section. The intention of this thesis is to further explore the nature of these modes by performing a dynamic configurational scan of the 0.5T plasmas in H-1. The helical current will be ramped over the duration of a plasma discharge to scan the magnetic configuration during a single shot. This the first time that a study of this kind has been performed.

Preliminary studies have revealed a phase “flip” between the Mirnov signals and the light signals obtained with a 16 channel photomultiplier tube (PMT) (a detailed discussion of the light imaging systems is given in section 2.6) about the resonances. The cause of this is of yet unknown. In order to get more information on the spatial structure of the modes and explore their behaviour at the resonances a second 16 channel light imaging system was installed. Computer simulations (section 2.7.2) showed that by breaking the symmetry of the plasma it would be possible to obtain information on the rotation of these modes. This system would therefore be best placed at a port with an asymmetrical view of the plasma at a different toroidal location than the initial PMT array. In this way we can simultaneously explore the toroidal structure of the modes along with their rotational behaviour.

1.1.1 Propagation and Structure of Alfvén Modes

As pointed out in 1.1 the dispersion relation of Alfvén waves in a uniform magnetic field with no boundary conditions (the slab or straight cylindrical geometry) is given by [11]

$$\omega = k_\parallel v_A. \quad (1.2)$$

For a homogeneous plasma in a uniform magnetic field equation 1.2 remains independent of position and as such the wave is dispersionless. When considering a sheared (twisted) magnetic field of toroidal geometry such as those in toroidal fusion devices (Figure 1.1) the waves must be both toroidally and poloidally periodic [11]. If the field rotates poloidally $\iota$ times per toroidal turn it is required that $k_\parallel = |n - m\iota|/R$ where $n$ and $m$ are the toroidal and poloidal mode numbers of the wave respectively, and $R$ is the major radius [5, 11]. The rotational transform $\iota$ depends upon radius which means that the dispersion relation (equation 1.2) is a function of radius for sheared toroidal magnetic fields [11]. The dispersion relation is now given by

$$\omega = \frac{|n - m\iota| B_0}{R} \frac{1}{\sqrt{\mu_0 \rho}} \quad (1.3)$$

Equation 1.3 describes the Alfvén continuum for a sheared magnetic field of toroidal geometry. Waves which satisfy this equation are said to lie within the continuum. From equation 1.3 it can be seen that as $\iota \to n/m$, $\omega \to 0$. This results in the zero frequency “resonances” shown in Figure 1.3 and Figure 1.4 which occur when the mode lies along the resonant field lines.

To create a simplified physical picture of this imagine two intertwined helices which are of equal radius and centred on the same axis but which have different pitch angles.
One helix represents the line of force (pitch angle of $\iota$) with which the mode is associated while the other represents the resonant mode structure (pitch angle of $n/m$). This is demonstrated in Figure 1.6 for straight magnetic field line coordinates. The two helices will intersect periodically which gives rise to a wavenumber $k_\parallel$ which depends upon the difference in pitch angles. The plasma cannot support a pressure perturbation along the lines of force [8]. The perturbations will therefore propagate toroidally along the lines of force as a transverse wave. As the Alfvén mode must remain resonant with itself it will rotate poloidally with a frequency given by equation 1.3 in order to retain its resonant integrity as it propagates toroidally. The direction of propagation is determined by the direction in which the destabilising driving force (section 1.1.2) is oriented.

Waves that satisfy equation 1.3 are said to lie within the continuum. Due to the local variation of the phase velocity (as the density and magnetic field are inhomogeneous) these Alfvén wave packets quickly disperse and are damped (section 1.1.2). This phenomenon is called continuum damping. It has been theorised that the plasma heating can occur when waves inside the continuum are excited and transfer their energy to the plasma through the damping processes [9, 11].

As the rotational transform varies with radius equation 1.2 now depends upon the radius and has a minimum at some point inside the plasma (Figure 1.7). It is possible for Alfvén eigenmodes with frequencies below this minimum to exist. These are the global Alfvén eigenmodes (GAE) and do not lie within the continuum. These modes therefore avoid continuum damping and can easily be excited by energetic particles [5, 9, 11]. GAE’s are localised at the minimum in the shear profile [11]. As such the mode is expected to move radially outwards with increasing $\kappa_h$.

Another effect of the toroidal geometry is the coupling of neighbouring poloidal modes. In a cylindrical geometry with no periodic boundary conditions counter-propagating waves can cross without interacting. In a toroidal geometry the periodicity of both the toroidal and poloidal modes causes the waves to mix and the frequency crossing is avoided, creating

---

**Figure 1.6:** The Alfvén mode (red) and the field line along which it propagates (blue) in straight field line coordinates. The pressure perturbation that the periodic intersecting and separating of the two lines creates propagates along the line of force. The mode structure must remain resonant and so it rotates poloidally to “keep up” with the magnetic field line.
§1.1 Alfvén activity in the H-1 Helia

Figure 1.7: Radial profiles of the rotational transform at different $\kappa_h$ values [3]. The magnetic flux surfaces for two of the configurations are also shown.

A gap in the continuum. Discrete Alfvén eigenmodes with frequencies that lie within the gaps can exist. Like the GAE these modes are not part of the continuum and as a result avoid the strong continuum damping [5, 11]. They are localised by the coupling of poloidal harmonics caused by the magnetic shear [11].

1.1.2 Excitation and damping

The commonly understood mechanism for the excitation of Alfvén eigenmodes is resonance with energetic particles travelling with a velocity $v_\parallel \geq v_A$. Under these conditions it is possible for the Alfvén wave to tap into the free energy provided by the energetic particle distribution. This mechanism applies to both energetic ions and energetic electrons in any magnetic configuration [11].

The condition for resonance between a particle and a mode of frequency $\omega$ is given by [9]:

$$n\omega_\phi + l\omega_\theta = \omega \quad (1.4)$$

where $n$ and $l$ are integers, $\omega_\phi = d\phi/dt$ is the angular frequency of the particle’s toroidal motion and $\omega_\theta$ is its poloidal angular frequency. These are given by $v_\parallel/R$ and $v_\parallel t/R$ respectively where $v_\parallel$ is the component of the particle’s velocity which is parallel to the magnetic lines of force.

Several damping mechanisms are theorised to exist for Alfvén eigenmodes. The opposite of the wave excitation occurs when the wave transfers energy to the particles. This occurs when particles travel at velocities slightly below the resonance velocity. If the majority of the particle velocity distribution is lower than the Alfvén velocity then an overall
damping effect is observed. This is the Landau damping and applies to both electrons and ions [11]. Other damping mechanisms include the electron collisional damping, or resistive damping, which is caused by collisions between electrons and ions [12]. These mechanisms combined with the strong dispersive effect (due to variation in $v_A$) of the continuum constitute the continuum damping [13].

1.1.3 Excitation of Alfvén modes in the H-1NF Heliac

As the H-1 Heliac is a small plasma research device which only confines low power plasmas it was not expected to contain the fast ions which are capable of exciting Alfvén instabilities. The Alfvén velocity in the 0.5T plasmas in H-1 is approximately $5 \times 10^6 \text{m/s}$ [5]. A hydrogen ion travelling at this velocity has an energy of $E \approx 130 \text{ keV}$ [5]. An ion with this level of energy has a gyro-radius which is a significant fraction of the minor radius of H-1 [5]. As such it is expected that these energetic ions will quickly escape confinement [5].

The significance of this, as the commonly understood excitation mechanism is resonance with fast particles travelling at velocities close to the Alfvén velocity, is that the presence of Alfvén waves was not expected in H-1. The ion temperature in the H-1 plasmas is several orders of magnitude below the energy required for Alfvén waves.

