A Study of MHD Activity in the H-1 Heliac Using Data Mining Techniques.

A thesis submitted for the degree of
Doctor of Philosophy of
The Australian National University.

by

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This thesis, to the best of my knowledge and belief, does not contain any results previously published by another person or submitted for a degree or diploma at any university except where due reference is made in the text.

David George Pretty
31 March 2009
to my parents
Abstract of the Thesis

The H-1 heliac is a three field-period helical axis stellarator [1] with major radius $R = 1 \text{ m}$, minor radius $\langle r \rangle = 0.2 \text{ m}$ and a finely tunable magnetic geometry. The very precise control of magnetic geometry allows the exploration of the relation between plasma behaviour and magnetic configuration.

Experimental scans through plasma configurations via the geometric parameter $\kappa_h$, which controls the rotational transform $\iota$ (twist of the magnetic field lines) and shear $\iota'$ (radial derivative of rotational transform), have produced diverse spectra of magnetohydrodynamic (MHD) activity. The MHD activity is recorded via two toroidally separated poloidal arrays of Mirnov coils (induction solenoids) which sample $dB/dt$ locally. In the main dataset presented in this thesis, 28 Mirnov coils are used for 92 distinct plasma configurations, resulting in more than 100,000 short time Fourier spectra.

In order to analyse the dataset, a novel data mining technique for the analysis of multichannel oscillatory timeseries data has been developed. The procedure is highly automated, and scales well to large datasets. The timeseries data is split into short time segments to provide time resolution, and each segment is represented by a singular value decomposition (SVD). By comparing power spectra of the temporal singular vectors, singular values are grouped into subsets which define fluctuation structures. Thresholds for the normalised energy of the fluctuation structure and the normalised entropy of the SVD are used to filter the dataset. We assume that distinct classes of fluctuations are localised in the space of phase differences $\Delta \psi(n, n + 1)$ between each pair of nearest neighbour channels. A clustering algorithm is used to group distinct classes of fluctuations, and a cluster tree mapping is used to visualise the results.
Using the classes of fluctuations found by the data mining algorithm, comparisons with several theoretical models are made in order to interpret the data. The dominant modes ($f \sim 20 - 60$ kHz) are shown to be related to low-order resonant surfaces in the plasma; these dominant modes show quasi-Alfvénic behavior, with a similar dispersion relation to the global Alfvénic eigenmode and non-resonant interchange modes. Scaling with plasma density and rotational transform is found to be consistent with these modes for most of the identified clusters. However, a frequency scale factor of $\lambda \sim 0.3$ is required to match the data, and the scaling with magnetic field strength is inconclusive. A pressure-induced variant of the Alfvén eigenmode is discussed as another possible candidate. Also seen are frequency modes ($f \sim 60 - 100$ kHz) which are in good agreement with helical Alfvén eigenmodes. Preliminary results for fully three-dimensional modelling of these modes are presented, and fast ions are considered as a driving mechanism for the modes.
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Regarding the notation used in this thesis, there is some inconsistency due to software imperfections resulting in $\iota$ being rendered as $\iota$ in figures and subfigure captions. Apart from in defining equation 1.4, all instances of $\iota$ should be interpreted as $\iota$. There are also figures in chapter 5 where the absolute value $|\iota - n/m|$ is displayed as $(\iota - n/m)$, it should be clear from the context where this is the case.
CHAPTER 1

Introduction

1.1 Thermonuclear Fusion

Nuclear fusion reactions are exothermic for light nuclei (lighter than iron). The nuclear binding energy per nucleon within each of the elements is shown in figure 1.1. When a nuclear reaction occurs, the difference in binding energy (or mass, $\Delta E = \Delta mc^2$) between the products and reactants is released as kinetic energy of the reactants. For a fusion reaction to occur, two nuclei require sufficiently high velocity to overcome their repulsion due to positive electric charge; and to extract useful amounts of energy, many particles are required. Such conditions require heating bulk quantities of fuel to temperatures sufficiently high for it to be in the ionised gaseous state known as plasma.

The cross-sectional area for particle interaction $\sigma = \sigma(v)$ is a measure of the probability of an interaction between particles occurring. For a plasma in thermal equilibrium, a Maxwellian velocity distribution can be assumed and, by integration over particle velocity, the fusion reaction rate $\langle \sigma v \rangle_f$ is found, where $n_an_b(\sigma v)_f$ is the rate of fusion reactions between ion species $a$ and $b$ per unit volume, and $n_{a(b)}$ is the density of species $a(b)$. Required ion temperatures for a useful fusion reaction rate of $\langle \sigma v \rangle_f = 10^{-22}\text{m}^3\text{s}^{-1}$ for the most attainable fusion reactions are shown in table 1.1.
Figure 1.1: Nuclear binding energy per nucleon. Data from reference [2], most abundant or greatest half-life isotope used for each element.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$T_i$ [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-T</td>
<td>9.3</td>
</tr>
<tr>
<td>D-³He</td>
<td>65</td>
</tr>
<tr>
<td>D-D</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 1.1: Required ion temperature for fusion reaction rates of $\langle \sigma v \rangle_f = 10^{-22}$ m$^3$ s$^{-1}$.

1 keV = $1.16 \times 10^7$ degrees Kelvin.
1.2 The Motivation for Thermonuclear Fusion Research

Modern economies of the developed world depend on the ready availability of fossil fuels. While there remains uncertainty in the quantity of current oil reserves, there is general consensus that the peak in oil production, known as Hubbert’s peak [3, 4], will occur during the next few decades. Current predictions range from now (2004 – 2008) [5] to the middle of this century[6]. Such is the current dependence on fossil fuels that it appears realistic to expect some major geopolitical consequences to arise when the supply falls short of demand.

Arguably, even more important is the effect of our fossil fuel reliance on the environment. While there may be debate in the public arena over the existence or cause of global warming, the vast body of scientific work shows that recent climate change is due to anthropogenic effects. The most recent assessment report from Intergovernmental Panel on Climate Change [7] states that “[w]arming of the climate system is unequivocal, as is now evident from observations of increases in global average air and ocean temperatures, widespread melting of snow and ice, and rising global average sea level” and that “[m]ost of the observed increase in global average temperatures since the mid-20th century is very likely due to the observed increase in anthropogenic greenhouse gas concentrations”.

It is clear that alternative energy sources need to be exploited. Energy sources available on Earth are limited to incident radiation (electromagnetic), tidal forces (gravitational), and terrestrial energy (nuclear energy; either in the form of fuel for nuclear reactions (e.g. uranium ore deposits) or geothermal energy from radioactive decay deep underground). Incident radiation is the dominant driving force of the planetary energy cycle, eg: weather and the biosphere. Of the 178,000 TW of incident radiation, 62,000 TW is reflected and 76,000 TW is immediately reradiated as heat at long wavelength. Evaporation of water accounts for 40,000 TW,
and relatively minor amounts are absorbed elsewhere: 3,000 TW in wind, 300 TW in waves and 80 TW in photosynthesis - however nearly all incident radiation is ultimately reradiated as heat. Geothermal conduction and convection contribute 32 TW and tidal power just 3 TW.\(^1\)

Globally, the average energy usage was 13.5 TW in 2001, and is expected to rise to 20.8 TW in the year 2025 [6]. According to the U.S. Department of Energy [6], there is currently 250 TW-years of proven oil reserves and an estimated 320 TW-years of undiscovered oil and efficiency savings from future technologies. As of January 2004, the worldwide natural gas reserves were 210 TW-years, and there is about 210 years supply of coal (black and brown combined) at current consumption levels.

Solar power appears to be a desirable candidate, although construction costs may be exorbitant with over 5600 km\(^2\) per TW collection area required (average power, 50% solar efficiency). Compared to fossil fuels, nuclear power is a longer term solution due to the quantity and energy density of fuel available. Uranium-235 supply for fission power is \(\sim 300\) TW-years, and U-238 and Th-232 supply for fission breeder reactions is more than \(3 \times 10^8\) TW-years. An estimated \(6 \times 10^8\) TW-years of Lithium for D-T fusion, and \(2 \times 10^{11}\) TW-years of deuterium for D-D fusion are contained in the oceans [8]. Fission power is unfavourable due to environmental impacts of waste disposal, so fusion along with renewables appear to be the ideal power sources of the future.

\(^1\)Planetary energy cycle data taken from reference [8].
1.3 The Physics of Magnetically Confined Plasma

1.3.1 The Plasma State

Plasma is generally known as the fourth state of matter. As temperature is increased, solid melts to liquid, liquid evaporates to gas. If the temperature is further increased gas molecules break up into their constituent atoms, and the atoms become ionised. It is the collective behaviour of particles under the influence of long range electromagnetic forces from all other particles in the system which distinguishes plasma from gas.

While nearly all the observable universe is made of plasma, terrestrial plasmas are relatively rare. Naturally occurring terrestrial plasmas include lightning and the aurora. Plasmas are used in industrial processes as well as domestically, e.g.: fluorescent lighting.

The characteristic length and time scales a plasma are given by the Debye length and inverse plasma frequency. The Debye length

$$\lambda_D = \sqrt{\frac{\epsilon_0 T_e}{n_e e^2}}$$

(1.1)

is a measure of the reduced extent of an electric field within the plasma due to the dielectric effect of the compensating space-charge redistribution. The plasma frequency

$$\omega_p = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

(1.2)

is the oscillation frequency of response to a small separation of charge in a quasi-neutral plasma, where \(n_e\) is the electron density, \(e\) is the electron charge, \(T_e\) is the electron temperature, \(m_e\) is the electron mass, and \(\epsilon_0\) is the permittivity of free space. The collective behaviour of a plasma can be observed on length \((L)\) and time \((\tau)\) scales which satisfy \(\lambda_D/L \ll 1\) and \(\omega_p\tau \gg 1\).
1.3.2 Plasma Confinement

For most purposes, a plasma needs spatial confinement to be useful. On the large scale, plasmas are confined by their own gravitational field. On a smaller scale, for industry and research, cold plasmas can be confined by a material container, which is ultimately electrostatic confinement at the atomic scale. However, plasmas with temperature required for fusion \( T \gtrsim 10 \text{ keV} \) are too hot to be confined by any known material.

The Lawson criterion, \( n\tau_E \gtrsim f(T) \), is a reference for the required plasma conditions for fusion reactions to occur. Here, \( n \) is the plasma density and \( \tau_E \) is the energy confinement time defined as the ratio of stored energy in the plasma to the net input power. For deuterium-tritium reactions, \( f(T = 15 \text{ keV}) \sim 3 \times 10^{20} \text{ sm}^{-3} \). The inequality suggests two approaches to fusion: high density or long confinement time. Inertial confinement fusion (ICF) research focuses on the high density approach, which is to heat and compress a small amount of solid fuel to a sufficient level in the time before the fuel expands and blows apart. The magnetic confinement approach aims for long confinement times which are desirable for a steady state reactor.

1.3.3 Toroidal Magnetic Geometry

The magnetic force on a particle with charge \( q \) is in the direction perpendicular to both the magnetic field \( \mathbf{B} \) and particle velocity \( \mathbf{v} \):

\[
\mathbf{F} = q \mathbf{v} \times \mathbf{B}.
\]  

(1.3)

This means that, within a plasma, a particle will travel along a path that is helical about a magnetic field line, with the presence of the field having no effect in the direction of \( \mathbf{B} \). The angular frequency and radius (often called the gyroradius)
of this cyclotron motion are $\omega_c = eB/m$ and $r_c = \omega_c^{-1}\sqrt{2T/m}$ respectively. The electron gyroradius in fusion relevant plasmas is of the order $r_{ce} \sim 10^{-5} - 10^{-4}$ m, with the ion gyroradius $r_{ci} \sim 10^{-3} - 10^{-2}$ m. When the gyroradius is small compared to variations in the field and the size of the plasma it is useful to neglect the cyclotron component of the particle trajectory and consider only the parallel motion, or guiding centre of the particle.

Any force $\mathbf{F}$ on the particles within a magnetised plasma gives rise to a transverse drift $v_d = (qB^2)^{-1} \mathbf{F} \times \mathbf{B}$. Consequently, if the force is charge independent then the electron and ion drifts are in opposite directions, inducing a separation of charge. It should be noted that despite the effect described in equation 1.1, an electric field in the direction perpendicular to $\mathbf{B}$ can be maintained within a magnetised plasma because the particles cannot freely travel in the direction parallel to $\mathbf{E}$ to compensate the electric field.

Linear magnetic fields have been shown to be inadequate for confining fusion relevant plasma due to fast particle loss at the ends of the confinement device. The connection of ends of a linear machine to form a toroidal shaped field appears to be the simplest method of overcoming the problem of fast particle loss. The closing of the field introduces curvature, and with it new problems related to the non-uniformity of the field. Field variation and curvature are responsible for most difficulties with the confinement of plasma in a toroidal field. These problems often scale with the inverse aspect ratio $r/R$, which is a measure of toroidicity where $R$ and $r$ are the major and minor radius of the plasma.

The largest field variation is a $B \propto 1/R$ effect which introduces a $\nabla B \times B$ force, giving rise to charge-dependent vertical particle drift, leading to a rapid loss of confinement. This problem is overcome with the introduction of a poloidal field component to counteract the vertical particle drift. The method of creating
a poloidal magnetic field leads to the distinction between the two main classes of toroidal confinement devices: the tokamak, which uses a toroidal current within the plasma, and the stellarator which employs external field coils to generate the poloidal field component. The tokamak benefits from toroidal symmetry, allowing theorists to work with two-dimensional models, and relatively simple design which leads to lower construction costs compared to a similarly sized stellarator. The absence of an externally induced toroidal current in the stellarator design reduces the problem of current-driven instabilities and can produce more stable plasmas suitable for a steady-state reactor. Issues of stellarator theory, design and construction are generally more complex than those of tokamaks.

With both toroidal and poloidal components, the field lines map out a set of nested flux surfaces. Because particles can move freely along the field lines, the flux surfaces are in effect surfaces of uniform equilibrium quantities, such as pressure and electric field. The ratio of poloidal to toroidal field is known as rotational transform,\(^2\)

\[
\iota = \frac{\iota}{2\pi} = \frac{1}{2\pi} \frac{d\psi_p}{d\psi_t}
\]  

(1.4)

and is a function of the flux surface. Here, \(\psi_{t(p)}\) is the magnetic flux in the toroidal (poloidal) direction within a specific magnetic surface, where the poloidal flux is calculated about the magnetic axis, or the surface at the zero-volume limit. Rotational transform is present in stability analysis as a measure of connection length between regions of different curvature and field strength within the plasma. The radial (or flux surface coordinate) derivative of rotational transform is called shear:

\[
\iota' = \frac{d\iota}{d\psi_t}.
\]  

(1.5)

\(^2\)Note that, due to typographical issues, \(\iota\) is presented as \(\iota\) in some figures and figure captions. \(\iota\) as defined in equation 1.4 is not used in this thesis, and instances of \(\iota\) should be interpreted as \(\iota\).
Shear is important in the stabilisation of instabilities triggered by radial perturbations, such as pressure driven and drift modes.

1.3.3.1 Rational Surfaces and Magnetic Islands

For magnetic surfaces with irrational $\iota$, the field line ergodically covers the surface such that it can be defined by a single integral $\int dl/B$. On a rational surface where $\iota = n/m$ a field line will map onto itself after $m$ toroidal orbits, where $n$ and $m$ are integers. A condition for plasma equilibrium is that $\int dl/B$ is constant for a surface with constant pressure. In general, for configurations with finite shear this condition is not met which results in a slight modification of confinement due to flattening of the pressure profile near the surface [9].

When a perturbation to the magnetic field exists, resonance with the main field geometry can cause magnetic islands which contain isolated magnetic surfaces that do not encompass the main magnetic axis. The field due to toroidicity, which arises when linear geometry is curved into a torus, can be considered such a perturbation, ensuring that islands can be present in any three-dimensional toroidal geometry. Other perturbation fields can arise from coil error fields, or currents in the plasma. While stellarators generally have no net toroidal current, there are currents which lie on a flux surface required to satisfy the MHD equilibrium condition $\mathbf{J} \times \mathbf{B} = \nabla P$ (see section 1.3.5.2). In the limit of small island width, the width of an island is estimated by [10]:

$$\Delta = \left( \frac{|B_{nm}|}{|e'|m} \right)^{1/2}$$

(1.6)

where $B_{nm}$ is the resonant field with toroidal and poloidal harmonics $n$ and $m$ respectively.
1.3.4 The Kinetic and Fluid Models of Plasma

The two models most widely used for describing plasma are kinetic and fluid theory. Both models are based on a distribution function description of particle dynamics in phase-space. The main distinction is that the fluid model involves an integration over velocity space, resulting in quantities, such as plasma density flow velocity, which are more easily obtainable in an experimental situation.

The starting point of both models are the equations of motion: \( \dot{x}_s = v_s \), \( \dot{v}_s = F/m_s \) where \( F \) is the Lorentz force \( F = q_s(E + v_s \times B) \) and \( s \) denotes the particle species. The gravitational force is assumed to be negligible. Given a large number of particles in the system, it is more convenient to use a smooth distribution function description in phase space, \( f = f(t, x, v) \). Conservation of phase-space gives the kinetic equation:

\[
\frac{\partial f_s}{\partial t} + v \cdot \nabla f_s + \frac{q_s}{m_s}(E + v \times B) \cdot \frac{\partial f_s}{\partial v} = C_s(f) + I_s, \quad (1.7)
\]

where \( C_s(f) \) is the collision operator, and \( I_s \) is a source term to account for ionisation, fusion reactions, etc. In general, the collision operator is rather complicated as it describes the many body, long range (Debye shielded) Coulomb interactions, and approximates the physics neglected by assuming a smooth distribution function rather than the exact distribution function (an intractable summation of \( \delta \)-functions in phase-space).

The Vlasov equation is obtained by setting the right side of equation 1.7 to zero, neglecting collisions and particle sources/sinks. The Vlasov description of a plasma introduces an important collisionless damping mechanism unique to the plasma state known as Landau damping, as shown in figure 1.2 for the case of electron Landau damping in a cold ion model. Conversely, in the case of a population of fast electrons with velocity \( v_{\text{beam}} \) (\( v_{\text{phase}} \ll v_{\text{beam}} \)) we have
Figure 1.2: The process of electron Landau damping: assume $v_{\text{phase}} \gg v_{T_e}$ such that the ions are effectively a stationary background to an electrostatic wave in an electron gas, and at time $t = 0$ the electrons are in thermal, Boltzmann equilibrium in the wave potential so that $n = n_0 \exp(-q\phi/k_B T)$. The net effect of electrons with $v > v_{\text{phase}}$ is to move into region B, where they decelerate and give up energy to the wave – the opposite applies for electrons with $v < v_{\text{phase}}$, they move into region A and take energy from the wave as they accelerate. Because $v_{\text{phase}} > v_{T_e}$, $\frac{df}{dv} |_{v=v_{\text{phase}}} < 0$ and so there are more electrons slower than the phase velocity than there are faster, giving a net damping effect.

\[
\frac{df}{dv} |_{v=v_{\text{phase}}} > 0
\]
giving rise to a growth of the wave, known as the *beam-plasma instability*.