As it stands no fast ions have yet been observed in H-1 (which is not to say they are not present). Investigations into fast ion driving sources are under way [7]. One possibility is that the electrons and ions near the radio frequency (RF) antenna (used to heat the plasmas) are being accelerated to velocities sufficient for exciting Alfvén waves by the high potential on the antenna [7]. Another theory is the excitation of these waves by the ion temperature gradient which has been observed in some cases [7, 14].

1.2 Structure of thesis

The intention of this thesis along with the concept of Alfvén waves has been introduced in this chapter. In chapter 2 the technical details of the instrumentation is presented, along with the experimental setup. Also covered are the principles behind the measurement of light emitted from the plasmas and what it reveals. The modelling mentioned in section 1.1 is discussed in greater detail along with the simulation results.

The results of the experiments are presented in chapter 3. Several questions are explored in this chapter. First the rotational behaviour of the modes is observed; of particular interest are the events that occur at the resonances. Then there is the toroidal structure and resonant integrity of the modes. Do these modes maintain there resonant structure (helicity) with increasing shear? What is causing the phase flip at the resonances?
MHD activity affects both the plasma and the magnetic field in which it is confined. When attempting to identify MHD type fluctuations such as Alfvén waves a sensible first step is to cross reference signals from the magnetic field with signals from the plasma and isolate the activity which is common to both. The diagnostic systems employed for this purpose must have the capability of acquiring detailed information on the magnetic and fluid behaviour of the fluctuations present in H-1 without adding any perturbations of their own. The only types of diagnostic systems capable of satisfying these conditions are remote diagnostic systems. These systems take measurements of the properties of the plasma from external locations, in most cases through viewing windows in the vacuum tank of the H-1 Heliac such as in the case of the optical emission and microwave interferometer (discussed in sections 2.3 and 2.2 respectively). Some challenges arise, however, from the nature of these external measurements. The microwave interferometer (and the imaging cameras) which views the plasma through a viewing window in the vacuum tank takes line-of-sight integrated measurements. The challenge here is to somehow reconstruct the spatial structure of the observed fluctuations from these line integrated measurements. This is discussed in section 2.7. For the magnetic pick-up coils discussed in section 2.1 which measure the induced emf due to $d\mathbf{B}/dt$, another challenge presents itself. The results obtained are distance-weighted volume integrated measurements which are local to the position of the coil. In this chapter the principles of the diagnostic systems used in the studies for this thesis are discussed along with some of their technical details. The intention is to correlate the results from the various imaging systems to construct an intuitively meaningful understanding of the physical nature of the modes in H-1.

### 2.1 Magnetic Pick-Up Coils

Measurement of changes in the confining magnetic field is a diagnostic method with one of the simplest operating principles. By placing a coil of conducting wire into the magnetic field it is possible to directly measure the induced emf caused by the changes in the magnetic field ($\varepsilon = d\Phi/dt = d(\mathbf{BA})/dt$). By installing an array of these coils (which are called Mirnov coils) at different toroidal and poloidal locations it is possible to obtain the toroidal and poloidal mode numbers $n$ and $m$ of a mode from the relative phase of the coil’s signals [9]. Nevertheless there is some ambiguity associated with the identification of the mode numbers as discussed further on.

The H-1 Heliac has two poloidal arrays of Mirnov coils, each with 20 poloidally arranged coils. These arrays are located at the toroidal angles $\phi = 44^\circ$ and $\phi = 284^\circ$ and were installed for studies of the MHD activity in H-1 performed by David Pretty in [5] (the
toroidal angles are relative to $\phi = 0^\circ$ position as shown in Figure 2.1). A linear array of 5

![Figure 2.1: The toroidal positions on H-1 of the diagnostic systems used for these studies. The image was developed using Boyd Blackwell’s BLINE code. The magnetic field lines are shown in blue.](image)

coils is also present at the $\phi = 35^\circ$ position. All the coils are located outside the plasma volume. Figure 2.2 shows an example of the results obtained with one of the Mirnov coils. This data was obtained during an rf heated discharge in a 0.5T magnetic field. The magnetic configuration parameter ($\kappa_h$) was varied during the discharge by changing the helical current over the life of the discharge. These are the first results presented from a dynamically varied magnetic configuration. Notice that the raw data is AC coupled (only sensitive to $d\Phi/dt$). The time-frequency domain (tfd) shows Alfvénic scaling (see section 1.1.1).

![Figure 2.2: Typical Mirnov data. The upper plot shows the raw signal of the coil. The lower plot shows its representation in the time-frequency domain.](image)

The ease of use of these coils is partially offset by some significant limitations. Firstly,
these coils can only be placed outside the plasma volume as placing them inside the plasma would potentially damage them and disturb the plasma. Add this to the fact that the measurements provided by Mirnov coils are semi-localised to their position (the signal strength falls off as $\tilde{B}/r^{-m}$ where $\tilde{B} = dB/dt$, $r$ is the distance from the fluctuation to the coil, and $m$ is the poloidal mode number of the mode [5]) and you end up with coils that are effective at picking up modes with global structures which cause large edge perturbations but are not very sensitive to the localised core modes [9].

Another issue arises from the geometry of the plasma itself. As shown in Figure 2.3 the plasmas in the H-1 Heliac have a bean-shaped cross-section. The modes observed are

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2_3}
\caption{Poincaré plots of the plasma in H-1 and the location of the Mirnov coils of one of the poloidal arrays as calculated by David Pretty [5].}
\end{figure}

Figure 2.3: Poincaré plots of the plasma in H-1 and the location of the Mirnov coils of one of the poloidal arrays as calculated by David Pretty [5]. For sensible phase comparisons between the different coil signals the positions of the coils have to be corrected to straight field line coordinates. Ideal helical structures in straight magnetic field line coordinates (which were discussed in section 1.1). In the straight field line coordinate system the mode has constant angular velocity which, due to the spatial warping of the lines of force, translates to a non-uniform angular velocity in the laboratory frame (or real space). As such the phase comparison of the fluctuations at different locations is sensitive to the effective angular location of the coils in the transformed straight field line coordinate system. To further complicate matters the effective coil positions change with magnetic configuration as shown in Figure 2.3.
2.2 Density Interferometers

One of the most fundamental properties of the plasma is its density. For density measurements the phase shift in an electromagnetic “probe” due to the refractive index of the plasma is observed using an interferometer. This is a non-perturbative technique with a simple interpretation.

At frequencies above the plasma frequency ($\omega_p = \sqrt{ne^2/m_e\epsilon_0}$ where $n$ is the number density, $e$ is the elementary charge, $m_e$ is the electron mass, and $\epsilon_0$ is the permeability) the magnetised plasma appears as standard birefringent dielectric material. As such an electromagnetic wave propagating through a plasma will undergo a phase shift given by

$$\phi_p = r_e \lambda \int_{L_p} n_e(\rho) \, dl$$  \hspace{1cm} (2.1)

where $n_e(\rho)$ is the electron density profile, $L_p$ is the path length through the plasma, $\lambda$ is the wavelength of the probe beam, and $r_e = e^2/4\pi \epsilon_0 m_e c^2$ is the classical electron radius [6]. An interferometer can be used to measure this phase shift by combining the phase-shifted wave with a reference wave to produce a signal given by

$$S = I(1 + \zeta \cos \phi_p)$$  \hspace{1cm} (2.2)

where $I$ is the combined intensity of the two beams, $\zeta$ is the fringe contrast, and $\phi$ is the phase difference between the two beams [15].

Extracting the phase shift from equation 2.2 can prove difficult as $I$ and $\zeta$ vary due to attenuation and absorption effects in the interferometer and plasma and are thus unknown. To resolve this a carrier wave is applied which is then phase-modulated by the plasma. At H-1NF the carrier wave is produced by using a swept IMPATT diode with a centre frequency of 141 GHz ($\lambda \approx 2$ mm) as the source of the probe (and reference) beam for a Michelson interferometer (Figure 2.4) [5]. By applying a bias to the diode it is possible to

![Figure 2.4: The layout of the Michelson interferometer used on the H-1 Heliac.](image-url)
sweep the output frequency and produce frequency difference \( \Delta \omega = 2\pi \Delta f \) between the probe and the reference (local oscillator) beams (Figure 2.5). The time delay between the probe beam and the local oscillator is given by \( \Delta t = \frac{L}{c} \) where \( L \) is the path difference between the interferometer arms and \( c \) is the speed of light. The frequency difference between these two beams is therefore given by

\[
\Delta \omega = \frac{d\omega}{dt} \frac{L}{c}
\]

The signal at the detector is now given by (in the absence of the plasma)

\[
S = I_0 (1 + \zeta \cos \Delta \omega t)
\]

As the voltage across the diode cannot be ramped indefinitely it is applied as a sawtooth wave with a frequency of 100 kHz (Figure 2.6). The gradient of the frequency ramp \((d\omega/dt)\) is chosen so that a full period of equation 2.4 is covered in the sweep period (Figure 2.6), creating a 100 kHz carrier wave. The activity in the H-1 Heliac occurs at frequencies below 100 kHz and so having a carrier signal at this frequency is ample for measurements of the electron density.

When the plasma is present \( \phi = \Delta \omega t + \phi_p \) which when substituted into equation 2.2 gives

\[
S = I_0 (1 + \zeta \cos (\Delta \omega t + \phi_p))
\]

which is simply a phase-shifted form of equation 2.4. As shown in Figure 2.4 the spatial position viewed by the interferometer can be varied on a shot-to-shot basis. This allows the line averaged electron density profile to be determined over a number of discharges (shots). For systematic studies, the step scanned interferometer requires that the discharges are reproducible. For fluctuation measurements the density perturbations must be referenced to a fixed timing “clock” such as the signal from a given Mirnov coil.

Figure 2.7 displays a typical raw density data trace along with its time-frequency domain contour from the same shot as the data in Figure 2.2. Some density measurements of the Alfvénic modes in H-1 have been obtained with the 2mm scanned interferometer in

\[\text{Figure 2.5: The frequency difference between the probe and local oscillator due to the frequency ramp. This frequency difference } \Delta \omega = 2\pi \Delta f \text{ is determined by the delay between the two arms and the gradient of the frequency ramp.}\]

\[\text{probe beam and the local oscillator is given by } \Delta t = \frac{L}{c} \text{ where } L \text{ is the path difference between the interferometer arms and } c \text{ is the speed of light. The frequency difference between these two beams is therefore given by}\]

\[\Delta \omega = \frac{d\omega}{dt} \frac{L}{c} \]

\[\text{The signal at the detector is now given by (in the absence of the plasma)}\]

\[S = I_0 (1 + \zeta \cos \Delta \omega t)\]
Figure 2.6: The sawtooth frequency sweep applied to the interferometer source. The black line shows the frequency ramp, the solid red line shows the resulting sine wave that reaches the detector. The dashed red line demonstrates the phase shift due to the plasma.

previous studies performed by David Byrne. To supplement the new optical data obtained for this thesis, density was recorded by an upgraded version of the interferometer which was developed by Greg Potter. This instrument has a higher phase sensitivity and is better aligned than its predecessor.

Figure 2.7: Typical electron density data. The upper plot shows the raw data as recorded by the interferometer. The lower plot shows its conversion to the time-frequency domain.

2.3 Visible Light Emission

Another diagnostic technique which takes line-of-sight integrated measurements is plasma spectroscopy. For spectroscopic measurements light from a selected spectral range is collected into an optical detection system (such as a photomultiplier tube or CCD array). The spectral region selected depends upon the species (ion or atom) under observation.
For plasmas in H-1 light is emitted from electronic transitions between atomic or ionic states [6]. The wavelength of the emitted light depends upon the energy between the states as described by

\[ \lambda = \frac{hc}{E_{ki}} \]  

(2.6)

where \( i \) and \( k \) denote the lower and upper states respectively and \( E_{ki} \) is the energy between them, \( h \) is Planck’s constant, and \( c \) is the speed of light.

It is possible to determine many of the properties of the ions (or atoms) under observation from the shape of their spectral line emission. The movement of the particles causes a shift in centre frequency (\( \nu_0 = c/\lambda_0 \)) and broadening of the line due to the Doppler effect. These give the drift velocity and temperature respectively of the selected species.

As will be discussed in section 2.4 the original intention for this thesis had been to take measurements of the Carbon II spectral line. The Carbon II lines are chosen for a few reasons; they are bright compared to other lines, they correspond to an ion species, and this ion species is light enough to reflect the behaviour of the hydrogen ions. However due to the difficulties encountered (discussed in sections 2.4.2 and 2.4.3) this approach was abandoned for this thesis in favour of broadband light measurements of mode structure (which are discussed in section 2.5).

### 2.3.1 Spectral Line Intensity

The intensity of a spectral line depends upon the population of the excited state \( n_k \) and the probability of spontaneous radiative decay \( A_{ki} \) (called the Einstein A coefficient between states \( k \) and \( i \)). For optically thin media such as the plasmas in H-1 the mean free path of the emitted photons is longer than the dimensions of the medium [16]. This means that all the photons will escape the plasma and as such induced radiative decay can be ignored. Under these conditions the intensity of the light is given by

\[ I_{ki} = \frac{A_{ki}n_khc}{\lambda_{ki}} \]  

(2.7)

where \( \lambda_{ki} \) is the wavelength of the light emitted from the transition.

The excited state population for H-1 plasmas is described by a rate-equation model called the coronal equilibrium model [6, 16]. This model assumes that the excited state is reached via collisional excitation and that the radiation is released via spontaneous emission [16]. The excited state population density is therefore given by

\[ n_k = \frac{n_e n_1 \langle \sigma_{1k} v_e \rangle}{\sum_{i<k} A_{ki}} \]  

(2.8)

where the subscript 1 denotes the ground state, \( n_e \) and \( v_e \) are the electron density and velocity respectively [16]. \( \langle \sigma_{1k} v_e \rangle \) is the collisional excitation rate coefficient as given by

\[ \langle \sigma_{1k} v_e \rangle = \frac{f_{1k} K_A}{E_{1k} \sqrt{T_e}} \exp\left(-\frac{E_{1k}}{k_B T_e}\right) \]  

(2.9)

where \( f_{1k} \) is the oscillator strength for the 1 to k transition, \( K_A \) is a constant, and \( k_B \) is the Boltzmann constant [16].
Substituting equations 2.8 and 2.9 into equation 2.7 gives
\[ I_{kl} \propto n_en_1 \exp\left(-\frac{E_1k}{k_BT_e}\right) \sqrt{T_e}. \] (2.10)
This will be used in section 2.5 in an attempt to interpret line integrated light intensity fluctuation measurements.

2.4 Quadrature Coherence Imaging System

The original approach to the studies for this thesis had been to measure the pressure fluctuations associated with the Alfvénic modes in the 0.5T plasmas in H-1. Two issues got in the way of this. The first being the limitations in the spectral filter that was being used to isolate the Carbon II spectral light as will be discussed in section 2.4.2. The second issue is the level of shot noise in the Carbon II spectral line. As discussed in section 2.4.3 the signal-to-noise ratio was not high enough for coherence imaging.