The fluid equations are generated by taking the velocity moments of the kinetic equation, where the $k^{\text{th}}$ velocity moment is $M_k = \int d^3v v^k f(x, v, t)$. The infinite set of velocity moments contains the same information as the kinetic equation. The benefit of taking velocity moments is the conversion of a particle distribution function in 6 dimensional phase-space to measurable quantities (such as density, flow velocity and pressure) in 3 dimensional space. Clearly, an infinite set of moment equations is not a useful substitute for the kinetic equation and approximations must be made to truncate the series of moments. This raises a point about closure, the process of finding a set of equations that, given initial and boundary conditions, will describe the time evolution of the plasma. The
fluid closure relies both on the truncation of the series of moments and asymptotic assumptions about scale lengths of plasma properties. It should be noted that some physical processes, such as Landau damping, require a kinetic approach as they cannot be described by any finite subset of moment equations.

The first three moments are shown below, with the first and second moments including a mass factor. The zeroth moment is:

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{V}_s) = I_{s0},$$  \hspace{1cm} (1.8)

the first moment ($\mathbf{v} \rightarrow m\mathbf{v}$) is:

$$m_s \frac{\partial n_s \mathbf{V}_s}{\partial t} + \nabla \cdot \mathbf{P}_s - e_s n_s (\mathbf{E} + \mathbf{V}_s \times \mathbf{B}) = \mathbf{F}_s + mI_{s1},$$ \hspace{1cm} (1.9)

and the second moment ($\mathbf{v}\mathbf{v} \rightarrow \frac{1}{2}m\mathbf{v}^2$) is:

$$\frac{\partial}{\partial t} \left( \frac{3}{2}p_s + \frac{1}{2}m_s n_s V_s^2 \right) + \nabla \cdot \mathbf{Q}_s - e_s n_s \mathbf{E} \cdot \mathbf{V}_s = W_s + \mathbf{V}_s \cdot \mathbf{F}_s + \frac{1}{2}mI_{s2}. \hspace{1cm} (1.10)$$

Several of the quantities in equations 1.8–1.10 are moments of the distribution function: the particle density $n_s(\mathbf{x}, t) = \int d^3vf_s(\mathbf{x}, \mathbf{v}, t)$, plasma flow velocity $\mathbf{V}_s(\mathbf{x}, t) = n_s^{-1} \int d^3vf_s(\mathbf{x}, \mathbf{v}, t)\mathbf{v}$, stress tensor $\mathbf{P}_s(\mathbf{x}, t) = \int d^3f_s(\mathbf{x}, \mathbf{v}, t)m_s\mathbf{vv}$, and energy flux density $\mathbf{Q}_s(\mathbf{x}, t) = \int d^3f_s(\mathbf{x}, \mathbf{v}, t)\frac{1}{2}m_s\mathbf{v}^2\mathbf{v}$. The first ($m\mathbf{v}$) and second ($\frac{1}{2}m\mathbf{v}^2$) moments of the collision operator are $\mathbf{F}_s$ and $W_s$ respectively, while the $k^{th}$ moment ($\mathbf{v}^k$) of the source term is given by $I_{sk}$.

The $k^{th}$ moment equation is coupled to the $k \pm 1$ equations, so any truncation approach to closure must include an assumption of higher order moments. Exact fluid closures are possible by setting pressure and viscosity to zero, as in the cold plasma model which describes phenomena with $v \gg v_T$, or by imposing an exact Maxwellian local distribution function, reducing the information of $f_s$ to $n_s, \mathbf{V}_s,$ and $T_s$ which can be fully described by three moment equations. The latter approach, combined with some asymptotic assumptions, is the one taken by ideal MHD.
1.3.5 Magnetohydrodynamics

Magnetohydrodynamic (MHD) models are ubiquitous in plasma physics. For plasma confinement, ideal MHD equilibrium is a primary requirement for a useful magnetic geometry. The analysis of linear stability of such equilibria is also a vital application of MHD.

A general definition of MHD is the asymptotic expansion of the fluid equations (velocity moments of the kinetic equation) about the small parameter \( \delta \equiv r_{cs}/L \ll 1 \) for all species \( s \), where \( r_c \) is the gyroradius and \( L \) is the scale length of the system. Drift motion across the field is assumed to scale as \( \delta \) and is therefore neglected, with the exception of \( \mathbf{E} \times \mathbf{B} \) drift. Maxwell’s equations and Ohm’s Law augment the information about the system, but are not sufficient to close the system of equations. The closure is done through some further assumption about the physics of the plasma – in the case of ideal MHD, that the plasma distribution function is Maxwellian and therefore the pressure tensor is contracted to a scalar. This assumption of isotropic pressure seems ridiculous for a strongly magnetised plasma, but ideal MHD has proven to be useful in regimes where one would expect it to be invalid.

1.3.5.1 Ideal MHD

A simplified description of a MHD plasma is that of a single-fluid, rather than maintaining separate fluid equations for each species. Here we use a centre of mass frame of reference, although in practise the assumption \( m_e = 0 \) in generally made. The change to centre of mass frame involves:

\[
\rho = \sum_s m_s n_s, \quad \mathbf{V} = \rho^{-1} \sum_s m_s n_s \mathbf{V}_s, \quad p = I p + \pi = \sum_s \int d^3v f_s m_s \mathbf{w} \mathbf{w},
\]

(1.11)
and \( \mathbf{J} = \sum_s q_s n_s \mathbf{V}_s \), where \( \mathbf{w} = \mathbf{v} - \mathbf{V} \), \( p \) is the scalar, isotropic pressure, and \( \pi \) is the anisotropic stress tensor.\(^3\) The assumption of isotropic pressure implies that \( \pi = 0 \).

The single fluid ideal MHD equations are:

\[
\begin{align*}
\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{V} &= 0, \\
\rho \frac{d\mathbf{V}}{dt} + \nabla p - \mathbf{J} \times \mathbf{B} &= 0, \\
\frac{dp}{dt} + \gamma_s p \nabla \cdot \mathbf{V} &= 0, \\
\mathbf{E} + \mathbf{V} \times \mathbf{B} &= 0, \\
\nabla \times \mathbf{B} &= \frac{\theta}{\mu_0} \mathbf{J}, \\
\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= 0,
\end{align*}
\]

where \( d/dt \equiv \partial/\partial t + \mathbf{V} \cdot \nabla \) is the convective derivative and \( \gamma_s \) is the ratio of specific heats. Equations 1.12 to 1.14 are the fluid equations, equation 1.15 is the MHD Ohm’s Law, equation 1.16 is Ampere’s Law and equation 1.17 is Faraday’s Law.

The assumption of no particle sources is made in ideal MHD, removing the source terms from the fluid equations. The collision terms also disappear from the right side of the single-fluid equations as they include a summation over species; conservation of momentum implies \( \sum_s \mathbf{F}_s = 0 \) and conservation of energy requires \( \sum_s (W_s + \mathbf{V}_s \cdot \mathbf{F}_s) = 0 \). In equation 1.13, the \( \mathbf{E} \) term is neglected because we are considering dynamics of a fluid element which is quasi-neutral, so it won’t feel any electric force. The general form of equation 1.15 is derived from the electron equation of motion: \( \mathbf{E} + \mathbf{V} \times \mathbf{B} + (\nabla p_e - \mathbf{J} \times \mathbf{B})/en = \eta \mathbf{J} \), where the terms in parentheses cancel in the small gyroradius limit, and the resistivity \( \eta \sim 0 \) under

\( ^3 \)Note that \( \pi \) is written here in the standard bold font notation for tensors, however in this font it is barely distinguishable from the unboldened \( \pi \).
suitable conditions which are discussed below. Ampere’s Law doesn’t include the
displacement current ($\partial \mathbf{E} / \partial t$) term because MHD activity is assumed to slow
compared to the speed of light. The remaining two Maxwell’s equations are not
included as they don’t provide any new information about the system given the
assumptions already made.

A brief overview of the neglected physics of ideal MHD is called for. The basic
assumption of quasi-neutrality places initial limits on the plasma dimensions:
$\tau \gg 1 / \omega_p$ and $L \gg \lambda_D$. The small gyroradius assumption removes drift motion
across the field, apart from $\mathbf{E} \times \mathbf{B}$ drift. This, along with the assumption of a
perfectly conducting fluid, are responsible for the frozen-flux effect, where the
magnetic flux through any surface that moves with the fluid cannot change. The
condition for $\eta$ to be negligible is $(m_e / m_i)^{1/2}(\nu_{ii} / \omega)(r_{ci} / L) \ll 1$, which shows
the ion collision frequency $\nu_{ii}$ must not be too high. On the other hand, the
collision frequency must be sufficient for a local Maxwellian distribution to be
valid, allowing pressure to be described by a scalar and omission of viscosity,
heat flow and other higher moment properties.

1.3.5.2 MHD Equilibrium

A system is in a state of equilibrium when there is a complete balance of forces.
Accessible MHD equilibrium is a requirement for all practical plasma confinement
devices; the stability properties of such MHD equilibria are considered in the
chapter 2. The MHD equilibrium equations are derived from the standard form
with $d/dt, \mathbf{V} \to 0$:

\[
\nabla p = \mathbf{J} \times \mathbf{B} \tag{1.18}
\]

\[
\mathbf{J} = \mu^{-1} \nabla \times \mathbf{B} \tag{1.19}
\]

\[
\nabla \cdot \mathbf{B} = 0. \tag{1.20}
\]

Equation 1.18 shows that, for a finite pressure gradient, there must be a non-zero perpendicular current density $\mathbf{J}_\perp$ which lies on the flux surface, where $\mathbf{J} = \mathbf{J}_\perp + \mathbf{J}_\parallel$. To maintain a divergence-free current density, $\nabla \cdot \mathbf{J} = 0$, there will also be a finite $\mathbf{J}_\parallel$ which is known as the Pfirsch-Schlüter current.

For an axisymmetric torus, the Grad-Shafranov equation can be used to find an equilibrium state. A general approach to finding MHD equilibrium in three-dimensional plasma is based on the variational principle. The variational principle involves reducing the plasma potential energy $W = \int \left( B^2/2\mu_0 + p/\left(\gamma s - 1\right) \right) dV$ to a stationary point $\partial W = 0$. The variation of the potential energy has the form

\[
\delta W = \int \mathbf{F} \cdot \xi dV,
\]

where $\mathbf{F} = \mu_0^{-1} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p$ and $\xi(x, t)$ is a perturbation vector. Widely used three-dimensional MHD equilibrium codes such as VMEC and BETA use this approach.

### 1.3.5.3 Other Fluid Models

Various other approaches to a fluid description of plasma are taken to overcome inadequacies of ideal MHD. Two of the most important are resistive MHD and the drift model. Finite resistivity is generally more crucial than other energy dissipation mechanisms, such as viscosity, because it allows modification of magnetic topology, introducing a new class of instabilities even for very small values of $\eta$. The approach to resistive MHD is to reintroduce the $\eta \mathbf{J}$ term on the right side of Ohm’s law, breaking the frozen-flux effect and allowing magnetic diffusion.
and reconnection.

The drift model introduces finite Larmor radius (FLR) effects to the fluid description. This approach is used to study a plasma evolving sufficiently slowly that effects neglected by MHD are important. The most prominent effect introduced is diamagnetic drift \( \mathbf{V}_d \equiv (qBn)^{-1}\mathbf{b} \times \nabla p \), where \( \mathbf{b} = \mathbf{B}/B \), which is assumed to be of the same order \( O(\delta) \) as \( \mathbf{E} \times \mathbf{B} \) drift. Scalar pressure cannot account for the gyratory motion considered in the drift model, so off-diagonal elements of the pressure tensor, \( \pi \), need to be included. The investigation of drift waves (ion acoustic waves propagating in an inhomogeneous plasma) as an explanation of anomalous transport is a common utilisation of this theory.

1.4 Data Mining

Data Mining is the process of extracting useful information from large databases. It is a recent field in which research into, and application of, new techniques has increased dramatically with the rise of the information age; although the field is generally considered a confluence of several longer standing fields of research including information technology, statistics and machine learning. Data mining techniques are becoming ubiquitous in business applications and have gained considerable attention when used by government intelligence agencies due to privacy concerns. Within scientific fields, perhaps the best known application is in bio-informatics research where data mining techniques are used to discover useful information in genetic code. Other applications are used in astronomy, geosciences and other fields with large datasets. Ever since computers have allowed the automated collection and storage of data, fusion researchers have collected as much information as possible about the plasmas they study. Much of this meta-dataset still exists in some form, and there is a possibility that applying
data mining techniques to this dataset may uncover useful information. In this thesis, data mining techniques are used specifically on data from the H-1 heliac device.

In general, the data mining procedure consists of data preprocessing, mining, and visualisation and interpretation. Preprocessing is the stage where existing data, possibly in several formats, is gathered, filtered and sampled to maximise the effectiveness of the main data mining algorithm. This may include converting between nominal and numeric data types, reducing the dimensionality of the data, taking a random sub-sample of the dataset, or filtering out data known to be noise. The method of data visualisation is specific to the problem at hand, but generally involves mapping the data to a reduced number of dimensions.

The main algorithm generally falls into three categories: association, classification, and cluster analysis [11]. Association rule algorithms are used to find non-trivial correlations between different parameters within the dataset. Classification algorithms use a training dataset to generate classification rules, which can then be applied to unseen data. Decision trees, Bayesian belief networks and neural networks are commonly used for this purpose. Classification is generally associated with discrete data; for continuous data types, regression analysis can be used. Cluster analysis is used to find similar types of data together by defining a metric in the multi-dimensional space of the dataset and grouping data points which are close to each other.

1.4.1 Software

Open source software has been utilised in most aspects of the work presented here. Digitised data from H-1 is stored in an MDSPlus database [12], while all other data, i.e.: from HELIAC code runs and data mining procedures, is
stored in MySQL databases [13]. The expectation maximisation (EM) clustering algorithm from the WEKA suite of data mining tools is used [14], for all other computations the Scipy [15] scientific libraries for Python [16] were used. Nearly all of the original figures are made with the Matplotlib libraries [17] for Python, with a few plots being made with the GRI language [18].

1.5 Aim and Scope of Thesis

The aim of the work presented in this thesis is to explore and understand the range of observable magneto-hydrodynamic fluctuation spectra in the H-1 heliac. Shortly prior to the start of my PhD candidature, an upgrade to the heating system on H-1 allowed access to more fusion relevant plasmas. This increase in performance, combined with the unique configurational flexibility of H-1, called for an investigation of MHD activity in the newly accessible regimes.

While the range of plasma configurations available in H-1 is quite broad, the scope of this thesis is limited to the study of the relation between MHD activity and rotational transform profile. The fine control of rotational transform in H-1 allows investigation of the dispersion relation of several classes of Alfvén mode, including the global Alfvén eigenmode, which can play an important role in energy retention in reactor scale fusion devices. In a sense, this work is exploratory: while MHD activity associated with low order resonances in rotational transform is expected, the scope of this thesis also extends to the analysis of any unexpected observations of MHD activity. Besides the construction and installation of Mirnov diagnostics in order to acquire the dataset, this work extends to the development of any computational or theoretical methods required for analysis of the experimental observations.
The remainder of this thesis is set out as follows. Chapter 2 presents an overview of the H-1 heliac; discussing the motivation for the heliac design as well as describing the H-1 experimental setup specifically, including magnetic configurations and diagnostics. A background of the MHD theory relevant to H-1 low $\beta$ plasmas is also provided. Chapter 3 lays out the data mining framework developed for this project. The preprocessing, clustering and visualisation stages are presented in a general way which does not limit the application to the dataset presented here. The separate parts of the data mining procedure are brought together into a simple test case at the end of the chapter.

Chapter 4 describes the application of the data mining technique introduced in chapter 3 to a specific H-1 dataset. Following a summary of the experimental procedure and the acquired dataset, the data mining method is used to extract different MHD modes from the Mirnov signals. Multiple clustering algorithms are shown quantitatively to produce very similar results. An analysis of the structure of the observed modes is also provided.

Chapter 5 gives an analysis of the MHD activity identified in chapter 4. After general observations of the dataset, the focus is shifted to the modes which are resonant about low order rational surfaces. Interchange and Alfvén modes are discussed as candidate models for the observed fluctuations, with neither providing an entirely satisfactory explanation. Other candidates for these modes are then discussed, along with consideration of higher frequency modes and possible instability driving mechanisms. Finally, a summary of the thesis and conclusions appear in chapter 6.
CHAPTER 2

The H-1 Heliac

2.1 The Heliac Concept

The design of the heliac resulted from the search for plasma confinement devices with improved stability properties. Since the earliest days of fusion research, it has been clear that the avoidance of MHD instabilities is an essential requirement for achieving effective magnetic confinement. There are two sources of instability in the MHD model: currents in the plasma and the plasma pressure gradient. The stellarator design benefits from having no requirement for a toroidal current, unlike the tokamak which utilises such a toroidal current to generate poloidal magnetic field required for confinement. In stellarators it is the instabilities due to the plasma pressure gradient, notably the interchange mode, which have the largest effect.

In the ground-breaking 1958 publication which detailed the discoveries of the previously classified Project Matterhorn [19], Lyman Spitzer, Jr. suggests that shear in rotational transform is sufficient to stabilise interchange modes if the field-normalised plasma pressure \( \beta = 2\mu_0 P / B^2 \) is less that some critical value \( \beta_c \), where \( P \) is the plasma pressure and \( B^2 / 2\mu_0 \) is the magnetic pressure. This stabilisation is due to the energy required by a radial perturbation to bend magnetic field lines which, in the presence of shear, are topologically different at different radii. In the ideal MHD model, which assumes infinite conductivity and
plasma to be ‘frozen’ to the magnetic flux, such a perturbation is not possible; however by 1963 [20] it was clear that the ideal MHD model is insufficient to properly describe the plasma. If resistivity $\eta$ is introduced into the model then instabilities with growth rate $\sim \eta^{1/3}$ can exist even in the presence of shear.

Another method for stabilising against interchange modes is the average magnetic well. A magnetic well describes the condition:

$$V''(\psi) < 0$$  \hspace{1cm} (2.1)

where $V(\psi)$ is the volume enclosed by the surface containing magnetic flux $\psi$ and the prime represents a derivative with respect to the argument. This represents a smaller magnetic flux per unit volume at the inner magnetic surfaces than at the outer surfaces. In a toroidal geometry, it is not possible to have a magnetic well (outward radially increasing magnetic field) everywhere; therefore it is desirable to have an average magnetic well configuration, where the average is taken along a flux surface field line. The magnetic well is related to the average curvature of the magnetic line of force, where we require the average of the curvature $\kappa = b \cdot \nabla b$ on a flux surface to have the same sign as the pressure gradient in order to obtain stability against interchange modes. The average magnetic well ensures that a radial perturbation with long toroidal wavelength will be influenced more by the regions of favourable rather than unfavourable magnetic curvature.

A standard, approximately circular stellarator geometry relies on toroidal curvature to generate magnetic well in vacuum [21]. However, a helical magnetic axis can produce a magnetic well even in the limit of large aspect ratio. The heliac configuration is produced by arranging planar toroidal field (TF) coils with helical displacement around a central conductor, as shown in figure 2.1. The central conductor, or poloidal field (PF) coil, significantly increases the rotational transform which decreases the distance between regions of favourable and un-
favourable curvature [22], reducing the upper limit on the toroidal wavelength of modes localised in unfavourable curvature regions.

In 1985, a modification to the basic heliac design was proposed [23]. The addition of an $l = 1$ helical winding around the main PF coil allows for a wide range of rotational transform and shear. This additional flexibility allows for exploration of a wide range of configurations, and can be used to avoid configurations with destructive low-order rational surfaces (see section 1.3.3.1).

The first operating heliac device was SHEILA (Canberra, Australia, 1984) [24], followed by TU-heliac (Tohoku, Japan, 1987) [25], H-1 (Canberra, Australia, 1992) [26] and TJ-II (Madrid, Spain, 1996) [27]. The TU-heliac ($R_0 : \bar{a} = 0.48 : 0.07 \text{m}$) is significantly larger than the SHEILA heliac ($0.2 : 0.03 \text{m}$). The H-1 heliac ($1.0 : 0.22 \text{m}$) was the first of the large flexible heliacs, followed by TJ-II
Aside from its $\beta = 0$ magnetic well stabilisation and highly configurable magnetic geometries, the heliac has the advantage of high ideal MHD stability limits (large $\beta_c$). The stability thresholds were found to be dependent on the shape of the plasma cross-section, with the distinctively bean-shaped cross section providing better stability than more circular cross sections. The effect of the plasma shape is due to its connection with the strength of the $\beta = 0$ magnetic well [21], especially via the favourable curvature of the indentation of the ‘bean-shape’.

\section*{2.2 MHD Activity in a Low $\beta$ Heliac Plasma}

In section 1.3.5.2 we discussed the equilibrium state of ideal MHD and noted the effects of resistivity and finite Larmor radius, both of which are excluded from the ideal MHD model. In this section we give an overview of the stability properties of a heliac plasma using the MHD model. The plasmas in H-1 are considered to be “low-$\beta$” as the plasma pressure has negligible effect on the magnetic surfaces. Analysis of experiments in other fusion laboratories with significantly greater heating power than is available to H-1 must take into account the effects of finite $\beta$ on the plasma equilibrium.

In reality a plasma, like any other physical object, is not a continuum as described by MHD, but has sub-structure at the atomic level with thermal vibrations at finite temperature. Thus, even in an equilibrium state, the plasma is prone to perturbations which may or may not be destabilising. The linear growth rate $\omega$ of a fluid perturbation $\xi(x,t) = \xi(x)e^{-i\omega t}$ is a primary quantity of interest when considering the effects of fluctuations in a plasma; perturbations
with \(\text{Im}(\omega) > 0\) have exponential instability while \(\text{Im}(\omega) < 0\) corresponds to exponential stability and \(\text{Im}(\omega) = 0\) is the case of marginal stability.\(^1\) The growth rate can be found by solving the linearised momentum balance equation as an eigenvalue problem.

When considering the overall MHD stability of a given magnetic configuration, little attention is generally given to fluctuations which do not lead to instability; though they may be important when considering transport properties. Because fusion research is mission orientated, a higher priority is given to the avoidance of instabilities than the investigation of properties of stable waves. One exception is that of Alfvén eigenmodes, discussed in section 2.2.3, where wave–particle interactions have the potential to reduce the efficiency of a burning plasma.

### 2.2.1 Ideal MHD Stability Limits

The conditions for the stability against interchange modes for a plasma in cylindrical coordinates can be expressed by the Suydam criterion, or, for toroidal coordinates, the Mercier criterion. Using the representation of reference [28], the ideal Mercier criterion predicts plasma stability when

\[
D_M = D_S + D_I + D_W + D_G > 0. \tag{2.2}
\]

The terms \(D_S\) and \(D_I\) scale as the square of the shear (radial gradient of rotational transform) and the net toroidal current respectively, and can be neglected for the low-shear heliac configuration. The remaining two terms describe the effect of magnetic well \(D_W\) and the total current averaged over a magnetic surface \(D_G\). The Mercier criterion can only indicate stability around rational surfaces,

\(^1\)Note that \(\omega\) and \(\gamma\) are used interchangeably for growth rate and frequency throughout the thesis. In chapters 3 and 4, \(\gamma\) is used mostly as the cross-correlation, while \(\gamma_s\) refers to the ratio of specific heats.
and it is correlated to the beta limit due to low-\( n \) interchange modes [29, 30]. The ideal interchange stability limit in the H-1 heliac was found [28] to be \( \langle \beta \rangle \sim 1\% \), varying from 0.6\% to 1.9\% depending on magnetic configuration and method of calculation. A study of \( \beta \) stability limits in the TJ-II heliac showed good agreement with the H-1 results [31]. It is not necessary for a rational surface to be present for an interchange mode to be destabilised [30, 32], in chapter 5 we consider a non-resonant interchange model as an explanation of observed MHD activity in H-1.

It is generally regarded that short-wavelength, pressure-driven ballooning modes are a likely limit to the obtainable \( \beta \) in toroidal fusion devices [33]. The high toroidal mode numbers \( n \) associated with the ballooning mode allow it to be localised in regions of unfavourable curvature, negating the stabilising properties of an average magnetic well. A second region of stability, at pressures above the onset of the ballooning mode, has been found for tokamak devices where the mode is stabilised by an increase in local shear, caused by sufficiently high plasma pressure gradients [34]. In the stellarator, however, the existence of a second stability regime is not clear cut; it has been shown that some devices allow access to second stability while others do not [35].

Although such numerical solutions are widely used in the study of stellarator configurations, even the existence of a three-dimensional toroidal equilibrium remains an unanswered mathematical question [36]. It may be that the model of a smooth pressure profile across the plasma volume is inaccurate, and that a treatment which includes the flattening of pressure gradient across rational surfaces is required.
2.2.2 Resistive MHD Stability Limits

By adding resistivity $\eta$ to the MHD model, the magnetic field has a finite coefficient of diffusion through the plasma, i.e.: the plasma is not ‘frozen’ to magnetic flux as it is within ideal MHD. This means that the stabilisation against interchange modes given by magnetic shear is no longer effective, as the perturbed plasma can interchange with a different topology of a neighbouring surface. The resistive effect is localised to a region near the resonant surface which depends on the relative resistive diffusion and Alfvén timescales [37, 38]. This skin depth $(r - r_s)$, where $r_s$ is the radius of the resonant surface, scales as

$$ (r - r_s) \sim \gamma_g \sim \eta^{1/3} \quad (2.3) $$

where $\gamma_g$ is the resistive growth rate.

Non-resonant pressure-driven resistive instabilities have also been found for stellarator configurations [39]. This mode is located near the magnetic axis and has ballooning structure, and is unstable in highly resistive plasmas ($S \leq 10^5$, where $S = \mu_0 L V / \eta$ is the magnetic Reynolds number and $L$ and $V$ are the characteristic length and flow velocity). As the resistivity is decreased, the mode undergoes a transition to a resistive interchange mode about a resonant surface.

2.2.3 Alfvén Eigenmodes

The Alfvén wave is a torsional wave which is analogous to the vibrations of a taut string. In ideal MHD, a magnetic flux tube with tension $B^2/2\mu_0$ will have an associated fluid mass density $\rho$ ‘frozen’ to it such that the string analogy gives a wave velocity of

$$ v_A^2 = \frac{B^2}{\mu_0 \rho}. \quad (2.4) $$
In a non-uniform magnetic field, there is damping of Alfvén waves within a continuum spectrum. However, certain Alfvén eigenmodes (AEs) can exist in toroidal geometries which can be excited by wave-particle resonance and reduce confinement by ejecting the resonant particles. Most importantly, this effect is expected for the $\alpha$-particle products of D-T fusion reactions.

If finite resistivity $\eta$ is added to the MHD model then damping of the wave is possible, with a damping distance of $z_0 = 2\mu_0 v_A^2 / \eta \omega^2$. Here, only waves with wavelength $\lambda \ll z_0$ can exist. Note that the ideal MHD case of $\eta \to 0$ gives $z_0 \to \infty$ so there is no upper limit on $\lambda$. If compressibility is considered then there are three modes of propagation: the fast and slow magnetosonic waves given by the two signs of the equation

$$\frac{\omega^2}{k^2} = \frac{1}{2}(v_s^2 + v_A^2) \pm \frac{1}{2}[(v_s^2 + v_A^2)^2 - 4v_s^2 v_A^2 \cos^2 \theta]^{1/2} \quad (2.5)$$

and the Alfvén wave given by

$$\frac{\omega^2}{k^2} = v_A^2 \cos^2 \theta, \quad (2.6)$$

where $v_s = (\gamma_s P_0/\rho_0)^{1/2}$ is the speed of sound and $\theta$ is the angle between the direction of propagation and the magnetic field. When $\theta = \pi/2$ only the fast mode exists, and is called the compressional Alfvén mode. The case of $\theta = 0$ gives Alfvén and sound waves.

### 2.2.3.1 The Alfvén Wave Continuum

The expressions for Alfvén waves shown above assume a homogeneous magnetic field. However, for toroidal plasmas we need to account for an inhomogeneous magnetic field as, for the simplest case, $B \propto R^{-1}$. Inhomogeneity in the magnetic field will clearly give local variation in the Alfvén velocity. As detailed in reference [40], an analysis of the dynamics of this system found that a singularity arises in
the wave equation at $\omega = k|v_A$ (often called the Alfvén resonance) and showed that there is no possibility of any dispersion relation. The equation and boundary conditions of this problem were found to be identical to those of the electrostatic plasma oscillation in a cold, inhomogeneous plasma, which has a singularity at the electron plasma frequency. This problem had previously been fully analysed and shown to produce continuum spectrum, a result which could be directly applied to the Alfvén problem.

Another feature of the electrostatic plasma oscillation which can be generalised to the Alfvén problem is the resonant absorption of externally driven oscillations, a process which used as a method of heating plasma. Although the ideal MHD equations do not contain any damping mechanism, if finite Larmor radius and finite electron mass effects are introduced, then the singularity disappears and a new form of electrostatic wave, the kinetic Alfvén wave, emerges with very short wavelength. The absorption process can then be described by the conversion of an externally driven Alfvén wave to a kinetic Alfvén wave, which then undergoes Landau or collisional damping [41].

2.2.3.2 Gaps in the Alfvén Wave Continuum

In toroidal geometry there can exist gaps in the Alfvén wave continuum. The periodic boundary conditions of a torus in the limit of large aspect ratio ($R/a \rightarrow \infty$) and rotational transform profile $\iota(r)$ give $k|| = |n - \iota m|/R$. The frequency $\omega(r) = k|v_A$ is a function of minor radius and has a global minimum at some location $0 \leq r \leq a$ within the plasma. Global Alfvén eigenmodes (GAE) can exist at frequencies slightly below this minimum, outside the continuum spectrum such that they avoid continuum damping, and can be excited via energetic particles. Alfvén eigenmodes have been observed in many experiments, usually through
matching Mirnov signals with frequencies expected from models incorporating $n$ and $B$ data [41].

The coupling of poloidal and toroidal field components within the plasma radius introduces gaps in the Alfvén wave continuum within which eigenmodes can exist. Following the notation of reference [42], we have, for the coupling between $n - mt = -(n + \delta_n N_{fp} - mt - \delta_m t)$, a gap location of:

$$\eta = \frac{2n + \delta_n N_{fp}}{2m + \delta_m}, \quad \omega = \left(\frac{v_A}{R}\right) \frac{\delta_m n - \delta_n m N_{fp}}{2m + \delta_m}. \quad (2.7)$$

Here, $\delta_{n(m)}$ denote integer mode displacements, and $N_{fp}$ is the number of field periods ($N_{fp} = 3$ in H-1). The different types of gap modes are listed in table 2.1.

The commonly understood mechanism for excitation of Alfvén eigenmodes is the resonance of energetic particles with velocity $v \sim v_A$ or a sideband resonance with $v \sim v_A/3$ [43]. However observations suggest that certain AEs can also be excited in the absence of fast particles via ion temperature gradient (ITG) turbulence [44, 45, 46, 47].

| Acronym | Name                           | $|\delta_m|$ | $|\delta_n|$ |
|---------|--------------------------------|-------------|-------------|
| GAE     | Global Alfvén eigenmode        | 0           | 0           |
| TAE     | Toroidal Alfvén eigenmode      | 1           | 0           |
| EAE     | Elliptical Alfvén eigenmode    | 2           | 0           |
| NAE     | Noncircular Alfvén eigenmode   | $\geq 3$    | 0           |
| MAE     | Mirror Alfvén eigenmode        | 0           | $\geq 1$    |
| HAE     | Helical Alfvén eigenmode       | $\geq 1$    | $\geq 1$    |

Table 2.1: Types of Alfvén gap modes.
2.2.4 The Beta-induced Alfvén Eigenmode

Beta-induced Alfvén eigenmodes (BAE) are eigenmodes which are situated in a low frequency gap in the Alfvén continuum which appears due to finite compressibility [48, 49]. A major effect of introducing acoustic coupling via finite compressibility is the strong mixing of the acoustic and Alfvén branches in the lower frequency range of the Alfvén continuum. Distinction between Alfvén and sound activity in this regime is somewhat artificial, a useful classification is the polarisation: $|\xi_\perp| > |\xi_{||}|$ for Alfvénic and $|\xi_{||}| > |\xi_\perp|$ for acoustic [49].

The BAE was first observed at DIII-D in 1993, appearing in place of beam-driven TAEs as beta was increased [50]. The frequency of the modes $f/f_{TAE}$ was found to have a stronger correlation with normalised beta $\beta_N \equiv \beta_t a B_t / I_p$ than with other plasma parameters, where $\beta_t$ is the toroidal beta, $a$ is the minor radius, $B_t$ is the toroidal field and $I_p$ is the plasma current. A linear scaling with $v_A$ was also observed, suggesting an Alfvén wave.

Subsequent to the DIII-D results, instabilities similar to BAEs have been observed in the Tokamak Fusion Test Reactor (TFTR) and Joint European Torus (JET) [51, 52], though there remain questions concerning whether or not these observations have the nature indicated by the original theoretical work (i.e.: references [48] and [49]). A paper titled *What is the “beta-induced Alfvén eigenmode?”*, published in 1999 [52], compared the “BAE” observations to four models: an Alfvén eigenmode, a kinetic ballooning mode, a mode that propagates at the ion thermal speed and an energetic particle mode. It was concluded that none of these models is adequate to explain the experimental observations.

While we are concerned with low-beta plasmas in H-1, we will see in section 5.6 that observed modes in H-1 are reminiscent of the “BAEs” observed elsewhere. As with other Alfvén eigenmodes, helical Alfvén eigenmodes (HAEs) have been
observed in the absence of a fast particle driving source [53], and have recently been seen in relatively low-\(\beta\) plasmas in JET [54].

### 2.2.5 The CAS3D Code

The nature of computational physics is that numerical codes, written for general or niche problems, evolve as processing power increases and numerical methods improve, in parallel with experimental and theoretical developments. The Code for the Analysis of the Stability of three-dimensional (3D) equilibria (CAS3D) [55] developed in the early 1990s by C. Nührenberg née Schwab uses the ideal MHD energy principle to find stability properties of fully three-dimensional geometries, rather than using the stellarator expansion [56] or other approximations.

The original version of CAS3D used a substitute matrix for the kinetic energy, sufficient to find the stability properties of a perturbation, in order to reduce computational complexity at the expense of physical values such as growth rates. The plasma was also taken to be incompressible; this does not modify the marginal stability points as the compressibility term in the plasma potential energy is stabilising. The code was improved in 1999 to include compressibility and the correct kinetic energy matrix [57] allowing the calculation of growth rates as well as compressional modes, e.g.: sound waves. This also confirmed that an incompressible model is sufficient to decide the question of stability or instability of a mode.

The CAS3D code has been applied various stellarator devices, notably the Wendelstein experiments in the vicinity of its author [57, 58, 59, 46]. An analysis of a high-\(\beta\) W7-AS configuration (shot #41618) in [59] shows the computed sound and Alfvén spectra, including gap modes, as well as higher frequency magnetosonic spectra. In [46] a TAE radial eigenfunction computed by CAS3D shows
very good agreement with experimental results. A recently initiated collaboration between the H-1 team and the CAS3D author has begun to yield full 3D ideal MHD spectra for H-1 plasmas, which are investigated in section 5.6.

2.3 The H-1 Heliac

The H-1 National Facility (H-1NF) heliac is a three-field-period helical axis stellarator with major radius $R = 1.0$ m and average minor radius $\langle r_a \rangle \approx 0.2$ m. The magnetic configuration is a flexible heliac [23, 26], in which toroidal field (TF) coils are helically displaced around a hard-core assembly comprising an axially symmetric circular (poloidal field, PF) coil and a helical control (HC) winding wrapped around it. An external vertical field (VF) is used to adjust magnetic well and major radius of the magnetic axis.

The work in this thesis is done with a base magnetic field of $B_0 = 0.46$ T, which is often referred to as 1/2 T operation. At these fields, the programmable control system allows for repetition rates of around 30 shots per hour, limited by data acquisition time and magnet cooling time. Plasma discharges (shots) of 60 ms duration are produced using 50 – 60 kW of 7 MHz ion cyclotron resonance frequency (ICRF) heating in a H:He = 3:2 mixture. The ICRF antennas are conformal helical picture-frame coils located 3 – 4 cm outside the last closed flux surface [60].