2.4.1 Concept

The quadrature switching system is a Fourier transform spectrometer which uses two ferroelectric liquid crystal (FLC) switches to step the interferometer delay about a fixed delay \( \phi_0 \). The fixed delay \( \phi_0 \) is chosen to maximise the sensitivity to variations in the species temperature or drift velocity [17].

The signal picked up by the detector is given by
\[ S = I[1 + \zeta \cos(\phi_0 + \phi_i)] \] (2.11)
where \( I \) is the intensity of the light, \( \zeta \) is the fringe contrast, and \( \phi_i \) is the delay introduced by the FLC switches (which is stepped). The quadrature system takes measurements that are averaged in time over the period of constant \( \phi_i \). This is done at four different delays producing four equations with three unknowns; \( I, \zeta (T_i) \), and \( \phi_i (v_\theta \text{ flow}) \). As the instrument sits at a constant delay step and synchronously integrates the light signals for the duration of the step. It is required that the properties of the plasma fluctuations do not change from one step to the next.

2.4.2 Filtering of the Spectral Light

The region of interest for the tests of the quadrature coherence imaging system was the Carbon II doublet at 658nm. This line was isolated by what should have been a CII interference filter. However as shown in Figure 2.8 the filter was also allowing H\( \alpha \) light through in the wings of its passband. This pollutes the results and as such the simple interferometric system cannot be used for the CII measurements. While a more sophisticated interferometer can manage the H\( \alpha \) contamination, a consideration of SNR issues as discussed in the next section led to the decision to abandon this aspect of the work for the purpose of this honours thesis.

2.4.3 Shot Noise and Integration Time

Shot noise is the unavoidable noise in the detection of light signals that occurs due to the statistical variation in the number of photons reaching the detector during a time period
2.4 Quadrature Coherence Imaging System

Figure 2.8: The transmission of the interference filter (red) and the positions of the H\(\alpha\) and CII spectral lines (blue). As can be seen the filter is transmitting light from the H\(\alpha\) line. Figure courtesy of John Howard.

\(\tau\). Shot noise is characterised by a Poisson distribution and so the variance in the photon count \(\sigma_n^2 = (n - \bar{n})^2\) is equal to the mean photon count \(\bar{n} = \Phi \tau\) (\(\Phi\) is the photon flux) \([6]\). The signal-to-noise ratio (SNR) is given by

\[
\text{SNR} = \frac{\bar{n}}{\sigma_n} = \sqrt{\Phi \tau}.
\] (2.12)

It is therefore possible to increase the SNR by integrating over a longer time period or by increasing the light flux.

The calculation of the signal-to-noise ratio in the Carbon II spectral region has to be high enough to resolve the coherent fluctuations in the signal. If the output voltage from the detector is proportional to the photon count as given by \(V = \alpha \bar{n}\) (\(\alpha\) is the constant of proportionality), and the variance is given by \(\langle V^2 \rangle = \alpha^2 \langle (n - \bar{n})^2 \rangle\), then the average photon count is given by

\[
\bar{n} = \frac{\langle V^2 \rangle}{\langle V^2 \rangle}.
\] (2.13)

The constant \(\alpha\) is given by

\[
\alpha = \frac{\langle V^2 \rangle}{V}.
\] (2.14)

Figure 2.9 shows the red light signals during a 0.5T rf-heated discharge. Figure 2.10 shows a plot of \(\alpha\) over the time of the discharge (Graph A) alongside the SNR (Graph B). As expected for shot noise statistics \(\alpha\) remains effectively constant over time. As can be seen from Graph B of Figure 2.10 the SNR peaks at around 15. To resolve a fluctuation with an amplitude which is 1% of the signal to a SNR of 1 then the signal has to have an SNR of 100. To achieve this the counting period would have to be increased by 45 times. The counting period used for the data in Figure 2.9 was 0.4ms, therefore the required counting period is \(\approx 20\text{ms}\). For coherence imaging a SNR of at least 1000 would be required, or a counting period of a couple of seconds (the plasma lasts for 100ms at most).

Alternatively, by referencing to a suitable clock (such as a Mirnov signal), synchronous detection (in which the pass band is effectively narrowed) can substantially increase the
2.5 Broadband Light Emission

The photon flux in the broadband region (all wavelengths) is high enough to produce an acceptable SNR at the expense of obtaining species specific information. As discussed in section 2.3.1 the light emitted from H-1 plasmas originates mainly from atomic line transitions. In the case of broadband light all of the emitting atoms and ions in the plasma contribute to the intensity (the hydrogen ions in H-1 do not radiate as they have no bound electrons) which gives

$$I_{\text{total}} \propto \int_L \frac{n_e}{\sqrt{T_e}} \sum_j n_j \exp\left(-\frac{E_j}{k_B T_e}\right) dl$$  

(2.15)

where $j$ denotes the species and $L$ is the line of sight along which the measurement is taken. Equation 2.15 demonstrates the complexity of the broadband light. It is impossible to obtain any specific information about the ions and atoms contributing to the intensity.

The spatial variation of $T_e$ has been observed to be fairly flat in H-1 and $T_e$ should be small as the thermal velocity of the electrons is greater than the Alfvén velocity (the
electrons are isothermal). Equation 2.15 therefore can be rewritten as

\[ I_{\text{total}} = \sum_j f_j(T_e) \int_L n_j n_e \, dl \]  

(2.16)

where \( f_j(T_e) = \exp(-E_j/(k_B T_e)) \). The fluctuating component of the intensity can be written in terms of fluctuating quantities \( \tilde{n}_e \) and \( \tilde{n}_j \) as

\[ \tilde{I} = \sum_j f_j(T_e) \int_L n_j n_e (\frac{\tilde{n}_e}{n_e} + \frac{\tilde{n}_j}{n_j}) \, dl \]  

(2.17)

For emission from atoms (which will be weakly coupled to the plasma instabilities) we take \( \tilde{n}_j \) to obtain

\[ \tilde{I} = \sum_j f_j(T_e) \int_L n_j \tilde{n}_e \, dl \]  

(2.18)

From this it can be seen that the broadband light intensity from atoms is a weighted (by \( n_j \)) line integral of the electron density.

The instrumentation used to collect and image the broadband light is relatively simple and easy to implement. The advantage offered by this technique is the availability of multiple fast, 16 channel imaging systems which can be placed at different toroidal and poloidal positions to allow the global (or localised) structure of the H-1 activity to be explored by obtaining radial profiles of the plasma in a single discharge.

### 2.6 The Imaging Cameras

Two multiple channel imaging systems were used to collect the broadband light emitted from the plasmas in H-1, along with a single channel system at the same viewing location as the density interferometer. The basic layout of the multi-array imaging cameras is shown in Figure 2.11. The detectors used are 16 channel linear array Hamamatsu photomultiplier (PM) tubes. The face of each detector is 15.8mm x 16mm with the channels being spread across the 15.8mm dimension. Each channel has an effective area of 0.8mm x 16mm and their centres are separated by 1.0mm. A slit is used to restrict the toroidal view (Figure 2.11).

Both cameras have anti-aliasing low-pass filters at 400kHz and are digitised at 1MHz.

#### 2.6.1 Camera 1

Positioned at the \( \phi = 312.5^\circ \) viewing port camera 1 offers a view from under the plasma as shown in Figure 2.12. It uses a 6mm wide slit which reduces the effective detector area to 15.8mm x 6mm (area per channel of 0.8mm x 6mm). The plasma is imaged onto the detector area using a Nikon camera lens of 135mm focal length (the detector is located at the focus). The distance from the plasma to the imaging lens is 2160mm. As such the detector is expected to be able to image an area of 237mm (poloidally) x 90mm (toroidally) with each channel viewing along a chord which sees a 12mm radial slice of the plasma. The separation between viewing chords is 15mm. Camera 1 samples at a rate of 640kHz giving a Nyquist frequency of 320KHz. The aliased high frequency noise does not enter the passband of interest (0-100kHz).
2.6.2 Camera 2

Camera 2 was installed for the work in this at the same toroidal position as the density interferometer ($\phi = 240^\circ$) giving a view from underneath the plasma as shown in Figure 2.13. A 4mm wide slit is used which reduces the effective detector area to 15.8mm x 4mm (area per channel of 0.8mm x 4mm). A Nikon camera lens of 200mm focal length is used to image the plasma onto the detector area. The distance from the lens to the plasma is 2320mm. This means that the area of the plasma being imaged is 167mm x 42mm with each channel seeing 8mm x 42mm. The separation between the viewing chords for camera 2 is 11 mm. Camera 2 samples at a rate of 500KHz giving a Nyquist frequency of 250KHz. This means that the light signals are slightly under-sampled which could lead to the aliasing of noise into the measurement frequency range. However, as the fluctuations only occupy the band $0 \leq f \leq 100$kHz, the aliasing is not an issue.

2.6.3 Single Channel System

In section 2.5 it was claimed that the light fluctuations were directly proportional to weighted line integrals of the density fluctuations. In order to acquire experimental evidence to support this an additional single channel imaging system was arranged to view the plasma along the same viewing chord as the density interferometer (Figure 2.14) in order that the light and density fluctuations can be directly compared.

The layout of the single channel imaging system is shown in Figure 2.15. The imaging lens focuses the incoming light onto an aperture opening. This arrangement restricts the range of angles at which light can strike the detector. The aim is to restrict the detector to seeing a collimated beam of light of a specific width (which can be changed by adjusting the aperture opening) which travels along the same viewing axis as the plasma arm of the density interferometer. This setup will allow direct comparisons between the broadband light emission and the electron density.

It can be seen in Figure 2.14 that the single channel imaging system as well as the beamsplitter and absorber limit the scanning range of the interferometer. As such this system had to be removed before the full radial profile of the electron density could be
§2.7 Interpretation of Line Integrated Measurements

Figure 2.12: The angle at which camera 1 views the plasma [6]. Each channel of the detector views an area of 12mm x 90mm with an inter-channel separation of 15mm.

obtained. The single channel imaging system was a temporary setup designed purely for comparing the electron density and light signals. The results of Figure 2.16 demonstrate that the light intensity is a weighted function of the electron density as asserted in section 2.5.

2.6.4 Camera Separation

The toroidal separation between camera 1 and camera 2 ($\Delta \phi$) is $72.5^\circ$. H-1NF is a three period Heliac and as such the plasma undergoes 3 complete poloidal turns every toroidal turn. One complete poloidal turn ($\Delta \theta = 360^\circ$) will occur over a $120^\circ$ transit in the toroidal direction. From this we can tell that the plasma will complete 0.6 poloidal rotations ($217.5^\circ$) in the $72.5^\circ$ toroidal separation of the cameras. Using the above information to project the viewing direction of camera 2 onto the cross-section of camera 1 (Figure 2.17) it can be seen that the poloidal separation between the two cameras is $\Delta \theta = 202.6^\circ$ (camera 1 views from the bottom along the axis while camera 2 views from the top at an angle of $22.6^\circ$ to the axis).

2.7 Interpretation of Line Integrated Measurements

Interpretation of the results obtained from the microwave interferometer and optical imaging systems presents a challenge as the results are line-of-sight integrated. One significant issue which complicates this further is the geometry of the plasmas in H-1. In real space (the laboratory frame) the viewing chords of the interferometer and imaging systems are straight lines through the plasma. Transformation to straight magnetic field line coordinates (hereafter referred to as flux space) distorts the viewing lines of each chord. Computer simulations of this have produced some interesting results as presented in 2.7.2. This section considers the projections of modes in flux space and the effect of transforming to real space for the plasmas in H-1.
2.7.1 Projections of a Mode in Flux Space

A mode structure in a unit circle can be described by a Fourier series of odd and even harmonics as given by

$$f(r, \theta) = \sum_{l=-\infty}^{\infty} f_l(r) \exp(il\theta) \quad (2.19)$$

where $f_l(r) = f_{-l}^*(r)$ is the radial variation of harmonic $l$, and $r$ and $\theta$ are the polar coordinates. The projection of this function can be described by

$$G(t, \phi) = \sum_{l=-\infty}^{\infty} g_l(t) \exp(il\phi) \quad (2.20)$$

where $g_l(t) = (-1)^l g_{-l}^*(t)$, $t$ is the impact parameter, and $\phi$ is the measurement angle. This is highlighted in Figure 2.18. $G(t, \phi)$ is called the sinogram of $f(r, \theta)$.

The harmonic $g_l(t) \exp(il\phi)$ is the projection or view of the Fourier component $f_l(r) \exp(il\theta)$ at measurement angle $\phi_l$. Note that the projection of an odd harmonic is odd and the projection of an even harmonic is even for the unit circle (which is a good representation of flux space). Cormack showed that the view (all line integrals at fixed measurement angle $\phi_l$) should have a minimum of $|l|/2$ zeros on $0 \leq t \leq 1$ [18]. When taking line integrated measurements of the plasma we look for parity properties and the number of zeros to aid in verifying the mode properties.
2.7 Interpretation of Line Integrated Measurements

Figure 2.14: The viewing mirror of the density interferometer. The linear scanner allows the radial density profile to be measured. The single channel imaging system receives the plasma light via the beamsplitter located directly below the viewing mirror.

Figure 2.15: The layout of the single channel imaging system. The limiting aperture is placed at the focal length of the imaging lens. This restricts the range of angles at which light reaches the detector.

2.7.2 The Effect of Transformation to Real Space

What may have been a pure harmonic in flux space becomes a mixture of harmonics when transformed to real space. This has some very interesting and useful effects on the appearance of the mode projections. To demonstrate this a model of the projections of the $\phi = 240^\circ$ cross-section of the H-1 plasmas has been constructed by John Howard for use in computer simulations. This forward modelling should give some idea of what to expect in the results.

The first instrument to be modelled is the density interferometer. The results for this are shown in Figure 2.19. As can be seen there is a symmetry to the plasma when viewed along interferometer lines of sight. As such the projections are similar to flux space predictions and the $m = 3$ mode shown in Figure 2.19 has an odd projection. Significantly the symmetry suppressed most of the rotation information. This was observed in the results obtained by David Byrne [13].

The next instrument under scrutiny is camera 2. This camera is located at the same
Figure 2.16: The electron density (red) compared with the light intensity (green) for a 0.5T plasma during a full range $\iota$ scan. The intensity is just a weighted function of the electron density.

Figure 2.17: The projection of the viewing axis of camera 2 onto the plasma cross section at $\phi = 312.5^\circ$. The poloidal separation is 22.6$^\circ$.

cross-section as the interferometer but with very different lines of sight. The results of the simulations for a $m = 3$ mode are presented in Figure 2.20. As can be seen the break in symmetry produces a shearing in the projections of the rotating mode. The direction of the shear depends upon the direction of the poloidal rotation of the mode. Notice also that the shearing causes the projection of this $m = 3$ mode to have approximately even parity. Further simulations were performed to explore this, this time with a $m = 4$ mode. The results are shown in Figure 2.21. As can be seen the projections for this $m = 4$ mode are approximately odd functions. Therefore for the projections seen by camera 2 the odd and even parities switch. Also the rotation of the mode is now detectable in the projections which was not quite the case for the interferometer (which views the plasma either side of its axis of symmetry). The setback is that the number of zeros in the projection no longer tells anything useful about the mode properties, we must rely on forward modelling or tomography.