2.3.1 Plasma Regime for 1/2 T H-1 Operations

For the basic assumption of a quasi-neutral plasma which can exhibit collective behaviour we require (see section 1.3.1) $\tau \gg 1/\omega_p$ and $L \gg \lambda_D$, where $\tau$ and $L$ are the time and length scales of the experiment, $\omega_p$ is the plasma frequency and
\( \lambda_D \) is the Debye length. In the present experiment we have electron temperature \( T_e \sim 10 - 20 \text{ eV} \) as measured by helium line ratio spectroscopy [61] and electron density \( n_e \sim 1 - 2 \times 10^{18} \text{ m}^{-3} \) measured by a 2 mm Michelson interferometer (section 2.3.3.1) giving \( \lambda_D \sim (1 - 5) \times 10^{-5} \text{ m} \), much smaller than the dimensions of the experiment \( L \sim 2 \langle r_a \rangle \sim 0.4 \text{ m} \). Of the dominant species present, singularly ionised helium gives the lowest plasma frequency \( f_{p,He} = \omega_{p,He} / 2\pi \sim 150 \text{ MHz} \); this is much greater than the frequencies we are observing here, which are below the digitisation Nyquist frequency \( f_N = 500 \text{ kHz} \), fitting the MHD requirement \( f \ll f_p \).

In the MHD regime, we require \( r_{c,s}/L \ll 1 \) for all species \( s \). For a plasma of singularly ionised particles in thermal equilibrium \( r_{cs} \propto \sqrt{m_s} \), therefore \( r_{c,He} \) is the largest in our H:He plasmas. We find \( r_{c,He}/L \sim 0.01 \), so the plasma is strongly magnetised and the MHD model is applicable. In the ideal MHD model plasma is effectively tied to magnetic field lines, so we expect to observe fluctuations in the plasma which correlate to the magnetic fluctuations observed by the Mirnov coils. This is the case with fluctuations in electron density observed with the electronically scanned interferometer as shown for the main spectral features of our configuration scan in figure 2.2.

### 2.3.2 Configurations of H-1

The primary and secondary power supplies of H-1 can be independently controlled to a resolution of 1 A. For the experiments presented in this thesis, the H-1 heliac was set up for variable \( \kappa_h \) operation. This configuration has the primary power supply connected in series to the toroidal, poloidal and vertical field coils, with the secondary power supply connected to the helical winding coil. This allows for a resolution in rotational transform of 1 part in 1000. The parameter \( \kappa_h \) is used
for experimental convenience, it is defined as $\kappa_h \equiv I_h / I_0$, where $I_h$ is the current in the helical winding and $I_0$ is the current in the other coils. A lower (non-zero) limit to the secondary current prohibits $1/2$ T operation at configurations in the range of $0 < \kappa_h < 0.16$. The upper limit of $\kappa_h = 1.07$ is set by the present operational limit of the helical winding current.

2.3.2.1 The HELIAC Code

The HELIAC code [62] was written in the early 1980s by A. Ehrhardt at the Princeton Plasma Physics Laboratory. The code models the vacuum magnetic field using the Biot-Savart law using coil filaments provided in an input file, and computed flux surface properties such as $\iota$ and magnetic well. The HELIAC code has been used for the computation of vacuum field properties throughout this thesis.

Concurrent work by T. A. Santhosh Kumar [63] on the mapping of error fields in H-1 using electron beam wire grid tomography has produced vacuum field models for the HELIAC code with a very high degree of accuracy. The three
Figure 2.3: Poincaré plots and \( z \) profiles for configurations in the \( \kappa_h \) scan.

models described in reference [63] will be referred to here as \( HM1 \), \( HM2 \) and \( HM3 \) for the basic, new and high field HELIAC models respectively. The HM1 basic model represents each of the 36 TF coils by a single circular filament and the PF coil by 36 circular filaments, with 6 vertically stacked layers of 6 concentric windings. Each of the vertical field coils is also modelled by a single circular filament while the helical winding is represented by piecewise linear filaments, one for each of the four windings.

The HM2 model matches more accurately the H-1 conditions by including error fields, coil offsets and cross-overs. Both HM1 and HM2 have been designed to match low-field conditions; for the \( B_0 \sim 0.5 \) T conditions used here a helical perturbation to the PF current was found to be necessary, the HM3 model in-
Rotational transform profiles and Poincaré plots produced with the HELIAC code for the range of $\kappa_h$ configurations used in this thesis are shown in figure 2.3. The Poincaré plots show where traced magnetic field lines puncture a given surface, here the surface in the poloidal plane at toroidal angle $\phi = 0$. We see for $\kappa_h \lesssim 0.4$ the $t$ profiles are monotonic, while for $\kappa_h \gtrsim 0.4$ H-1 has a reversed-shear geometry. As $\kappa_h$ is increased, the Poincaré plots show greater indentation with the field becoming more ‘bean-shaped’.

Figure 2.4 shows $\int dl/B$ and magnetic well $V''(\psi)$ produced by the HELIAC code for $0.2 \leq \kappa_h \leq 1$ plotted against normalised average minor radius $\langle \rho \rangle = \langle r \rangle / a$, where $a = 0.2$ m. We see that as $\kappa_h$ is increased the magnetic well ($V''(\psi) < 0$) deepens over most of the plasma radius, while a region with magnetic hill ($V''(\psi) > 0$) near the plasma edge is reduced in width from $0.7 \lesssim \langle \rho \rangle \leq 1$ at $\kappa_h = 0.2$ to $0.9 \lesssim \langle \rho \rangle \leq 1$ at $\kappa_h = 1$. There is some uncertainty in the boundary of the magnetic hill region because of a stray vertical field component [63]; the
sensitivity of the boundary location to the vertical field is stronger at low $\kappa_h$ where the magnetic well is shallower.

### 2.3.2.2 Parametrisation of $\iota$ Profiles

The parametrisation of field properties is useful for the analytic calculations in chapter 5. Using the HM3 $\iota$ profiles, we find a least-squares power series approximation of the form:

$$
\iota(x_1, x_2) = \sum_{j_1,j_2=0}^{N_j} a_{j_1,j_2} \prod_{k=1}^{2} (x_k - c_k)^{j_k},
$$

(2.8)

where $x_1 = \langle r \rangle$, $x_2 = \kappa_h$ and $N_j$ is the order of the power series. The coefficients of the $N_j = 4$ power series

$$
\begin{pmatrix}
  a_{0,0} & \cdots & a_{0,4} \\
  \vdots & \ddots & \vdots \\
  a_{4,0} & \cdots & a_{4,4}
\end{pmatrix}
$$

(2.9)

are:

$$
\begin{pmatrix}
  1.24 & 2.99 \times 10^{-1} & -4.18 \times 10^{-2} & 1.14 \times 10^{-2} & 1.37 \times 10^{-2} \\
  -6.44 \times 10^{-2} & 1.77 \times 10^{-1} & -5.68 \times 10^{-3} & 1.14 \times 10^{-1} & 9.81 \times 10^{-2} \\
  1.68 \times 10^{-1} & -4.11 \times 10^{-1} & 1.36 \times 10^{-3} & 2.39 \times 10^{-1} & 2.29 \times 10^{-1} \\
  -2.16 \times 10^{-1} & 1.62 \times 10^{-1} & 5.84 \times 10^{-2} & 1.88 \times 10^{-1} & -2.16 \times 10^{-1} \\
  1.27 \times 10^{-1} & -5.45 \times 10^{-3} & -3.21 \times 10^{-2} & -5.12 \times 10^{-2} & 7.17 \times 10^{-2}
\end{pmatrix}
$$

(2.10)

with $c_1 = -2.59 \times 10^{-1}$ and $c_2 = 3.48 \times 10^{-1}$. The power series expression is compared to the raw HELIAC output in figure 2.5.
2.3.3 H-1 Diagnostics

A wide variety of plasma diagnostics have been installed on H-1; most of which are, for various reason, rarely used. For 1/2 T operations, the ‘always on’ diagnostics are the 2 mm interferometer (electron density) and the Mirnov coil arrays (magnetic fluctuations). Often used diagnostics include the electronically scanned interferometer (ELSI) [64], the LARRY spectrum analyser and the residual gas analyser (RGA). In general, Langmuir and Mach probes are only used at lower field operations to avoid possible damage to the probes.

2.3.3.1 Density Interferometers

Line average electron density is measured for every shot using a Michelson interferometer. The interferometer source is a swept IMPATT diode with centre
frequency 141 GHz ($\lambda \sim 2$ mm). The system has a frequency response of 85 kHz and $n_e$ resolution of $5 \times 10^{16}$ m$^{-3}$. Figure 2.6 shows the line-averaged electron density as measured by the 2 mm interferometer for all H-1 shots with the same plasma species and heating power as the main dataset for this thesis (see section 4.2). We see regions of poor confinement in the vicinity of the $4/3$ and $5/4$ resonances which are highly reproducible.

The electronically scanned interferometer (ELSI) provides radial profiles of $n_e$. The ELSI utilizes a backward-wave oscillator (BWO) capable of sub-1 ms electronically controlled frequency sweep of the range 180 – 280 GHz. In the present setup, the plasma radius is swept at a frequency of $\sim 500$ Hz; however, indirect measurement of frequencies up to $f \sim 40$ kHz is possible through interpretation of spatial frequencies in the swept profile. All ELSI data in this thesis is provided
by David Oliver.

2.3.3.2 Mirnov Coils

For the work presented here, two poloidal arrays of Mirnov coils have been designed and installed, complementing an existing linear array of 5 coils. The two poloidal arrays are identical in design and are toroidally separated by one field period, located at $\phi = 44^\circ$ and $\phi = 284^\circ$. Each array has 20 coils, as shown in figure 2.7. The coils are 50 turns with diameter of 3.2 mm, electrostatically shielded and isolated from the vacuum region by a bean-shaped stainless steel tube of diameter 12.7 mm and wall thickness of 1.2 mm. The skin-depth frequency $f_\delta$ of the tubing is 130 kHz, and the digitisation Nyquist frequency $f_N$ is 500 kHz. The frequency response of the coils within the metal tubing is shown in appendix A.

A third set of Mirnov coils is a 5-coil linear array above the plasma at $\phi = 35^\circ$ which was installed before the commencement of the present work. Here the coils have same geometry but lie in a thinner stainless steel tubing with $f_\delta \sim 220 \text{ kHz}$. Figures 2.7(a) and 2.7(b) show the locations of the Mirnov coils in the poloidal arrays and computed flux surfaces for the $\kappa_h = 0$ and $\kappa_h = 1$ configurations respectively. The positions of the coils have been chosen so that they encompass the last closed flux surface (LCFS) for all accessible magnetic geometries. As a result, for most configurations there are a few coils in sub-optimal locations.

2.3.4 Calculation of Magnetic Angles

We use Poincaré plots generated by the HELIAC code to calculate the magnetic angles of the Mirnov coils for a given configuration. A HELIAC code generated Poincaré plot for a given toroidal angle $\phi$ is in Cartesian coordinates $(R, z)$. From the value of rotational transform $\iota$ for a given surface, we find the poloidal
magnetic angle for each puncture point. A $10^4$-point cubic spline interpolation of 250 puncture points is used to generate the surface. For each Mirnov coil, we find the point on the surface at minimum distance from the coil and take a linear interpolation, along the spline, between the nearest puncture points.
(a) The $\kappa_h = 0$ configuration.

(b) The $\kappa_h = 1$ configuration.

(c) Poloidal magnetic angles of Mirnov coils, evaluated at LCFS, for the parameter scan. Angles are measured clockwise, with $\phi(\text{coil1}) = 0$ for $\kappa_h = 1$.

Figure 2.7: The location of Mirnov coils and flux surfaces in H-1.
CHAPTER 3

Data Mining Geometrically Ordered Timeseries

3.1 Introduction

A useful analysis of a large dataset will involve looking for underlying structure, i.e.: a lower dimensional model of the data. In this chapter, we detail an algorithm for discovering classes of fluctuations for a set of geometrically ordered timeseries. We do this without any a priori assumptions about the structure of the fluctuations. By this process, we uncover underlying structure in the data through the fluctuation cluster definitions.

In this chapter, we do not limit the application of the data mining process to fusion specific datasets. The algorithm is sufficiently generalised to be useful in a wide range of other applications which share similar types of datasets, ie: multiple data channels with a common timebase. Indeed, the geometric ordering of the data channels is not a strict requirement, but is a condition which facilitates the interpretation of nearest neighbour channels and the underlying structure of fluctuations. An example of non-geometric datasets which could be analysed using this technique are financial or economic data where channels might be share prices of individual companies evaluated on the same timebase.

In section 3.2 we discuss the preprocessing stage; setting out the basic requirements for the dataset, introducing fluctuation structures and methods for filtering. Two different clustering methods are discussed in section 3.3, and a
A convenient method of visualising the results is introduced in section 3.4, with a discussion of the data mining procedure following in section 3.6.

3.2 Preprocessing

3.2.1 Data Preparation

We assume that each set of timeseries data can be represented as a $N_c \times N_s$ matrix:

$$S = \begin{pmatrix}
  s_0(t_0) & s_0(t_0 + \tau) & s_0(t_0 + 2\tau) & \ldots & s_0(t_0 + N_s \tau) \\
  s_1(t_0) & s_1(t_0 + \tau) & s_1(t_0 + 2\tau) & \ldots & s_1(t_0 + N_s \tau) \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  s_{N_c}(t_0) & s_{N_c}(t_0 + \tau) & s_{N_c}(t_0 + 2\tau) & \ldots & s_{N_c}(t_0 + N_s \tau)
\end{pmatrix} \tag{3.1}$$

where $\tau$ is the inverse of the sampling frequency, $N_c$ is the number of channels and $N_s$ is the number of samples. In our example dataset, the signal amplitudes depend on the plasma-coil distance which is a function of the plasma shape (magnetic configuration) controlled by $\kappa_h$. To reduce any configurational bias on $S$ we normalise each channel to its variance.

To achieve time resolution $\Delta t$, we split $S$ into short time segments $S$ with shape $N_c \times N'_s$, where $N'_s = \Delta t / \tau$. We also assume there are an arbitrary number of $S$ relating to the same system, e.g.: an experiment repeated under different conditions. At this stage there is no need to distinguish between the $S$ from different $S$, although we implicitly retain sufficient information to map the $S$ back to their original parameter sets.
3.2.2 The Singular Value Decomposition

Each $S$ is represented by a singular value decomposition (SVD) \[65\]

$$S = U AV^*$$

(3.2)

where the columns of $U$ and $V$ contain the spatial (topo) and temporal (chrono) singular vectors respectively, $V^*$ denotes the conjugate transpose of $V$, and the diagonal elements of $A$ are the $N_a = \min(N_c, N_s')$ non-negative singular values. The set of topos (chronos) are an orthonormal basis of $\mathbb{R}^{N_c(N_s')}$. The convention is for the singular values to be sorted in decreasing monotonic order meaning that $A$ is independent of the ordering of the channels within $S$. Shown in figure 3.1 are singular values from a typical H-1 Mirnov dataset. From the chronos power spectra we see that there are two dominant modes, each with two singular values suggesting they are both travelling waves, as discussed below. We also see that the variance-normalisation of each channel decreases the signal to noise ratio of the system, which can also be described in terms of the normalised entropy.

We calculate the normalised entropy $H$ of the singular values $a_k$ in $A$:

$$H = -\frac{\sum_{k=1}^{N_a} p_k \log p_k}{\log N_a},$$

(3.3)

where $p_k$ is the dimensionless energy:

$$p_k = \frac{a_k^2}{E}, \quad E = \sum_{k=1}^{N_a} a_k^2.$$

(3.4)

To some extent the scalar quantity $H$ can be used as a measure of how physically interesting the signals in $S$ are without any further investigation into the structure of $S$. The high entropy case ($H \to 1$) represents a high degree of disorder, or noise, in the system. Conversely, the low entropy case ($H \to 0$) occurs when the system is well ordered. In effect, we have a measure of the signal to noise ratio
Figure 3.1: Example of chronos power spectra and singular values. Singular values from both normalised (o) and unnormalised (x) $S$ are shown. $C_0, C_1, \ldots, C_5$ denote the chronos from the normalised singular value $0, 1, \ldots, 5$. There are two distinct modes, one at $f \sim 45$ kHz described by $SV_0$ and $SV_1$; the other, weaker, signal is at $f \sim 29$ kHz and is described by $SV_2$ and $SV_3$. The data is from H-1 shot #58122 at $31 < t < 32$ ms.

of the system $S$ as a whole, without needing to know what the signal is. There are subtleties involved in this interpretation, e.g.: when several modes exist in $S$, which will be investigated below.

In the maximal entropy case ($H = 1$) all singular values are equal, this occurs when $S$ consists only of noise in the limit of large $N_a$. This is shown in figure 3.2 for the geometric mean of $1 - H$ from 100 instances of $S$ with Gaussian distributed noise. The geometric mean

$$\mu_g = e^{\frac{1}{n} \sum_{i=1}^{n} \ln x_i}$$

(3.5)

is used because normally–distributed noise in $S$ gives rise to a log–normal distribution of $H$. For a fixed number of channels, a minimum in $H$ occurs when $N_a = N_c$. As the number of samples is increased the data better approximates a Gaussian distribution and $H$ asymptotically approaches unity. We see that for typical $N_c$ (10-100) the condition $N_a \gg 100$ should be applied to ensure that
1 − \( H \ll 0.1 \) for a purely noisy system and effects of the finite size of \( S \) can be neglected.

The other extreme, \( H = 0 \), occurs when there is only one non-zero singular value, meaning \( S \) has separable spatial and temporal singular vectors (i.e.: a standing wave) and the system is noiseless. Shown in figure 3.3(a) is a signal with separable spatial and temporal components with varying amounts of noise. Here, we use the standard definition of the signal to noise ratio (SNR):

\[
SNR = 10 \log_{10} \left( \frac{P_{\text{signal}}}{P_{\text{noise}}} \right) = 20 \log_{10} \left( \frac{A_{\text{signal}}}{A_{\text{noise}}} \right)
\]

(3.6)

where \( A \) and \( P \) are the RMS amplitude and power respectively. A SNR of 3 dB corresponds to \( H \sim 0.5 \), independent of the structure of the topo or chrono. For \( SNR \gg 3 \) dB we see that \( \log(H) \propto 1/SNR \), and for \( SNR \ll 3 \) dB we have \( \log(1 − H) \propto SNR \) for large \( N_s \). It is here that we can clearly see the one-to-one correspondence of the normalised entropy and the signal to noise ratio. Here we have used the same SNR in each channel; for generality, we also consider the case where individual channels have different signal to noise ratios. Figure 3.3(b) shows the normalised entropy for \( j \) channels with \( SNR = 0 \) dB and \( N_c − j \) channels with \( SNR = 20 \) dB for \( 0 \leq j \leq N_c \) with \( N_c = 20 \) and \( N_s = 10, 100 \) and 1000.