The last instrument to be modelled is camera 1. This camera is not located at the $\phi = 240^\circ$ cross-section and as such its view was projected back to this position from
Figure 2.18: The projection of $f(r, \theta)$ onto a measurement angle $\phi_i$ and range of impact parameters $-1 \leq t \leq 1$. The structure is rotating to give the projection $G(t, \phi_i)$.

$\phi = 312.5^\circ$ (as discussed in 2.6.4). The results of the simulation are shown in Figure 2.22.

For camera 1 it can be seen the phase of the mode oscillation is not sheared across the viewing array as was the case for camera 2. This, our choice of positioning for camera 1 ensures that we can use the 16-channel data to unambiguously identify the mode rotation direction. MHD activity is expected to rotate in the ion diamagnetic ($B \times \nabla p / (q_nB^2)$) direction and so determining the rotation direction is a step towards identifying the mode.
Figure 2.19: The simulation results for a $m = 3$ mode as viewed by the interferometer. (a) shows the geometry of the plasma cross-section with the viewing lines overlayed (the interferometer views from the right), (b) shows the profile of the light fluctuations. The mode is rotating in the ion diamagnetic drift direction (clockwise). For this viewing angle the projections are similar to what would be expected for flux coordinates (odd function, odd projection).
Figure 2.20: Simulation results for a $m = 3$ mode rotating in the ion diamagnetic direction as viewed by camera 2. The figure layout is the same as for Figure 2.19. As can be seen there is shearing in the projections. The direction of this shear depends upon the direction of the poloidal rotation of the mode.
Figure 2.21: The simulation results for a $m = 4$ mode rotating in the ion diamagnetic direction as viewed by camera 2. The figure layout is the same as for Figure 2.19. There is again a shear in the projections that depends upon the direction of rotation. This time the projections appear to be approximately odd.
\section*{Interpretation of Line Integrated Measurements}

Figure 2.22: The simulation results for a $m = 3$ mode rotating in the ion diamagnetic drift direction for camera 1. The projections in this case appear similar to the predictions for flux space. No shearing is seen for these projections.
Chapter 3

Results and Interpretation

Previous studies into the nature of the instabilities in the H-1 Heliac have tentatively identified them as torsional global Alfvén eigenmodes in a number of cases and have made estimates of their poloidal and toroidal mode numbers [7]. These results were obtained using data mining techniques to search for patterns in data obtained from hundreds of discharges taken under conditions of fixed magnetic configuration. However the advanced H-1 power supplies and control system now allow the possibility to change the magnetic configuration \((\iota)\) dynamically. This allows a large compression of the data. This chapter presents and discusses the results obtained from the dynamic configuration scans in the H-1 Heliac. This is the first time that a study of Alfvén activity in a dynamically changing magnetic configuration has been performed. Such continuous scans provide more information than previous shot-by-shot static scans, because more modes can be tracked continuously as they coalesce and split.

As discussed in section 2.1, the Mirnov coils which were used to obtain the mode number estimates have some degree of uncertainty associated with their measurements [5]. As such the toroidal mode numbers have yet to be confirmed. The modes have been observed to rotate in the same poloidal direction as the ion diamagnetic drift [5]. In order to explore this further a 16 channel PMT imaging system was constructed and installed to view the plasma from an angle that breaks the up/down symmetry as discussed in section 1.1 and provide information about radial mode structure unavailable from existing systems. The poloidal rotation of the mode is predicted from the computer simulations discussed in section 2.7.2 to cause phase shearing in the projections seen by this system. In addition to supplying information on the rotation direction of the modes this imaging system (camera 2), if toroidally displaced by less than a full period from its counterpart (camera 1), can be used in conjunction with camera 1 to explore the toroidal periodicity of the modes. Another interesting observation is the phase “flip” between the light signals and the Mirnov coils observed in preliminary studies as discussed in section 1.1.3. Multiple 16-array light imaging systems are cross-referenced with the Mirnov coils to obtain some insight on what is behind this phase behaviour.

3.1 Experiments

The experiments undertaken for this thesis involved scanning through a range of magnetic configurations by linearly ramping the current in the helical winding in H-1 over the duration of a single shot (100ms). The range of the configuration scan can be changed by adjusting the range of the current ramp applied to the helical control winding. For example, with \(I_{\text{main}} = 6500\) A (current required to produce an on-axis magnetic field
strength of 0.5T), it is possible to ramp the helical current from 1000A to 7000A in 100ms to dynamically vary the configuration parameter between $\kappa_h = 0.15$ and $\kappa_h = 1.08$, thereby varying the on-axis rotational transform continuously between 1.16 and 1.46.

There are some important experimental considerations when performing these dynamic scans. First, the plasma is a conducting medium and as such there will be a time delay ($L/R$, where $L$ is the plasma inductance and $R$ is the resistance of the plasma) which slows the change of the magnetic field in the plasma volume by inducing toroidal currents in the plasma. This delay will cause a time shift in the effective magnetic configuration parameter which depends on the scan rate and direction (increasing or decreasing helical current). In other words, while $\kappa_h$ remains given by $I_h/I_{\text{main}}$, the instantaneous rotational transform experienced by the plasma corresponds to the $\kappa_h$ value delayed in proportion to the $L/R$ time. Typically this delay can be $\approx 5$-10ms. The data shown here are plotted against the instantaneous $\kappa_h$ parameter resulting in an apparent shift in the position of the various transform resonances.

A related issue is that the distribution of currents in the plasma induced by the changing magnetic field can affect the plasma magnetic configuration (in addition to the change in $\kappa_h$) and could in principle disturb both the structure of the Alfvén wave modes and/or their stability. The instantaneous size of the induced current depends on the plasma resistance (which depends on temperature and effective atomic number $Z_{\text{eff}}$), its internal inductance and the mutual inductance between the helical conductor and the plasma. Many of these quantities are difficult to measure and beyond the scope of this thesis. However, we can make the following simple estimate. For the helical winding mutual inductance of $L_h \approx 8\mu\text{H}$, the toroidal induced emf is $L_h dI_h/dt \approx 0.48\text{V}$. At a temperature of 10eV, the plasma internal resistance is estimated to be $R = 1\text{m}\Omega$ leading to a steady dc current of $\approx 500\text{A}$.

To empirically confirm that the inductance effects do not affect the mode behaviour, scans were performed over different $\kappa_h$ ranges, and in different directions. The results, which are discussed in the next section, reveal the presence of the inductive delay, but otherwise no significant effect on the mode structure or stability.

### 3.2 Light Projections

In this section the results from the two 16 channel imaging systems will be considered. Except where stated otherwise the results presented have been referenced to the same Mirnov coil signal to allow phase comparisons.

#### 3.2.1 Shot Reproducibility

For any experiment, results are considered to be reliable when they are reproducible. This means that when the exact conditions of the experiment are repeated similar results are obtained. For the experiments performed at H-1, especially when the configuration is changed dynamically, we want to confirm reproducibility in the plasma discharges. That is, shots fired under the same conditions should exhibit the same behaviour. Some of the diagnostic techniques, such as using the interferometer to construct the electron density profile on a shot-to-shot basis, require that the shots are reproducible. Figure 3.1 demonstrates the reproducibility of the results obtained from H-1 plasmas, both for a full $\kappa_h$ scan and a partial scan. Figure 3.2 shows a shot for a reverse scan (decreasing $\kappa_h$). As can be seen, reversing the scan changes the results quite significantly. Nevertheless, for $\kappa_h$
scans taken in the same direction, and over the same $\kappa_h$ range, the results are substantially reproducible (see Figure 3.1.

Figure 3.1: Comparison of the data from camera 2 for different shots. The graph to the left shows an overlay of two shots (distinguished by colour) from a full $\kappa_h$ scan. The graph to the right shows the same for a partial $\kappa_h$ scan.

Figure 3.2: The signal from camera 2 for a reversed $\kappa_h$ scan.

3.2.2 Inductance Effects

In this section the effects of the inductive delay discussed in section 3.1 are considered. The following discussion refers to Figures 3.3, 3.4, and 3.5 which show the results from camera 1 for a full configuration scan, a partial scan, and a reversed scan respectively. The figures have the same layout. (a) shows the time evolution of the cross-power between a mid-radius PMT channel and the Mirnov coil. (b) shows the relatively normalised light intensity contours for all of the 16 channels, smoothed to eliminate the fluctuating component (level $\approx 1\%$). (c) shows the time evolution of the fluctuation power for all 16 channels. (d) is the profile of the power-weighted phase (referenced to the Mirnov signal)
across the channels. The power-weighted-phase is given by

\[
\langle \phi_i \rangle_j = \left[ \frac{\int P_i(\omega)\phi_i(\omega)d\omega}{\int P_i(\omega)d\omega} \right]_{\tau_j}
\]  

(3.1)

where \(i\) denotes the channel and \(j\) is the time window. The power-weighted-phase emphasises the phase of the strong harmonic components but is only meaningful when there is one dominant frequency component. This is not always the case as can be seen from the cross-power contours shown in Figure 3.3(a), for example the \(\kappa_h\) range \(\sim 0.48\) or \(0.59\). To compensate for multiple signals the pass band has to be narrowed to include only the signal of interest. It should be noted that this more detailed analysis was not performed for the work in this thesis.

Returning to the issue at hand it can be seen by comparing Figure 3.3(a) to Figure 3.4(a) that the location of the \(5/4\) resonance (occurring at \(\kappa_h \approx 0.45 \pm 0.01\) and \(\kappa_h \approx 0.43 \pm 0.01\) respectively) departs from the expected value of value of the resonance (\(\kappa_h = 0.4\)) in proportion to the range of the configuration ramp. Comparing with Figure 3.5 on the other hand, where the scan direction is reversed, as expected the apparent location of the resonance occurs at \(\kappa_h = 0.37\).

![Figure 3.3:](image)

**Figure 3.3:** The results from camera 1 from a \(0.23 \leq \kappa_h \leq 1.08\) scan in a 0.5T plasma discharge (shot no. 66676). Going from top to bottom: 1. time evolution of the cross-power between channel 6 of camera 1 and the Mirnov coil, 2. the normalised dc light emission intensity profile, 3. the light-Mirnov cross-power profile, 4. the power-weighted-phase profile. The resonance points are where the frequency of the mode almost reaches zero (\(\kappa_h \approx 0.45, 0.77\)). Note the power-weighted-phase jumps by \(180^\circ\) at these points.

The observed time delay in reaching a certain configuration can be used to estimate
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Figure 3.4: The results for camera 1 from a 0.23 ≤ κ_h ≤ 0.61 scan in a 0.5T plasma discharge (shot no. 66684). Going from top to bottom: 1. time evolution of the cross-power between channel 6 of camera 1 and the Mirnov coil, 2. the normalised dc light emission intensity profile, 3. the light-Mirnov cross-power profile, 4. the power-weighted-phase profile. Notice the slight difference in the resonance position (at κ_h ≈ 0.43 compared to 0.45 for Figure 3.3 results. The power-weighted-phase again jumps at the resonances

the resistance of the plasma and hence the current induced during the ramp. Results from previous studies using the Mirnov coils showed that the n/m = 5/4 resonance should occur at κ_h ≈ 0.4 [7]. From Figure 3.4 it appears that the 5/4 resonance is occurring at κ_h ≈ 0.43. The scan is performed over a duration of 100ms which gives a delay time of

$$\frac{L}{R} \approx \left(\frac{0.43 - 0.40}{1.08 - 0.23}\right) \times 100ms \approx 3.5ms.$$  

(3.2)

The plasma external inductance is $L \approx 10\mu H$, which gives a plasma resistance of $R \approx 2.8m\Omega$. This agrees fairly closely with the estimate of a resistance of 1m\Omega based on the Spitzer formula for plasma resistance [19]. The mode structure itself does not appear to be affected by the induced current although there are some significant changes in the relatively normalised light intensity between forward and reverse scans. The light emission of the reversed scanned plasma almost drops out completely at κ_h ≈ 0.6. This difference between the light intensity evolution between the forward and reverse κ_h scans may be due to the strong effect of configuration resonances on plasma confinement and the fact that particle confinement times are of order 5ms. On the other hand, apart from frequency and opposite sense of configuration shift changes (due to different $n_e$ evolution), the fluctuation structures persist regardless of ramp direction.
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Figure 3.5: The results for camera 1 from a 1.08 ≥ $\kappa_h$ ≥ 0.18 (reversed) scan in a 0.5T plasma discharge. Going from top to bottom; 1. time evolution of the cross-power between channel 6 of camera 1 and the Mirnov coil, 2. the normalised dc light emission intensity profile, 3. the light-Mirnov cross-power profile, 4. the power-weighted-phase profile. Note the significant change in resonance location ($\kappa_h$ ≈ 0.37, 0.71). Also note the phase jumps at these locations.

3.2.3 Relative Phase Between $\tilde{B}$ and $\tilde{n}_e$

One very interesting phenomenon observed when referencing the light signals to a Mirnov signal is the 180° relative phase shifts which occur across the resonances. These can be seen clearly in the power-weighted-phase contours for both camera 1 (at $\kappa_h$ ≈ 0.45, 0.77 for Figure 3.3, $\kappa_h$ ≈ 0.43 for Figure 3.4, and $\kappa_h$ ≈ 0.37, 0.71 for Figure 3.5) and camera 2 (Figure 3.6, resonance at $\kappa_h$ ≈ 0.43). To better understand the cause of these phase shifts we carefully examine the Mirnov and light signals at a resonance. Figure 3.7 shows one of the resonances for a full range (1.08 ≥ $\kappa_h$ ≥ 0.15) reverse configuration scan. The light signals (red) are compared to the Mirnov signals (black) over the 5/4 resonance (which occurs at $\kappa_h$ ≈ 0.37 for this shot). By looking closely at the fluctuations in the 0.08s ≤ t ≤ 0.081s time range just before the 5/4 resonance it can be seen that the light signals are leading the Mirnov signals by a small amount. Because of this there is a phase difference between the signals. However as the resonance (at around t ≈ 0.082s) is approached the oscillations begin to line up until at the resonance there is no phase difference between them. On the other side of the resonance the Mirnovs start leading the light fluctuations, causing the phase difference to change sign (“flip”). The 5/4 resonant mode structure resonates with the magnetic field lines at an apparent $\kappa_h$ = 0.43 (Figure 3.4). Either side of the resonance the mode helix (using the simple picture described in chapter 1) becomes out of phase with respect to the background field. The sense of the poloidal magnetic
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Figure 3.6: The results for camera 2 from a $0.23 \leq \kappa_h \leq 0.61$ scan in a 0.5T plasma discharge (shot no. 66684). Going from top to bottom: 1. the time evolution of the cross-power between channel 6 of camera 1 and the Mirnov coil, 2. the relatively normalised smoothed light emission intensity profile, 3. the fluctuation power profile, 4. the power-weighted-phase profile. Note the phase jump at the resonance location ($\kappa_h \approx 0.43$.)