We now examine a more general case, where \( S \) describes the function

\[
f(\theta, t) = \sum_{i \in \{0, 1\}} A_i \sin(m_i \theta + \omega_i t + \phi_i),
\]

(3.7)

where \( \theta \) is a periodic variable with each channel \( l \) having an associated value \( \theta_l \). The effect of variation in the ratio of amplitudes \( A_2/A_1 \) is shown in figure 3.4, where we have averaged over random \( m_i \) and \( \omega_i \) and have used a signal to noise ratio of 20 dB. Each mode which has non-separable spatial and temporal components is represented by two singular values, relating to a pair of orthonormal
Figure 3.2: Entropy of singular values for $N_c = 10, 25, 50, 100$ channels of Gaussian noise and $\Delta t/\tau$ samples. Error bars are $\pm \sigma_g$, where $\mu_g$ and $\sigma_g$ represent the geometric mean and standard deviation respectively.

chronos (and topos) which are $\pi/2$ out of phase. We see that $H$ has an increase around $A_1 = A_2$, i.e.: multiple signals with the same individual SNR will have an increased entropy, or decreased effective SNR, when combined. From this we deduce that, if $H$ is to be used as an effective measure of the noise, the number of modes $N_m$ in the system should be significantly fewer than the number of channels, $N_m \ll N_c$. When the structure of the two modes is similar, ie: $m_2 - m_1, f_2 - f_1 \to 0$, the modes share singular vectors which reduces the size of one pair of singular values, which manifests as a decrease in $H$, as shown in figure 3.5. From equation 3.7 we see that figure 3.5 is identical for $m_2 - m_1$ and $\omega_2 - \omega_1$. Clearly, the SVD cannot be used to resolve distinct modes with identical spatial or temporal structure.
(a) Normalised entropy of singular values $H$ vs. signal to noise ratio for a set of signals with separable time and spatial structure.

(b) $H$ for $j$ channels with SNR= 0 dB and $N_c - j$ with SNR= 20 dB.

Figure 3.3: Normalised entropy for a mode with separable temporal and spatial components with varying levels of noise. $N_c = 20$ and $N_s = 10, 100, 1000$.

Figure 3.4: The dependence of $H$ on the amplitude ratio of two non-separable modes, averaged over 100 signals arrays with randomised $m, f$ and phase offset. $N_c = 20$, $N_s = 1000$, $SNR = 20$ dB.
3.2.3 Fluctuation Structures

Recognising that a travelling wave structure consists of a pair of singular values which naturally belong together we group similar singular values, defining a fluctuation structure $\alpha$ as a subset of singular values which have chronos with similar power spectra. We measure the similarity between two chronos $c_1$ and $c_2$ with the normalised average of the cross-power spectrum $\gamma_{c_1,c_2}$:

$$\gamma_{c_1,c_2} = \frac{G(c_1, c_2)^2}{G(c_1, c_1)G(c_2, c_2)},$$  \hspace{1cm} (3.8)

where $G(a, b) = \langle |F(a)F^*(b)| \rangle$, $F$ is the Fourier transform, and $\langle \ldots \rangle$ represents the spectral average.

When allocating singular values to fluctuation structures, the observation:

$$\gamma_{a,b} > \gamma_{\min} \quad \text{and} \quad \gamma_{a,c} > \gamma_{\min} \quad \Rightarrow \quad \gamma_{b,c} > \gamma_{\min},$$  \hspace{1cm} (3.9)

suggests that we should not simply seek to require $\gamma_{a,b} > \gamma_{\min}$ for each pair of singular values $a, b$ within a structure, instead we follow the process in algorithm
1. In doing so, we therefore require that each constituent singular vector has sufficient $\gamma$ with the dominant singular vector of the structure. Algorithm 1 describes the process of grouping singular values together into fluctuation structures: starting with the full set of singular values, the set of singular values whose chronos have sufficient cross-power with the chrono with the largest singular value is defined as a fluctuation structure; the process is iterated for the remaining singular values (using the largest remaining singular value as reference) until all singular values have been allocated to a fluctuation structure.

**Algorithm 1** Building fluctuation structures $\alpha_i$ from singular values $a_j$. The largest unallocated singular value $a_\xi$ will always be allocated to $\alpha_i$ because $\gamma_{\xi,\xi} = 1$.

\[
\text{while Number of unallocated singular values > 0 do}
\]
\[
\text{Define a new fluctuation structure as an empty set of singular values: } \alpha_i = \{\}
\]
\[
\text{Denote the largest unallocated singular value by } a_\xi
\]
\[
\text{for Every unallocated singular value } a_\zeta \text{ do}
\]
\[
\text{if } \gamma_{\zeta,\xi} > \gamma_{\min} \text{ then}
\]
\[
\text{Allocate } a_\zeta \text{ to fluctuation structure } \alpha_i
\]
\[
\text{end if}
\]
\[
\text{end for}
\]
\[
\text{end while}
\]

Various possible fluctuation structures for the dataset of figure 3.1 are shown in figure 3.6 as a function of the threshold value $\gamma_{\min}$. At $\gamma_{\min} = 0$, all singular values are grouped together as a single fluctuation structure, while at $\gamma_{\min} = 1$ each fluctuation structure contains one singular value. The key features are the two fluctuation structures $\alpha_0 = \{a_0, a_1\}$ and $\alpha_1 = \{a_2, a_3\}$ which coexist for $0.50 < \gamma_{\min} < 0.87$. After application of such analysis to a suitably sized sample of short time segments, a threshold of $\gamma_{\min} = 0.7$ was found to be appropriate for
Figure 3.6: The possible fluctuation structure groupings according to their energies as defined by algorithm 1 through the range of $\gamma_{\text{min}}$. The dataset is the same as in figure 3.1. We see that $\gamma_{\text{min}} \lesssim 0.4$ allows unrelated singular values to be included within a fluctuation structure, whereas with $\gamma_{\text{min}} > 0.87$ algorithm 1 will not recognise the similarity between $a_2$ and $a_3$. our dataset.

3.2.4 Data Filtering

Filters are applied to the dataset in order reduce its size and to remove noise. Two values which can be used to quantify the quality of the data are the normalised energy $p$ and normalised entropy $H$. The normalised energy $p$ of a fluctuation structure is defined as the sum of the normalised energies of its constituent singular values from equation 3.4. The nature of these thresholds is quite different; $H$ thresholds will act upon the entire short time segments, whereas $p$ thresholds affect individual fluctuation structures.

Using a normalised energy threshold value $p'$ allows filtering out of low energy noise. As seen in our example dataset (figure 3.8), there appears to be a clear
distinction between higher energy fluctuations \((p \gtrsim 0.6)\) and lower energy noise \((p \lesssim 0.2)\). The use of \(H\) thresholds is not always appropriate, especially if spectra are present with several distinct fluctuations; in such cases if the dataset needs to be reduced in size it is preferable to simply use a random subset of the data. Entropy filtering is generally more useful when a significant number of \(S\) contain only noise.

An alternative to using a hard \(p\) threshold uses an energy threshold defined as a fraction of the possible range of normalised energy for a given singular value, before constructing fluctuation structures. This method is more sensitive to modes which have reduced \(p_k\) due to the coexistence of other modes. The \(n^{th}\) largest singular value in a given short time segment has, by definition, a maximal normalised energy of \(1/n\). We then apply a factor \(p^*\), where \(0 < p^* < 1\), and, starting with the smallest singular value, retain singular values with \(p_n \geq p^*/n\). Any point retained brings in all larger singular values from the same time segment, trumping the \(p_n \geq p^*/n\) condition for lower \(n\). The requirement for bringing in larger singular values is easily understood by considering the case of a mode having two singular values of almost equal energy, it is possible for the lower energy value to exceed the threshold with the higher value below the threshold.

A comparison of \(p'_{SV}\) and \(p^*\) filtering on singular values with normalised energy is shown in figure 3.7. We use the subscript \(SV\) to distinguish from \(p'\) used to filter fluctuation structures; here \(p'_{SV}\) is simply a threshold value where we retain singular values with \(p > p'_{SV}\). For each method, the number of singular values and the sum of their normalised energies are plotted. There is a clear elbow in the \(p^*\) energy plot at \(p^* \simeq 0.38\), for \(p^* \lesssim 0.38\) the filtering affects mostly noise, while for \(p^* \gtrsim 0.38\) it is the stronger signals which are most affected. For the \(p'_{SV}\) filter, the energy plot is essentially linear for \(p'_{SV} \lesssim 0.5\) and lacks a similar
feature to distinguish noise and signal filtering.

3.2.5 Mapping of Fluctuation Structures into $\Delta\psi$-Space

We regard each fluctuation structure as a point in the space $[-\pi, \pi]^{N_c}$, an $N_c$-dimensional torus of length $2\pi$ which we will call $\Delta\psi$-space. In this application $\psi$ represents the electrical phase of the reconstructed fluctuation structures at the positions of the coils. Fluctuation structures which are close in $\Delta\psi$-space can be considered the same type. This interpretation arises from the expectation that the possible waves have various phase velocities and mode numbers due to the periodic boundary conditions of the physical plasma torus in which they propagate. It is also applicable to the more general case where waves are spatially localised within the system and do not have well defined mode numbers.

For each fluctuation structure $\alpha_l$ we take the inverse SVD to get $S_l$:

$$S_l = U A_l V^*,$$  \hspace{1cm} (3.10)
Figure 3.8: A 10% random sample of a dataset. The left panel shows $p$ and $H$ for the fluctuation structures. The middle panel shows the number of fluctuation structures $N_{\alpha}$ within $\delta p = 0.01$. The right panel shows $pN_{\alpha}$, which is effectively the density of normalised energy; while this is not physically meaningful because the normalisation factor is dependent on short time segment, it is a useful guide to the energy distribution among fluctuation structures.

where the elements of $A$ not in $\alpha_l$ are set to zero to form $A_l$. The rows of the matrix $S_l$ contain the timeseries relating to $\alpha_l$ for each channel. In general, the power spectra of the topos in $\alpha_l$ are peaked around a single frequency $\omega_l$. The phase differences $\Delta \psi_{a,b}(\omega = \omega_l)$ between channels $a$ and $b$ evaluated at $\omega = \omega_l$ are used to define the coordinates in $\Delta \psi$-space. Using phase differences between each pair of channels would result in a $\frac{1}{2}N_c(N_c - 1)$-dimensional space; instead we use the $N_c$-dimensional space of only nearest neighbour channels. Note that in our example dataset the actual phase difference between channels depends on $\kappa_h$ so we map the phase differences to a coordinate system which is independent of $\kappa_h$, namely the $\kappa_h$-averaged magnetic angles of the Mirnov coils.
3.3 Clustering

We aim to discover any underlying lower-dimensional model of the dataset; that is, groups of fluctuation structures which are similar throughout some range of short time segments. As discussed in section 3.2.5, we assume that a class of fluctuations is localised in the $N_c$-dimensional $\Delta\psi$-space. For example, it is simple to understand such localisation in terms of a simple cylindrical geometry with poloidally equispaced measurements, where each mode with poloidal mode number $m$ will be located at $\Delta\psi = 2\pi m/N_c$ in each dimension. However, we assume a generalised case in which the fluctuation may have arbitrary, including localised, structure.

We use a clustering algorithm to locate the classes of fluctuations, here we describe and compare two different algorithms. The agglomerative hierarchical clustering algorithm does not include any assumption of the cluster shapes, how-
ever it scales poorly ($O(n^2)$), or quadratic computational complexity) with the size of the dataset. The expectation maximisation algorithm scales better with the size of the dataset, however the clusters are assumed to have a Gaussian shape and issues with the averaging of the periodic variable $\Delta \psi(n, n + 1)$ need to be addressed.

### 3.3.1 Agglomerative Hierarchical Clustering

Hierarchical clustering involves generating new clusters from pre-existing clusters, where the starting condition is either where the entire dataset is a single cluster, or where each datapoint defines a cluster. The agglomerative hierarchical (AH) method is the latter, where existing clusters are merged to form larger clusters.

We define the distance, or metric, between a pair of fluctuation structures as

$$\delta(\alpha_a, \alpha_b) \equiv \sum_n (\Delta \psi'_b(n, n + 1) - \Delta \psi_a(n, n + 1))^2$$

(3.11)

The notation $\Delta \psi'$ denotes a possible $2\pi$ phase shift such that $|\Delta \psi'_b - \Delta \psi_a| \leq \pi$. This phase shift allows fluctuation structures at $\Delta \psi = -\pi$ and $\Delta \psi = \pi$ to be mapped to the same cluster. We have found this metric to be sufficient for our application, however alternative metrics may be favourable for other cases. For example, a metric might adapt to some functional dependence on channel to allow better resolution of localisation in specific dimensions; or a channel dependent signal amplitude factor.

As an initial condition, each $\alpha$ defines a cluster. These give the initial set of $N_{\text{Cl}} = N_\alpha$ cluster definitions which fully describe the filtered dataset, where $N_{\text{Cl}}$ is the number of clusters and $N_\alpha$ is the number of fluctuation structures. First the distance is calculated between each of the $N_\alpha(N_\alpha - 1)/2$ pairs of fluctuation structures $\alpha_a, \alpha_b$, and the list of pairs is sorted in order of increasing distance.
We then iterate through the increasing $\delta(\alpha_a, \alpha_b)$; in the case where $\alpha_a$ and $\alpha_b$ belong to separate clusters we join those two clusters and record the new set of $N_{Cl}$ cluster definitions, continuing until we reach $N_{Cl} = 1$.

### 3.3.2 Expectation Maximisation Clustering

The expectation maximisation (EM) algorithm is a method for estimating the most likely values of latent variables in a probabilistic model [66]. Here we assume that each type of fluctuation can be described by a $N_c$-dimensional Gaussian distribution in $\Delta \psi$ space. The latent variables are the mean $\mu_i$ and standard deviation $\sigma_i$ for each cluster $i$, where $i = 1, 2, 3, \ldots, N_{Cl}$ and $N_{Cl}$ is the number of clusters.

Given the initial conditions, in the form of random initial $\mu_i$ and $\sigma_i$ values for a prescribed number of clusters, the EM algorithm consists of two steps which repeat until a convergence criterion is met. Firstly, the expectation step assigns to each datapoint a probability, or expectation value, of belonging to each cluster which is calculated with the Gaussian distribution function. Secondly, $\mu_i$ and $\sigma_i$ are recalculated using the new expectation values as weight factors.

We use the EM algorithm as implemented in the WEKA suite of data mining tools [14]. The WEKA algorithm does not operate with toroidal data, so we map the $\Delta \psi$-space from the $N_c$-dimensional torus to a $2N_c$-dimensional cube $[-1, 1]^{2N_c}$ by taking the $\sin(\Delta \psi)$ and $\cos(\Delta \psi)$ components.

The 10-fold cross-validated log-likelihood ratio is used as a measure of how well the cluster assignments fit the data. The cross-validation process involves partitioning the dataset into random sub-samples and comparing results from each subset to avoid oversensitivity to outliers in the data. The likelihood is the conditional probability of obtaining the cluster means and standard deviations
given the observed data.

The EM procedure is better suited to data which may have localised fluctuations than is the AH method because a cluster can be defined with an arbitrarily large $\sigma_i$ in a subset of dimensions where a fluctuation might be undefined. For a given set of initial conditions, the EM procedure requires $N_{\text{CI}} \times N_{\alpha} = O(N_{\alpha})$ distance calculations per iteration. Because the algorithm can only guarantee a local maximum in likelihood we require a Monte Carlo approach, with multiple repetitions with different randomised initial conditions. For the cases we have studies with $N_{\alpha} \lesssim 2000$, the time complexity is greater for EM than AH while the space complexity, or memory requirement, is greater for AH.

### 3.4 Visualisation Using Cluster Trees

The identification of the correct number of clusters, or of those which are important, is a task that is by no means trivial to automate. We have found inspection of a dendrogram, or *cluster tree*, mapping (figure 3.10) to be a practical method for identifying the important clusters. The cluster tree displays clusters for each $N_{\text{CI}}$ below some maximum value $N_{\text{CI,max}}$, with all clusters for a given $N_{\text{CI}}$ forming a single column. Each child cluster is mapped to the cluster on the parent level with which it has the largest fraction of common datapoints. Cluster branches which do not fork over a significant range of $N_{\text{CI}}$ are deemed to be well defined, and the point where well defined clusters start to break up suggests that $N_{\text{CI}}$ is too high. While this procedure is clearly a subjective one, it is effective and does not depend on the type of clustering algorithm used.

For AH clustering the clustering algorithm naturally develops a tree structure; however, there are as many branches as there are datapoints and so a method
is required to prune the tree so that only interesting clusters remain. Even for low $N_{\text{Cl}}$ we often find clusters defined by only one or two datapoints. By imposing a minimum population for the clusters in the tree, we can effectively compress the tree to focus on the dominant clusters. If a cluster in the $N_{\text{Cl}}$ set has fewer than the minimum required number of datapoints then we map $N_{\text{Cl}} + n \rightarrow N'_{\text{Cl}}$, increasing $n$ until we have the required number of clusters with sufficient populations.

### 3.5 Test Case

In order to test the data mining process described in this chapter we apply it to an artificial dataset. Ten detectors are distributed at fixed radius in polar coordi-
nates, equispaced with Gaussian noise (0.1 radian standard deviation) added to angular coordinate: \( \theta = [0, 0.553, 1.266, 1.934, 2.480, 3.232, 3.758, 4.338, 5.033, 5.703] \). Over a period of 80 ms, three modes are detected at a sample rate of 1 MHz: from \( t = 0 \) to 20 ms an \( m = 1 \) mode at 20 kHz, from 20 ms to 40 ms an \( m = 3 \) mode at 73 kHz and for 40 ms < \( t < 60 \) ms a 136 kHz \( m = 2 \) mode. The modes have Gaussian envelope (standard deviation of 500 Hz) sidebands with random phase, and normally distributed noise with standard deviation a quarter of the RMS signal amplitude is added to the signal. Data for \(-2 \text{ ms} < t < 0 \text{ ms}\) is included as a reference for the noise level. A spectrogram of the first channel at is shown in figure 3.11.