Field component that accounts for the mismatch between the magnetic field line and the mode helix reverses about the resonance and is a likely explanation for the observed phase change between the magnetic and density fluctuations. Note the propagation of torsional (or shear) Alfvén waves along a magnetic line of force is due to the interchange of magnetic ($\Delta B^2/2\mu_0$) and kinetic pressures ($\Delta p = \Delta nkT$). The magnetic restoring force is independent of the sense of $\Delta B$.

3.2.4 Mode Rotation and the Presence of Sound Waves

As can be seen in Figure 3.7 the Mirnov fluctuations almost die out at the resonance yet the light signals grow in amplitude. This suggests the presence of low-frequency sound modes at the resonances. Recent studies conducted at the W7-AS have shown that it is possible to simultaneously excite Alfvén waves and sound waves which rotate in the ion diamagnetic direction and the electron diamagnetic direction respectively [20]. It is therefore possible that as the torsional Alfvén mode in H-1 approaches resonance ($\omega \rightarrow 0$) the sound wave begins to dominate.

While previous studies have shown that the magnetic perturbation associated with Alfvén modes in H-1 rotate in the ion diamagnetic direction [5] the rotation of the fluid (density and light) fluctuations has yet to be explored. In order to obtain some information on the mode rotation camera 2 was set up to view the plasma from an angle that broke the
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Figure 3.7: Light signals from camera 1 (red) compared with Mirnov signals (black) for a full range $1.08 \geq \kappa_h \geq 0.15$ reversed configuration scan (shot no. 66667). Before the resonance ($\kappa_h \approx 0.37$) the light signals lead the Mirnov signals. After the resonance the opposite is true. The light signals increase at the resonance while the Mirnovs almost disappear completely.

symmetry as discussed in chapters 1 and 2. Using computer models we have found that this will make camera 2 sensitive to the direction of rotation of the mode (section 2.7.2). The rotation should produce shearing in the intensity fluctuation ($\tilde{I}/I$) profiles. Figures 3.8 and 3.9 show the $\tilde{I}/I$ profiles for camera 1 and camera 2 respectively.

The profile for camera 1 shows some shear (sloping to the right when going from channel 1 to channel 16). This was not predicted by the simulations described in section 2.7.2. However, the model for the simulations was made based on the geometry of the plasma at the location of camera 2 ($\phi = 240^\circ$). Camera 1 was modelled by projecting its view from $\phi = 312.5^\circ$ to the position of camera 2. This assumes that the geometry of the plasma at $\phi = 240^\circ$ is the same as the geometry at $\phi = 312.5^\circ$. In reality this is not strictly true. The geometry at camera 1 may differ to the geometry at camera 2. An important consideration

Figure 3.8: The $\tilde{I}$ profile from camera 1 in the region $0.42 \leq \kappa_h \leq 0.46$. Note the slight shear in the projections. This indicates both a rotating mode and a slight inaccuracy in the model when applied to camera 1 (camera 1 is located at a different poloidal cross-section than the $\phi = 240^\circ$ cross-section).
Figure 3.9: The $\tilde{I}$ profile from camera 2 in the region $0.42 \leq \kappa h \leq 0.46$. The shear is quite apparent before and after the resonance ($\kappa h \approx 0.45$). The mode becomes dominated by a low frequency component at the resonance.

Figure 3.10: The profile obtained from computer simulations of a $m = 4$ mode structure rotating in the ion diamagnetic direction. Comparison with Figure 3.9 demonstrates that the observed Alfvén modes are rotating in the ion diamagnetic direction.

at this point is whether or not the shear in the profile for camera 1 agrees with camera 2. From Figures 3.8 and 3.9 it can be seen that they do (remember that channel 16 is the outer channel for camera 1 and channel 1 is the outer channel of camera 2). The next task is to determine the direction of the poloidal rotation by matching the model to the results. Figure 3.10 shows the results from the simulations for a $m = 4$ mode rotating in the ion diamagnetic direction. The general direction of the shear matches that of most of the profile for camera 2 (before and after the resonances). This then confirms that the observed Alfvén modes are rotating in the ion diamagnetic drift direction. However, at the resonance ($\kappa h \approx 0.45$) a low frequency mode appears which almost reverses the direction of the shear in the profile at the resonance. This could be the sound mode rotating in the electron diamagnetic direction as discussed in [20].

The modes around the resonances have been identified as global Alfvén eigenmodes in previous studies [5]. As such they are expected to be localised to an extent at the minimum in the rotational transform profile (Figure 3.11). As the minimum in the transform moves radially outwards with increasing $\kappa h$ the GAE is also expected to move radially outwards as $\kappa h$ increases. Thus we expect to see the modes located around the resonance points to move towards the outer channels of the power-weighted-phase contours for both cameras as $\kappa h$ increases. Camera 1 and camera 2 have their outer channels at channel 16 and 1 respectively. The global modes should therefore move up on the camera power-weighted-phase contours and down on the camera 2 contours. The results from camera 1 (Figures 3.3 and 3.4) show the mode moving gradually towards the outer channel (radially outwards) in the regions $0.32 \leq \kappa h \leq 0.52$ and $0.65 \leq \kappa h \leq 0.8$. The modes located in these regions are the $5/4$ and $4/3$ modes respectively [5, 7]. The movement of these modes is consistent with what is expected for global Alfvén eigenmodes. In the results from camera 2 (Figure 3.6) the mode around the resonance appears to move slightly towards channel...
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Figure 3.11: Radial profiles of the rotational transform at different $\kappa_h$ values \cite{3}. The magnetic flux surfaces for two of the configurations are also shown.

1 (the outer channel) in the $0.34 \leq \kappa_h \leq 0.53$ region, although it does not appear to be moving by much. It is possible that the angle at which camera 2 views the plasma could be making it less sensitive to the radial localisation of the mode (it can only sense movement perpendicular to its viewing chords). Away from the resonances there appear to be higher frequency, higher order modes (for example in the $\kappa_h < 0.3$ and $\kappa_h > 0.8$ regions, they can be identified by the higher number of zero crossings in the power-weighted-phase contours). It is possible that these could be the higher order gap modes such as helical or elliptical Alfvén eigenmodes.

Both toroidally displaced cameras show a steady structure over a wide range of rotational transform, clearly indicating that an eigenmode of fixed helicity but with rotational transform dependent frequency as given by equation 1.3, persists in the regions close to rational configurations.

3.3 Conclusions

A new array of photomultiplier tubes were installed onto the H-1 Heliac at a location at which a completely different view of the plasma was available. This was intended to break the up/down symmetry of the views seen by most of the other imaging systems and hence make the array sensitive to the rotation of the modes observed. By using the advanced H-1 power supplies and control system to perform dynamic scans of the magnetic configuration data from a wide range of configurations was able to be collected from a single discharge. The data from these shots were then compared to modern theory.

The results presented were in agreement with the simulation results in that the new imaging system was sensitive to the rotation of the mode. The rotation direction of the mode was confirmed to be in the ion diamagnetic drift direction. A phase reversal across the resonances at rational ratios of mode numbers $n/m$ was observed between the magnetic and light fluctuations. Although the exact cause of this remains unknown.
Finally an estimate of the plasma resistance and inductance through the observed delay in the magnetic field due to induced plasma currents showed an example of a recent development known as “Alfvén Spectroscopy”, the use of Alfvén emission to determine plasma parameters.