Fluctuation structures are generated using time segments of 1024 samples and coherence threshold of \( \gamma_{\text{min}} = 0.7 \). Of the 728 fluctuation structures produced, 60 are retained after an energy threshold of \( p > 0.5 \) is applied (see figure 3.12). The EM clustering algorithm with \( N_{\text{Cl}} = 3 \) clearly separates the three modes (lower left subplot, figure 3.12). Phase-angle representations of the clusters are shown in the lower right subplot, where the small error bars indicate that the clusters are well defined. The slight deviation from linear phase is due to the random input noise. Note that, due to under-representation of data, we expect that this test case will not perform as well as for a larger dataset with the same number of channels. Here, the ratio of the number of datapoints (fluctuation structures) to the number of dimensions (twice the number of channels) is approximately 3, whereas for the dataset in the following chapter the ratio is an order of magnitude greater.
Figure 3.11: Spectrogram of one channel from artificial dataset. Three modes are present: $0 \text{ ms} < t < 20 \text{ ms}$ $m = 1$ at 20 kHz, $20 \text{ ms} < t < 40 \text{ ms}$ $m = 3$ at 73 kHz, and $40 \text{ ms} < t < 60 \text{ ms}$ $m = 2$ at 136 kHz

### 3.6 Discussion

The data mining algorithm presented here is potentially useful in numerous other domains where spatio-temporal data is used. However, there is a limitation on the nature of the fluctuations amenable to this analysis due to the SVD. The SVD is not effective in distinguishing different modes coexisting with the same frequency or spatial structure because the modes would share a chrono or topo, whereas the SVD requires orthogonal components to distinguish modes. The assumption that such coexisting modes are not present is also important in assigning a single frequency $\omega_l$ to a fluctuation structure, i.e.: two modes with the same spatial structure will also share chronos, but only one frequency would be recorded.

In considering which other systems may be amenable to analysis using the data mining technique presented here, we recognise a possible limitation to those
Figure 3.12: Preprocessing and clustering of artificial dataset. Upper left: single channel time series showing the pure signal before (blue) and after (cyan) the addition of noise. The signal with noise is used in remaining plots. Upper right: distribution of fluctuation structures shown in terms of energy and entropy; middle left: time-frequency plot showing all fluctuation structures; middle right: time-frequency plot showing fluctuation structures with $p > 0.5$; lower left: resulting clusters for EM algorithm with $N_{Cl} = 3$; lower right: clusters shown in phase-angle plot using the mean phase differences for each channel pair, which is the final result of the procedure. In the simple case with synthetic data and small number of fluctuation structures the cluster widths are small, so the cluster variances shown here have been exaggerated (10 standard deviations) for visibility.
for which coherent phase between detector channels can reasonably describe or
identify the state of the system. One possibility is the application to other multi-
channel plasma diagnostics. Such diagnostics often require signal reconstruction
for analysis, whereas, in principle, the data mining technique may be applied to
the raw data allowing identification and classification of modes of the system.
When working with large datasets, this could dramatically reduce the work re-
quired in signal processing as only a single reconstruction per identified mode
would be required.

Other possible applications may arise in, for example, chemistry, engineering,
or seismology where repeated measurements of the same system under varying
conditions may occur. The assumption that separate observations of the same
state of the system will be localised in phase space for clustering limit the appli-
cability to cases where the system itself is not changing in any way which cannot
be accounted for through preprocessing of the dataset. As an example of prepro-
cessing specific to an application; for the case of seismology, earth tremor signals
could be filtered to include only those distant enough such that the waveforms
are almost sinusoidal and sufficiently strong to reach many detectors.

The EM clustering method described here relies on the assumption that clus-
ters can be described by a Gaussian distribution. To check if the imposed Gaus-
sian distributions significantly influence the cluster outcomes, we have also used
the agglomerative hierarchical (AH) clustering algorithm [67] which does not
make such an assumption. The initial condition for AH clustering is that each
fluctuation structure defines a cluster. Using a suitable metric the two closest
clusters are combined, iterating the process until we have $N_{C1} = 1$ gives a (pro-
hibitively large) cluster tree. Compression of the AH cluster tree can be achieved
by filtering out clusters with small populations, allowing for a visualisation sim-
ilar to figure 3.10. We show in chapter 4 that our results from EM and AH clustering methods are very similar.

It is important to note the scalability of the algorithm. Given fixed values of $N_c$ and $\Delta t$, the size of $S$ remains constant and the preprocessing stage has complexity $O(N_S)$, where $N_S$ is the number of timeseries datasets $S$. The scalability of the clustering stage depends on the algorithm used, for the EM case we have $O(N_{Cl}N_\alpha)$, which gives $O(N_S)$ for constant $N_{Cl}$. The AH clustering algorithm is less desirable as it has complexity $O(N_\alpha^2)$ due to distance calculation between each pair of fluctuation structures.

We have implemented the preprocessing and visualisation stages using the python language with the Scipy and Matplotlib libraries [16, 15, 17]. The preprocessing of the dataset used in chapter 4, 4600 $S$ arrays (28 by 1000), takes around 2 hours using a 1.9 GHz Intel Pentium M processor. The results are stored in MySQL tables; a table of fluctuation structure properties excluding $\Delta \psi$-space mapping is around 5 Mb in size, with the $3.6 \times 10^6$ rows of the $\Delta \psi$ mapping table taking around 30 Mb, using optimal data types. For clustering, we have used the EM algorithm from the WEKA suite of data mining tools [14] which runs at about $0.05 \times N_{Cl} \times N_\alpha$ CPU seconds using 2.2 GHz AMD Opteron processors. For each $N_{Cl}$, 100 randomised initial conditions were used; the results with maximal log-likelihood are selected as the best clusters. The WEKA algorithm does not operate with toroidal data, so we map the $\Delta \psi$-space from the $N_c$-dimensional torus to a $2N_c$-dimensional cube $[-1,1]^{2N_c}$ by taking the sin($\Delta \psi$) and cos($\Delta \psi$) components. For the present work, no specific efforts were made to optimize the clustering process; more efficient clustering routines exist, including, for example, genetic algorithms for faster convergence.
CHAPTER 4

Results from Mining H-1 Data

4.1 Introduction

In the previous chapter we described an automated process for detecting distinct types of fluctuations, classified by their spatial phase structure, and locating them in parameter space. For the examples given, the parameter space is spanned by $\kappa_h$, frequency and time. In the present chapter the data mining process is specifically applied to a $\kappa_h$ configuration scan, and the structure of the resultant classes of fluctuations are analysed. The investigation into the physical nature of the fluctuations is detailed in the next chapter.

4.2 Experiment Setup

A scan through magnetic geometry was performed by varying $\kappa_h$. The range of $0 < \kappa_h < 1$ with $\Delta \kappa_h = 0.01$ corresponds to a range of rotational transform at the axis (edge) of $1.12 \ (1.28) < \tau_0(a) < 1.43 \ (1.44)$ with $\Delta \tau_0(a) = 0.0031 \ (0.0016)$. The transform profile changes from monotonic positive shear ($\nu > 0$) at $\kappa_h = 0$ to central reversed-shear at $\kappa_h = 1$. The magnetic well increases from 0.4% to ~5.0% over this range, with a local magnetic hill at the outer edge for low $\kappa_h$, as shown in figure 2.4(b). As mentioned in section 2.3.2, the range $0 < \kappa_h < 0.16$ is not accessible at 1/2 Tesla operation, however the $\kappa_h = 0$ configuration (standard...
configuration) is accessible and is included in the present dataset.

The dataset consists of 92 shots with distinct magnetic configurations and, for each shot, data for 28 Mirnov channels. Figure 4.1(a) shows the time of each shot in the campaign, with the scan executed in two interleaved sweeps. Although controllable parameters were maintained constant throughout the campaign, some conditions in the device may vary slightly during the period of operations. The largest effect is likely to be a change in impurity levels within the vacuum vessel, through the impurity levels were not measured during this set of shots. Other effects may be due to thermal loading on the magnetic field coils and the antenna.

In figure 4.1(b) we see the RF heating power is effectively constant, importantly so between the two interleaved sweeps. The magnetic field $B_0 = 0.46$ T, besides the varying helical component, is also kept constant; figure 4.1(c) shows the main current to be effectively constant across the configuration.

Shots of 60 ms duration were produced using 50 – 60 kW of 7 MHz ICRF in a H:He = 3:2 mixture. The plasma density is shown in figure 4.2; line-average electron density is $n_e \lesssim 1.5 \times 10^{18}$ m$^{-3}$ and the $\kappa_h$ dependence is consistent between the two interleaved scans and matches that observed in previous campaigns (see

Figure 4.1: Experimental parameters for the $\kappa_h$ configuration scan.
Figure 4.2: Line-averaged electron density from 2 mm interferometer, showing the similarity between the two interleaved $\kappa_h$ scans. Circles (triangles) represent the first (second) scan shown in figure 4.1(a). The poor confinement configurations due to proximity of low order rational transform have been observed in previous campaigns (see fig. 2.6).

Plasma temperatures are $T_e \sim 10 - 20$ eV and $T_i < 100$ eV, and the normalised plasma pressure $\beta = 2\mu_0 p/B^2$ is quite low, $\ll 1\%$, so that the magnetic flux surfaces are assumed to be those of the vacuum configuration.

For the present analysis, we normalise the Mirnov signal amplitudes (equation 4.1) and classify fluctuations by their phase structure, taking into account the shifting magnetic coordinates of the coils through the configuration scan as shown in 2.7(c). The set of available Mirnov coils for this campaign was $M_{44} = [1, 2, 3, 4, 7, 8, 9, 10, 15, 16, 17, 18]$, $M_{284} = [1, 2, 3, 5, 7, 8, 9, 10, 15, 17, 18, 19, 20]$, and $M_{35} = [2, 4, 5]$. 

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4.3 Mining Data from H-1 Configuration Scans

4.3.1 Overview of the Dataset

We wish to find the maximum range of parameters within which a dataset suitable for mining can be defined. We know from the previous section that, apart from $\kappa_h$, externally controllable parameters are essentially constant. General plasma properties such as density and temperature are not included in the clustering process, however we need to define $\Delta t$ (and through it, $\Delta f$) and consider any assumptions we make about the spatial properties of the fluctuations. In section 4.3.2 we determine appropriate values and ranges of $\Delta t$, and in section 4.3.5 we detail our assumptions about spatial properties of the fluctuations.

4.3.2 Temporal Evolution of a Typical Shot

Figure 4.3 shows a shot typical of the configuration scan. The top panel of figure 4.3(a) shows the Mirnov signal for a single coil and its spectra is shown in the bottom panel. The top panel of 4.3(b) shows the fluctuation structures for the shot, here the frequencies are taken from SVD processed data so they relate to globally observed oscillations rather than a single coil. Various values of normalised energy are used as thresholds to distinguish stronger and weaker signals. We have used $\Delta t = 1$ ms for the fluctuation structures, while this is appropriate for the stable frequencies at $t \gtrsim 10$ ms we see that it is too large to capture the behavior of the fluctuations at $t \lesssim 10$ ms with higher $|df/dt|$.

The bottom panel of figure 4.3(b) shows the electron density profile evolution as measured by ELSI.

Considering all configurations (figure 4.4) we see that figure 4.3 is indeed representative of all shots, where larger $|df/dt|$ mostly occurs at $t \lesssim 10$ ms. Note
Figure 4.3: A typical shot in the configuration scan (shot 58048, $\kappa_h = 0.24$).

that most of the scattered sequences of higher $|df/dt|$ at $t \gtrsim 10$ ms are artifacts due to co-existing modes with similar amplitude, with the selection of a single dominant mode alternating between them. Hence, we consider the two plasma regimes separately: the $t \lesssim 10$ ms heating phase and the $t \gtrsim 10$ ms steady state phase.

4.3.3 An Example of the Singular Value Decomposition

The SVD results for a typical short time segment are shown in figure 4.5. In this case there are two dominant singular values whose temporal basis vectors are highly correlated and out of phase by approximately $\pi/2$; together they represent a travelling wave in the plasma fluctuating at $f \sim 30$ kHz with around 97% of the signal energy. Only coils from one array are used here, facilitating the visualisation of the spatial basis vectors. From figure 4.5(d) we see clearly a travelling wave between coils 5 and 13, with the other coils having more complex
 behaviour. Here, the remaining singular values may be attributed to noise in the system.

4.3.4 Poloidal Component of Wave Propagation

Before proceeding with the data mining analysis of the structure of observed fluctuations we consider the direction of wave propagation in the poloidal direction. This can be important in determining the nature of the fluctuations, especially if drift motions are important. As can be seen from figure 4.5(d), the irregular spacing of the coils in the poloidal arrays complicates the problem of finding a spatial phase difference between two topos belonging to the same fluctuation. Rather than attempting to quantify the phase differences by using an interpolation between coils we assume a qualitative approach will suffice and simply cross-correlate the topos, keeping the coil ordering but disregarding their angular locations. This is generally sufficient to find the sign of the phase offset.
and, combined with phase offset of the chronos, gives the direction of poloidal rotation.

Here we consider the fluctuations from a single poloidal Mirnov array from $\Delta t = 1$ ms samples at $t = 10, 20, 30, 40, 50$ ms. For each time segment we take the SVD and combine singular values, using an average cross-power threshold of $\gamma_{\text{min}} = 0.7$. We denote the phase difference between chronos (topos) by $\chi_{c(t)}$; fluctuations with $\chi_c \times \chi_t > 0$ are in the electron diamagnetic direction, and those with $\chi_c \times \chi_t < 0$ are in the ion diamagnetic direction. Taking only fluctuation structures with multiple singular values and normalised energy $p > 0.2$ we find that of the 436 fluctuation structures 86% are in the ion diamagnetic direction, with the remainder apparently in the electron diamagnetic direction, as shown in figure 4.6. On inspection of the fluctuation topos, it is found that those counted as travelling in the electron diamagnetic direction have generally been falsely classified due to the irregular shape of the topos. We therefore assume that
all non-stationary fluctuations observed in the configuration scan travel in the ion diamagnetic direction.

4.3.5 Preprocessing

4.3.5.1 Amplitude Normalisation

The inverse problem of determining the structure of a fluctuation from the amplitudes of external magnetic coils signals cannot be solved without additional information about the system. If we have knowledge about how the electromagnetic radiation propagates from the fluctuation then it would be possible to back-project the amplitude information from the Mirnov coils and resolve the spatial structure of the fluctuation. The simplest such model of a fluctuation would be cylindrically symmetric multipole radiation in free-space with radial
localisation on a single magnetic surface. In such a model, the attenuation of radiation is proportional to $d^{-m}$, where $d$ is the radial distance between the fluctuation surface and magnetic coil, and $m$ is the poloidal mode number. However the complex geometry of the plasma and the likely non-local radial structure makes such a treatment of the problem beyond the scope of this thesis.

Our aim here is to determine what can be inferred about the fluctuations through data mining techniques without any a priori information about the structure of the fluctuations. Hence, in this chapter we utilise the phase information between the coils and discard the amplitude information. For each short-time segment, we use RMS normalisation on the signal amplitudes from individual coils $s_i(t)$:

$$s_{RMS}(t) = \frac{1}{\sqrt{\mu(s^2)}} s(t).$$

(4.1)

### 4.3.5.2 Coil Angles

As mentioned above, we rely on spatial phase structure information to distinguish types of fluctuations. In the present case of a scan through varying magnetic geometries, the effective magnetic angles (see section 2.3.4) of the coils are dependent on $\kappa_h$. It is therefore insufficient to simply use the phase differences between the coil channels as $\kappa_h$ will enter as a hidden variable. Instead, we map the phase differences to $\kappa_h$-averaged (virtual coil) angles instead. Because the effective magnetic angle of a coil is a function of magnetic surface, when we consider the magnetic angles we make in implicit assumption about the radial location of the fluctuations. Unless stated otherwise, the effective magnetic angles will be evaluated at the last closed flux surface. The limitations of this assumption are discussed in section 4.6.
4.3.5.3 Filtering

The dataset is filtered to reduce both noise and the size of the dataset. In both the \( t < 10 \text{ ms} \) and \( t > 10 \text{ ms} \) cases, we use the same dataset for the expectation maximisation (EM) and agglomerative hierarchical (AH) clustering procedures, where a normalised cross-correlation threshold of \( \gamma_{\text{min}} = 0.7 \) and normalised energy threshold of \( p' = 0.2 \) have been applied.

Because its Mirnov spectra show a small number of distinct features, the \( t < 10 \text{ ms} \) dataset has been treated more as a test case to check whether or not the clustering process will group together the fluctuation structures one would expect. As well as limiting normalised entropy to \( H' < 0.5 \), we have filtered out fluctuations with \( f < 50 \text{ kHz} \) where there are no clear features (see figure 4.7). This reduced dataset has 1152 datapoints, and is considered in section 4.3.6.1. The \( t > 10 \text{ ms} \) dataset is limited in size to 2000 datapoints by a 500 Mb memory limit of the computer used for AH clustering; the 2000 datapoints in the subset are randomly selected and no entropy filtering is used.

4.3.6 Clustering

We now consider the results of using the EM clustering algorithm on the magnetic fluctuation data. The EM algorithm is preferable to the AH algorithm because of its lower computational complexity and well-defined scalar cluster parameters (cluster mean and standard deviation). As described in section 3.3.2, the implementation of the EM algorithm used here has clusters defined by Gaussian distributions in the \( 2N_c \)-dimensional cube \([-1, 1]^{2N_c} \) of \( \sin(\Delta \psi) \) and \( \cos(\Delta \psi) \). Each datapoint (fluctuation structure) has a calculated probability of belonging to each cluster and is allocated to the cluster with which it has highest probability. The terminology for cluster referencing used here is (clustering algorithm): (figure
where cluster is first shown): (cluster number in figure); for example, EM:4.8:11 refers to cluster 11 in figure 4.8 which was produced with the EM algorithm.

4.3.6.1 Magnetic Fluctuations in the \( t < 10 \text{ ms} \) Heating Stage

Because of the large \(|df/dt|\) in the \( t < 10 \text{ ms} \) heating stage, we use a smaller time window size of \( \Delta t = 0.1 \text{ ms} \). Shown in figure 4.7 are the fluctuations which occur at \( t < 10 \text{ ms} \): there are four clear ‘V’ shapes in the \( f-\kappa_h \) plot, each associated with a low order rational value of \( \iota \). With the cluster tree shown in figure 4.8 we see the expected clusters defined by the ‘V’ shapes. Here the tree has been truncated at \( N_c = 5 \), after which most of the branch splitting occurs out of the residual cluster EM:4.8:12. Clusters EM:4.8:(14, 11, 13 and 15) are centred around \( \iota (\iota' = 0) = 4/3, 5/4, 6/5 \) and \( 7/5 \) respectively. Both the ELSI and 2 mm interferometers provide unreliable measurements of \( n_e \) in this regime because the density is quite low \( (n_e \ll 10^{17} \text{ m}^{-3}) \) so it is not possible to do a detailed analysis of the physics of these modes.

4.3.6.2 Magnetic Fluctuations in \( t > 10 \text{ ms} \) Steady State Plasmas

The magnetic fluctuations observed in the \( t > 10 \text{ ms} \) steady state plasmas are shown in figure 4.9. The main features are the two whale-tail structures, so called because their representation in \( \kappa_h - f \) coordinates are reminiscent of the tail fin of a diving whale. These are centred on \( \kappa_h \sim 0.40 \) and \( \kappa_h \sim 0.74 \), where the 5/4 and 4/3 rational surfaces occur on the zero-shear \( (\iota' = 0) \) flux surface.

The cluster tree representation of the \( t > 10 \text{ ms} \) data is shown in figure 4.10 in the frequency and magnetic configuration \( (\kappa_h) \) coordinates, with an alternative cluster tree representation shown in appendix B. Here the time window for the fluctuation structures is set to \( \Delta t = 1.0 \text{ ms} \). At \( N_{Cl} = 3 \) we have the well defined
Figure 4.7: High frequency modes. $t < 10 \text{ ms}, H < 0.5$. A random 25% subsample of the full dataset is displayed here. The size and colour of the fluctuation structures in the middle panel are proportional to the normalised energy $p$.

Clusters EM:4.10:5 and EM:4.10:6 centred on the $\iota(\iota' = 0) = 4/3$ and $5/4$ configurations respectively. The branch stemming from EM:4.10:9 at $N_{Cl} = 4$ contains modes with $(n, m) = (0, 0)$ (see section 4.5). At $N_{Cl} = 10$ the $\iota(\iota' = 0) = 7/6$ and $6/5$ configurations are represented by EM:4.10:54 and EM:4.10:50 respectively, while EM:4.10:51 is associated with $\iota \simeq 7/5$.

The clusters at the truncation level of $N_{Cl} = 10$ in figure 4.10 mostly appear well-defined in $\kappa_h - f$ coordinates, with the exception of EM:4.10:49 and EM:4.10:52 which seem relatively unrefined. In order to observe their further breakdown, these two residual clusters are combined to form the base cluster EM:4.11:1. We note that we again see clusters located around the lowest order rationals (EM:4.11:(18 and 19)) and at low frequencies at $\iota(\iota' = 0)$ resonance configurations (EM:4.11:16).
Figure 4.8: Cluster tree representation of the $t < 10\text{ ms}$ magnetic fluctuation data.
Figure 4.9: Mirnov spectra at $t > 10 \text{ ms}$. In the top panel, the size and colour of the fluctuation structures are proportional to the normalised energy $p$. An energy threshold of $p' = 0.2$ has been used.

4.4 A Comparison of Clustering Methods

4.4.1 AH Clustering for the $t < 10 \text{ ms}$ Heating Stage

We now consider an alternative method for generating clusters in order to corroborate the results of section 4.3.6. The AH clustering algorithm (see section 3.3.1) does not use a statistical distribution to define the clusters, and so it is a suitable method for checking for effects of imposing Gaussian distributions on the phase-space structure of clusters, as is done in the EM procedure. We first consider the $t < 10 \text{ ms}$ dataset.

The AH clustering method is initiated with $N_{\text{Cl}} = N_{\alpha}$, which means that it can produce a prohibitively large cluster tree with $N_{\alpha}$ branches. An alternative visualisation to the cluster tree is shown in figure 4.12, where the population of each cluster in the large AH tree is displayed; the horizontal axis $N_{\text{Cl}}$ corre-
Figure 4.10: Cluster tree representation of the $t > 10 \text{ ms}$ Mirnov data. The sum of clusters at each level is the same for all levels in the tree. An alternative representation of this cluster tree is shown in appendix B.
Figure 4.11: Residual cluster tree, where cluster 1 is the sum of clusters EM:4.10:49 and EM:4.10:51.
sponds to the number of clusters in the tree at a certain depth and the vertical axis shows the populations of the clusters. At $N_{\text{Cl}} = 1$ there is a single cluster with population of 1156 and as $N_{\text{Cl}}$ is increased the main cluster gradually loses some its constituents to small-population clusters. At $N_{\text{Cl}} = 167$ there is a large discontinuity where the main cluster has split into two clusters with significant size, after which there is a cascade where the smaller clusters split up in turn. Assuming that well defined clusters are those which do not undergo major splitting as we traverse a range of levels in the cluster tree, we see that while the largest cluster is well defined over a large region ($167 \leq N_{\text{Cl}} \leq 896$), the set of the 5 largest clusters are reasonably well defined over the range $320 \leq N_{\text{Cl}} \leq 402$. At the maximal value $N_{\text{Cl}} = 1156$ each cluster is populated by a single fluctuation structure.

In general, we are more concerned about finding the range of $N_{\text{Cl}}$ where clusters are stable and at which points they split than we are about knowing how many datapoints they contain. Figure 4.13 shows the derivative $\Delta N_{\alpha}/\Delta N_{\text{Cl}}$ for the largest clusters. Again, we see the 5 largest clusters to be reasonably stable over the same range of $320 \leq N_{\text{Cl}} \leq 402$. Note that there is no implication that any subset of datapoints exists that belongs to the label of $n^{th}$-largest cluster; for example, if the largest cluster splits in two, then what was the second largest cluster may become the first, third, fourth, or remain the second largest cluster depending on the resultant population sizes.

The clusters in figure 4.14 are chosen by selecting $N_{\text{Cl}} = 330$ and $N_{\alpha} > 10$. The $t < 10\,$ms clusters as defined by the two clustering methods are nearly identical, as shown in the comparison matrix in table 4.1. The sub-matrix of the well defined clusters (all rows except cluster EM:4.8:12 and all columns except AH:4.14:remainder) is diagonal, meaning that there is no cross-contamination be-
Figure 4.12: Cluster populations at each level of the $t < 10 \text{ ms}$ cluster tree. Each point represents a cluster with its population $N_\alpha$ plotted on the vertical axis and tree depth $N_{Cl}$ on the horizontal axis.

tween clusters defined by the different methods. The residual cluster EM:4.8:12 is about 15% larger AH:4.14:remainder; the difference between the residual clusters is that while the EM cluster is defined by a single broad Gaussian, AH:4.14:remainder is a collection of many small clusters which have been filtered out by the cluster tree compression.

4.4.2 AH Clustering for the $t > 10 \text{ ms}$ Steady State Plasmas

We now turn to the $t > 10 \text{ ms}$ data; shown in figure 4.15 is the AH clustering data in the same format as figure 4.12. Here the overall structure is more complicated than the $t < 10 \text{ ms}$ case. The two regions where we see few major discontinuities among a significant number of clusters are $737 \leq N_{Cl} \leq 890$ and $1000 \leq N_{Cl} \leq 1191$. To retain the largest fraction of the dataset, we choose the lowest value
Figure 4.13: Alternate representation of the $t < 10$ ms cluster populations. Derivative of cluster populations $\Delta N_{\alpha}/\Delta N_{\text{Cl}}$ for largest clusters, showing clearly the cluster splitting and cascades.

Figure 4.14: AH clusters for $t < 10$ ms data selected with $N_{\text{Cl}} = 330$ and $N_{\alpha} > 10$. 
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Table 4.1: A comparison of clustering methods for $t < 10$ ms data: the rows are the EM clusters EM:4.8:X and columns are the AH clusters AH:4.14:X.

$N_{Cl} = 737$ within these ranges which, for a minimum cluster population of 30 gives the clusters shown in figure 4.17. From table 4.2 we see the similarity between the EM and AH clusters represented by a sparse matrix, though in this case we have more deviations from the diagonal than in table 4.1. While there is only a small amount of cross-contamination between clusters, the two algorithms split clusters differently such that 2 clusters defined by one method may correspond to a single cluster defined by the other. For example, both clusters EM:4.10:48 and :55 are contained within AH:4.17:3258; similarly, EM:4.10:50 and :51 are each spread over two AH clusters (not including remainder). This difference in how fluctuation structures are distributed among clusters is partly controlled by the $N_{Cl}$ limit chosen for AH clustering.

### 4.5 Phase Structure of the Fluctuation Clusters

We have shown that the classes of fluctuations distinguished by the clusters are essentially the same regardless of which clustering algorithm is used. We now
look at the characteristic phase structure of the clusters, restricting the analysis to the $t > 10\text{ ms}$ results from the EM algorithm.

4.5.1 Phase Difference Between Nearest Neighbour Coils

As mentioned in section 3.3.2, the EM algorithm used here is implemented using sine and cosine phase components. A Gaussian distribution in phase space $\theta$ will be poorly represented by a Gaussian distribution in the projection $\sin(\theta)$ as $|\sin(\theta)| \to 1$. While this is not a problem in the high-dimensional space used for clustering, the information loss in the projected Gaussian fit is undesirable when considering the phase structure of the clusters. Rather, we return to the phase information of the constituent fluctuation structures in a cluster and find the mean and standard deviation of the phase angle for each nearest neighbour.
Figure 4.16: Alternate representation of the condensed cluster tree for \( t > 10 \) ms. Derivative of cluster populations \( \Delta N_\alpha/\Delta N_{Cl} \) for largest clusters, showing clearly the cluster splitting and cascades.

Figure 4.17: AH clusters for \( t > 10 \) ms data selected with \( N_{Cl} = 737 \) and \( N_\alpha \geq 30 \).
Table 4.2: A comparison of clustering methods for $t > 10$ ms data: the rows are the EM clusters EM:4.10:X and columns are the AH clusters AH:4.17:X.

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Although one of the advantages of this data mining procedure is that the
fluctuations can have arbitrary phase structures, it is instructive to show several cases where the phase structure has a simple interpretation. Figure 4.19 shows phase angle plots for poloidal Mirnov array #1 for four clusters from figure 4.10. The centre line corresponds to the cumulative mean phase of the coil pairs. The lines above and below are the cumulative cluster standard deviations of the coil pairs, where $\sigma^2_{1,n} = \sum_{j=1}^{n-1} \sigma^2_{j,j+1}$ for $\psi_{1,n} = \sum_{j=1}^{n-1} \Delta \psi_{j,j+1}$. Here, the magnetic angles have been evaluated for a flux surface at $r = 0.1 \, \text{m}$. We see from figures 4.19(b) and 4.19(d) that the clusters around the $\iota = 5/4$ rational surface have $m = 4$, and from figure 4.19(c) that the large cluster around the $4/3$ resonance has $m = 3$.

### 4.5.2 Fourier Analysis of Poloidal Mode Structure

To identify the nature of fluctuations with poloidal phase structure more complicated than those shown in the phase-angle plots of the previous section, the
Figure 4.19: Phase-angle plots for poloidal Mirnov array #1 for four clusters. The centre line corresponds to the cumulative mean phase of the coil pairs. The lines above and below are the cumulative cluster standard deviations of the coil pairs, where $\sigma_{1,n}^2 = \sum_{j=1}^{n-1} \sigma_{j,j+1}^2$ and $\psi_{1,n} = \sum_{j=1}^{n-1} \Delta \psi_{j,j+1}$.
non-uniform discrete Fourier transform (NDFT) for time sample \( t \) is used:

\[
P_t(m) = \sum_q p_{t,q} e^{-im\theta_q},
\]

where \( \theta_q \) is the poloidal magnetic angle of virtual coil \( q \), \( m \) is the poloidal mode number and \( p_{t,q} \) is the fluctuation structure signal at sample time \( t \) and virtual coil \( q \). The magnetic angle of a coil is a function of the minor radius at which it is evaluated (see section 2.3.4). Here we have evaluated the poloidal magnetic angles at a fixed minor radius of \( \langle r \rangle = 0.15 \text{ m} \).

Shown in figure 4.20 are the two clusters associated with the 4/3 resonant surface, both clusters have a strong \( m = 3 \) component, while cluster EM:4.10:55 also has large \( m = 0 \). Here the poloidal Fourier components are summed over all time samples within each 1 ms fluctuation structure, and the closest 10% of fluctuation structures to the cluster means are shown.

The clusters near the 5/4 rational surface are shown in figure 4.21, where we see that the \( m = 4 \) component dominates. Cluster EM:4.10:53, figure 4.21(a), shows a discrepancy between the two Mirnov arrays with an \( m = 0 \) component significant only in array 1. A large \( m = 7 \) component is detected by Mirnov array 1 in cluster EM:4.10:47, figure 4.21(b), which may be due to the presence of the 9/7 surfaces, however it is not prominent in array 2. The discrepancy here is likely due to the different coils sets used in the two arrays; while the non-uniformity of the coil distribution means we don’t have a clear-cut spatial Nyquist limit on the poloidal wavelength or mode number, we can estimate it by \( N_{\text{coils}}/2 \). Given \( N_{\text{coils}} = 12 \) and 13 for beams 1 and 2 respectively, we expect results for \( m > 6 \) to have diminished accuracy.

Besides the \( m = 0 \) cluster (EM:4.10:46, figure 4.22(f)), the remaining clusters in figure 4.22 do not show such well defined poloidal mode structure. This demonstrates the usefulness of this data mining technique in its ability to isolate
classes of fluctuations whose nature is not easily determined by a standard Fourier harmonic analysis, either due to the complexity of the geometry or localisation of the fluctuation.

4.5.3 Toroidal Phase Information

The analyses of the previous sections have focused solely on the poloidal phase structure of the fluctuations, although the cluster definitions include the phase differences between coils in different Mirnov arrays. Unfortunately, unresolved issues with the calibration of the linear array precludes the resolution of conclusive toroidal mode numbers. The remaining two bean shaped Mirnov arrays are separated by a field period and cannot resolve between toroidal mode numbers \( n \) and \( n \pm 3 \).

In figure 4.23 we see the toroidal phase difference calculated between the common coils \([1, 2, 3, 7, 8, 9, 10, 15, 17, 18]\) of Mirnov beans 1 and 2. Notwith-
standing the degeneracy in observable toroidal mode numbers, we see that the peak toroidal phase for clusters EM:4.10:5 and EM:4.10:6 are consistent with $n = 4$ and $n = 5$ expected for the $i = 4/3$ and $5/4$ respectively. In both cases there is a small offset between the observed peak in the phase distribution and the expected phase difference for the mode. For EM:4.10:46 there is a clear peak at $n = 0$; when combined with the poloidal phase information shown in figures 4.19(a) and 4.22(f) we find this cluster to have a $(n, m) = (0, 0)$ mode structure.

### 4.5.3.1 Origin of the Toroidal Phase Offset

To investigate the nature of the toroidal phase offset we consider the toroidal phase independently for each coil, and also look at its dependence on magnetic configuration. This is shown in figure 4.24 for two of the clusters in figure 4.23. For cluster EM:4.10:5, figure 4.24(a), most coil pairs have $\Delta \phi \simeq 2\pi/3$, as we expect from figure 4.23. The clear exception is coil 15, with coils 2 and 3 having
Figure 4.22: Poloidal Fourier components for other EM:4.10 clusters.

a broad range of values for some configurations. We see the same behavior of these coils for cluster EM:4.10:6, figure 4.24(b), with the addition of a phase offset in coil 17 and an offset and poorly defined phase, for different configurations, in coil 10.

We rule out any contribution to the phase offset from approximations or errors in the calibration of magnetic angles, as we only compare coils with ostensibly identical poloidal positioning. More likely, the observed difference between phase and expected value is due to positioning errors or a slight slippage of the beans relative to one another within the machine. Assuming a unidirectional poloidal phase velocity, a relative translation of the arrays, i.e.: a small vertical or horizontal displacement, will result in observed phase shifts of opposite signs for opposite
Figure 4.23: Toroidal phase differences between common coils in Mirnov bean arrays for clusters EM:4.10:5, EM:4.10:6 and EM:4.10:46.

sides of the array. Note that in figure 4.24(a) coils 17 and 18, and possibly 15, have a positive phase offset while the remainder have a negative offset. This is commensurate with a vertical displacement, although for confirmation we require the same behavior to be observed in figure 4.24(b) which does not hold for coils 1, 2 and 3, suggesting that such a simplistic model is not adequate. However figure 4.24 provides significant circumstantial evidence for the expected toroidal mode numbers despite complexity of the geometry.
4.6 Discussion

Perhaps the most obvious complication in the adaptation of the data mining procedure of chapter 3 to the configuration scan data is the treatment of the magnetic coordinate system. As discussed in section 4.3.5.2, we interpolate the fluctuation phase structure to ‘virtual coil’ angles which are independent of configuration to avoid $\kappa_h$ appearing as a hidden variable in the clustering results. In assigning magnetic coordinates to the Mirnov coils it is also necessary to assume a fixed magnetic surface (due to the bean shape changing with radius), in most cases here we have used the last closed flux surface, which will not necessarily correspond to the radial location of the fluctuation. The clustering process will be relatively insensitive to coordinate error due to radial displacement as long as the error is constant. For a fluctuation whose radial location varies with $\kappa_h$ we can expect coordinate errors to propagate through to the cluster assignments. In such a case, the mode structure interpretation of the cluster is more sensitive to coordinate errors; the high degree of noise in the poloidal and toroidal mode
structure plots of this chapter is likely due both to inaccuracies in the coordinate system and the finite radial width of the fluctuations.
CHAPTER 5

Analysis of Fluctuations in RF Powered Heliac Plasmas

5.1 Introduction

In the previous chapter we applied our data mining technique to magnetic fluctuation data from a magnetic geometry configuration scan in the H-1 heliac. We investigated the phase structure of the different classes of fluctuations found, but did not consider in any depth the physical nature of the modes. In this chapter we incorporate density profile information with the Mirnov signals to look more closely at our results. In section 5.2 we make several general observations of the plasma properties, such as $B$– and $n_e$–scaling and rotation. The interchange mode and global Alfvén eigenmode, sections 5.3 and 5.4 respectively, are considered as candidate models for the observed Mirnov fluctuations. We find that these models show some qualitative agreement with the Mirnov fluctuations, but require a scale factor for quantitative agreement; we therefore canvass further alternative models in section 5.5. Initial results from fully three-dimensional modelling of H-1 are discussed in section 5.6.
5.2 General Observations

It is important to consider the dependence of the observed Mirnov oscillations on plasma parameters, namely magnetic field, electron density and plasma rotation. Several scans of magnetic field were carried out in an experimental campaign separate to the $\kappa_h$ campaign that has been the main focus of this thesis. In one such scan the magnetic field was varied in the range $0.42 \leq B \leq 0.54$ T for three configurations: $\kappa_h = 0.28, 0.60, \text{ and } 0.90$.

It is not feasible to change the heating frequency, therefore a variation in $B$ cannot be decoupled from variation in the heating mechanism, due to the changing ion cyclotron resonance frequencies. The effect on confinement due to an off-resonance magnetic field has been detailed elsewhere [1, 68], where the electron density is shown to decrease rapidly as the field is moved away from resonance ($B_0 = 0.46$ T).

Shown in figure 5.1 are the observed Mirnov power spectra for the aforementioned $B_0$ scan. The time evolution of the line-averaged electron density (2 mm interferometer) is also shown for shots where the $n_e$ data is reliable. Inverted density profile and temperature information is not available for this scan. In general, frequency features appear to be inversely correlated with the field strength, and have no clear correlation with $\tilde{n}_e$. However, because other factors such as the location of the heating resonance layer in the plasma may be important in this scan, we take these results as inconclusive and state that a linear relation between $f$ and $B_0$, as assumed in this chapter for Alfvénic activity, remains plausible.

Using coherence imaging Doppler spectroscopy techniques [69], the bulk rotation in a weakly argon-doped hydrogen/helium plasma was measured to be $v_\theta \simeq 400$ m/s. This corresponds to a poloidal rotation of $f_\theta = v_\theta/(2\pi a) \simeq 320$ Hz.
and a Doppler shift between plasma and lab reference frames of $f_{\text{Doppler}} = mf_{\theta} \sim 1 - 2 \text{kHz}$ for the low-$m$ modes observed in H-1. The Doppler frequency is of the order of the frequency resolution of the fluctuation structures used here, and can be safely neglected in the analysis of this chapter.
Figure 5.1: Magnetic field scan of $B_0 = 0.42 \rightarrow 0.54$ T, top panels show power spectra of largest chrono from SVD of 11 channels for $20 < t < 40$ ms; line averaged electron density $\bar{n}_e$ is shown in bottom panels. Shots with missing $\bar{n}_e$ plots have unreliable $\bar{n}_e$ data.
5.2.1 Electron Density

The fluctuation structures for $30 < t < 50$ ms from our main dataset are shown in figure 5.2 with corresponding $\bar{n}_e$. Later in this chapter we will consider MHD activity which has frequency $f_{\text{MHD}} \propto |n - \bar{n}_m|$; we see in the bottom panel of figure 5.2 that the fluctuation frequency appears to show such linear behaviour when scaled to the Alfvén transit time

$$\tau_A = R_0 \sqrt{\mu_0 \rho / B_0}, \quad (5.1)$$

where $\rho = m_i \bar{n}_e$ and $m_i$ is the ion mass. Throughout this chapter we use $\mu = m_i / m_p = 2.5$ to represent the standard H-1 hydrogen/helium plasma with central normalised pressure $\beta_0 = 10^{-4}$, assuming $T_i = T_e = 20$ eV and $n \equiv n_i = n_e$ with $n_0 = 1.5 \times 10^{18} \text{ m}^{-3}$.

While the configuration scan overview appears to show Alfvénic density scaling, such behaviour is not always clear from the temporal evolution within individual shots. In part, this is due to a generally small value of $df / dt$ for a given mode in the $t \gtrsim 10$ ms plasmas, as well as variation in the density profile throughout a shot. Furthermore, the absence of a direct measurement of the radial structure of the fluctuation means that there is no clear choice of radial location for where the density profile should be sampled. In the following we compare a parametrised frequency scaling with data for a range of assumed radial locations. From the inverted ELSI electron density profiles\(^1\) we have temporal information about $n_e$ for 10 approximately equispaced surfaces in the plasma. For each surface we can look for localised Alfvénic scaling; assuming the relation

$$f_M = A n_e^x, \quad (5.2)$$

\(^1\)ELSI data provided by D. Oliver.
Figure 5.2: Mirnov frequency (centre panel) and line-averaged electron density (top panel) for H-1 configuration scan for $30 \text{ ms} < t < 50 \text{ ms}$. In the bottom panel the Mirnov spectra are shown normalised to the Alfvén transit time $\tau_A = R_0 \sqrt{\mu_0 \rho / B_0}$, with reduced vertical range to show the linear behaviour.
we find the exponent $x$ which minimises the residual error of a least squares fit.

Shown in figure 5.3 are fluctuation structures belonging to clusters EM:4.10:5 and EM:4.10:6 at $\kappa_h = 0.60$ and 0.47 respectively. For $\kappa_h = 0.60$ we see that for assumed localisation on surfaces $0.05 \, \text{m} \lesssim \langle r \rangle \lesssim 0.12 \, \text{m}$ we clearly have a best fit of $x = -1/2$, while for $\kappa_h = 0.47$ the surfaces $0.08 \, \text{m} \lesssim \langle r \rangle \lesssim 0.12 \, \text{m}$ appear similarly Alfvénic.

We use the ELSI density inversion data cautiously; the resulting inverted profiles are quite sensitive to boundary conditions, and ambiguities are present due to the similar time scales of the interferometer sweep (1 ms) and density profile evolution when density varies quickly. The resolution of these issues is the focus of another PhD (D. Oliver); here we will simply note when necessary that care should taken in the interpretation of the inversion data.

### 5.2.2 Features of Mirnov Spectra in the $\kappa_h$ Scan

The data mining treatment of the magnetic fluctuation $\kappa_h$ scan dataset in chapter 4 revealed an assortment of modes, many of which are associated with low-order $(n, m)$ resonances in rotational transform. We now give an overview of the features of the Mirnov spectra in the $\kappa_h$ scan.

There are several modes which appear to belong to the same family as the main $(4, 3)$ and $(5, 4)$ ‘whale-tail’ resonances. The clusters containing these modes are shown in the top panel of figure 5.4. Clusters EM:4.10:48 and EM:4.10:55 belong to the $(4, 3)$ resonance, with clusters EM:4.10:47 and EM:4.10:53 belonging to the $(5, 4)$ resonance. Cluster EM:4.10:50 contains fluctuations associated with the $(6, 5)$, $(9, 7)$ and $(7, 5)$ resonances, these modes have similar coordinates in $\Delta \psi$-space and are seen to split into separate clusters at a level of the cluster tree higher than is shown in figure 4.10. Cluster EM:4.10:54 is associated with the
Figure 5.3: Best fit of electron density to $f_M$ using equation 5.2 for each surface in the reconstructed density profile. For each surface the least squares fit for $f_M = A n_e^{-1/2}$ (-) is compared to $f_M (x)$. 

(a) Shot 58066, $\kappa_h = 0.60$. Mirnov frequencies are those in cluster EM:4.10:5 for this shot.

(b) Shot 58063, $\kappa_h = 0.47$. Mirnov frequencies are those in cluster EM:4.10:6 for this shot.
Figure 5.4: Classes of modes in the $\kappa_h$ configuration scan. Top panel shows the ‘whale-tail’ type resonance modes, the middle panel shows other well defined modes.

$(7, 6)$ resonance. The characteristic features of these modes are that they have $f \sim |\nu - n/m|$ and, as shown in section 4.5.2, they generally have a dominant poloidal mode number $m$ corresponding to the rotational transform resonance.

In the middle panel of figure 5.4 are the two remaining well-defined clusters from figure 4.10. Cluster EM:4.10:46 contains the $(n, m) = (0, 0)$ modes which generally have low frequency and are associated with configurations near zero-shear rational surfaces which have poor confinement (see figure 2.6). Cluster EM:4.10:51 has a different spectral signature to the modes in the top panel – it appears associated with the $(7, 5)$, resonance however it lacks the $f \sim |\nu - n/m|$ behaviour and has distinct frequencies either side of the zero-shear resonance configuration at $\kappa_h \simeq 1.03$.

The magnitude of the magnetic fluctuations at the location of coils 1 and 8 in Mirnov array 1, $dB_{\text{Mirnov}}$, is shown in figure 5.5. For coil 1 we see that the fluctuations are, in general, approximately 20 mG; while at coil 8, which is closer
to the plasma in all configurations the fluctuations are around 100 mG. If we assume the magnetic perturbation to be an order of magnitude greater within the plasma than at coil 8, we have \( dB_{\text{plasma}} \sim 1 \text{ G} \), giving \( dB/B \sim 2 \times 10^{-4} \). While an absolute measurement of \( dB \) in the plasma is not possible, the direct ratio \( dn_e/dB \) is more than an order of magnitude greater for low frequency modes \((f \sim 1 - 10 \text{ kHz}) \) than for the ‘whale-tail’ modes shown here [68], suggesting the low frequency modes show drift-wave characteristics compared to the higher frequency MHD activity.

### 5.3 The Interchange Instability

The presence of interchange mode activity is a matter of concern for all magnetic confinement devices with significant pressure gradient. In this section we follow the procedure for the analysis of ideal interchange instabilities described in reference [30]. We first consider an analytic case made tractable by omitting the stabilising shear and magnetic well, followed by numerical simulations which...
restore these features in a cylindrical H-1 model. We then examine the radial structure of the resonant and non-resonant modes, as well as dependences on \( \beta \) and pressure profile. Finally, we discuss the viability of the interchange mode as a candidate model for the fluctuations observed in H-1.

5.3.1 Analytic Case for Stellarator Field with Zero Shear

We now follow the procedure in reference [30] for a stellarator field with zero shear using H-1 parameters. We use the ideal reduced MHD equations with normalised variables (see appendix C for details) to obtain the ordinary differential equation:

\[
\frac{d^2 u}{dr^2} + \left( \frac{1}{r} - \frac{2m' (n - mu)}{\gamma^2 + (n - mu)^2} \right) \frac{du}{dr} - \left\{ \frac{m^2}{r^2} + \frac{1}{\gamma^2 + (n - mu)^2} \right\} u = 0
\]

(5.3)

where \( u \) is stream function with mode numbers \( m \) and \( n \), \( \gamma \) is the growth rate, and \( D_s \) and the averaged helical curvature \( \Omega \) are:

\[
D_s = -\frac{\beta_0}{2\epsilon^2} \rho' \Omega',
\]

(5.4)

\[
\Omega = \epsilon^2 N \left( r^2 + 2 \int r dr \right),
\]

(5.5)

where \( \epsilon \) is the inverse aspect ratio, \( N \) is the number of field periods and the prime represents derivative with respect to the argument.\(^2\)

In the case of zero shear, \( \epsilon = 0 \), and parabolic pressure profile \( p = p_0 (1 - r^2) \) an analytic solution to 5.3 exists in the Bessel function form \( u \propto J_m(\tilde{r}) \) with growth rate given by

\[
\gamma^2 = \frac{\tilde{D}_s m^2}{Z^2(m, k)} - (n - mu)^2.
\]

(5.6)

\(^2\)When no argument is given assume \( x' = x'(r) \).
and where a radial transformation has been used:

\[ \tilde{r} \equiv \{ \tilde{D}_s m^2 / \left[ \gamma^2 + (n - m\epsilon)^2 \right] \}^{1/2} r \]  

(5.7)

where \( \tilde{D}_s = 4\varepsilon_0 N \), and \( Z(m, k) \) is the \( k \)th zero of the Bessel function of the first kind \( J_m(\tilde{r}) \).

We now consider two simplified H-1 models with zero shear, one where we take the rotational transform to be the value on axis with the axis \( \epsilon_0 = \epsilon(r = 0) \), and the other where we take the minimum value in the \( \epsilon \) profile. For non-monotonic profiles this corresponds to the value at the zero shear radius \( \epsilon_{zs} = \epsilon(\epsilon' = 0) \).

Figure 5.6 shows the zero-shear analytic solution for the \((n, m) = (4, 3), (5, 4)\) and \((6, 5)\) interchange modes; from \( \text{Re}(\gamma) \) we see that the modes are unstable in a region about \( \epsilon = n/m \), while further from the resonance configuration they are marginally stable and the frequencies (see \( \text{Im}(\gamma) \)) show the familiar ‘V’-shaped resonance structure seen in figure 5.2.

As discussed on page 26, the addition of shear has a stabilising effect on interchange modes because of the energy required to bend the field lines as flux tubes are interchanged. In the H-1 configurations considered here the magnetic shear approaches zero at the axis and, for the non-monotonic \( \epsilon \) profiles at \( \kappa_h \gtrsim 0.3 \), there is a zero-shear location away from the axis, within the plasma volume. Although the interchange mode is not shear-stabilised at these zero-shear surfaces, we see from figure 5.7 that for the configurations used here, the zero-shear surfaces remain in regions stabilised by the magnetic well.

### 5.3.2 Ideal Interchange Modes in a Cylindrical H-1 Model

We now use a power series expansion of the actual H-1 \( \epsilon \) profiles computed with the HM3 model (see section 2.3.2.2) and a more accurate expression for helical
Figure 5.6: Ideal interchange spectrum for H-1 configurations with $\iota' = 0$. Values are taken from transform at axis $\iota_0 = \iota(r = 0)$ (solid lines) and zero shear $\iota_{zs} = \iota(r = r(\iota' = 0))$ (dashed lines).
Figure 5.7: Radial locations of the magnetic well/hill boundary (o) and outermost zero shear surface (>). The region where the zero-shear surface approaches the axis is quite sensitive to the HELIAC model used, we see from the transform profiles, e.g.: figures 2.3 and 2.6, that a large centre region has very low shear. Therefore we cannot clearly define the configuration where the $\iota$ profile transitions from monotonic to reverse-shear, though evidently it is in the region $0.3 \lesssim \kappa_h \lesssim 0.4$. 
curvature to compute the ideal interchange spectra across the configuration scan.

We express the average helical curvature $\Omega$ as [70]:

$$\Omega' = \epsilon r B_0^2 V''(\psi),$$  \hspace{1cm} (5.8)

where $V'(\psi) = \int dl/B$ is the specific volume enclosed by a flux surface. The specific volume and magnetic well ($V''$) as calculated with the HM3 model are shown in figure 2.4

The spectra for a parabolic pressure profile were calculated using the 4th-order Runge-Kutta technique with the shooting method; the resulting eigenvalues are shown in figure 5.8. There are no unstable modes for these conditions. The $\text{Im}(\gamma)$ eigenmodes spread to continuous spectra due to finite shear, with a non-resonant mode sitting just outside the continuum spectrum. For the case of the (4,3) resonance shown in figure 5.8(a), the non-resonant mode has a frequency offset at $\kappa_h = 0.8$ of $\Delta \gamma \approx 1.5 \times 10^{-3} i$ below the continuum.

The (4,3) non-resonant modes for $\kappa_h = 0.5$ and $\kappa_h = 0.8$ are shown in figures 5.9(a) and 5.9(b) respectively, with several modes within the $\kappa_h = 0.8$ continuum shown in figure 5.9(c). On comparison with figure 5.7 we see that the eigenmodes of figure 5.9(a,b,d) are peaked at the zero-shear radius. The range of the continuum is defined by the zero denominator in equation 5.3 at $\gamma^2 + (n - m \iota)^2 = 0$, and is therefore defined by the range of $\iota$ in the plasma radius. We note that for the case of unstable modes where $\gamma^2 > 0$ (not seen here) this continuum is not present as $(n - m \iota)^2$ is positive definite. Shown in figure 5.9(d) are non-resonant eigenmodes for the (5,4) and (6,5) modes at $\kappa_h = 0.50$ and $\kappa_h = 0.35$ respectively.

On comparison with figure 5.6 we see that the eigenmodes seen in figure 5.6 are similar to the zero-shear modes found analytically using $\iota_{zs}$. In both cases
the mode frequencies are around three times greater than the frequencies of the observed Mirnov fluctuations, as shown in the bottom panel of figure 5.2. The same scaling problem is found in the Alfvén eigenmode analysis of section 5.4 and is discussed in detail in section 5.4.1.

5.3.3 $\beta$ Thresholds of the Resonant Ideal Interchange Mode

We now examine the thresholds for stability of the ideal spectra investigated above. It can be shown [30, 38] that, on a resonant surface, the indicial equation for equation 5.3 provides the Suydam criterion for instability $\gamma^2 > 0$

$$\frac{D_s}{\nu^2 r_s^2} < \frac{1}{4}, \tag{5.9}$$

where $D_s$ and $\nu'$ are evaluated at the resonant surface $r = r_s$. Within the magnetic well ($V''(\psi) < 0$) region we see from equations 5.4 and 5.8 that $D_s < 0$ for a plasma with monotonic pressure profile, and so the resonant modes are stable. Figure 5.10 shows the H-1 configurations in which low order rational surfaces exist in the magnetic hill region where they can be destabilised. From this it is clear that resonant interchange modes are not able to provide the symmetry about resonant values of $\kappa_h$ required for our observed “whale-tail” structures; however,
we proceed to investigate their $\beta$–limits for comparison with the non-resonant
modes.

Rearranging equation 5.9 we find the Suydam stability threshold $\beta_s$

$$\beta_s = -\frac{e\ell^2 r_s}{2\rho'B_0^2 V''(\psi)}.$$  \hspace{1cm} (5.10)

A comparison between the analytic (equation 5.10) and numerical thresholds for
the (4,3) mode at $\kappa_h = 0.5$ is shown in figure 5.11(a) and shows good agree-
ment. A small positive growth rate for the numerical model is found for $\beta_0$
about 20% higher than the analytic threshold, and above this the growth rate in-
creases markedly. As the value of $\beta_0$ in the numerical model is reduced to $\beta_S$
the eigenmode tends toward a delta function about the rational surface, which raises
difficulties in numerical modelling but makes the analytic evaluation especially
convenient.

In figure 5.11(b) the analytic value of $\beta_S$ is given for low order resonant modes
through the relevant range of $\kappa_h$. As $\kappa_h$ is increased, the increasing magnetic
hill magnitude at the plasma edge decreases $\beta_S$ while the increasing magnetic
shear increases $\beta_S$; the general trend of decreasing $\beta_S$ with $\kappa_h$ shows that $V''(\psi)$
increases faster than $\epsilon^2$. The decreasing span of each mode in configuration space
as $\kappa_h$ is increased is due to the narrowing radial width of the magnetic hill region,
cf. figure 5.10.

5.3.4 $\beta$ Thresholds of the Non-resonant Ideal Interchange Mode

We now consider the $\beta$ limits for stability against the non-resonant interchange
mode. On comparison of the unstable modes from the analytic interchange case
(top panel, figure 5.6) and the observed dependence of confinement on $\kappa_h$ (figure
2.6), we note that the non-resonant mode is a candidate for the poor confinement
in configurations where the zero-shear rotational transform value is close to rational. In figure 5.12(a) the dependence of growth rate on $\beta_0$ for the $(4,3)$ mode at $\kappa_h = 0.75$ is shown where we see that the threshold value is $\beta_0 = 3.1 \times 10^{-2}$, much higher than the pressure achieved in H-1 in these experiments. The configurational dependence of the $\beta$-threshold for the $(4,3)$ and $(5,4)$ non-resonant modes are shown in figure 5.12(b), along with the Suydam limits for the corresponding resonant modes. We see that the non-resonant mode generally has a greater threshold than the resonant mode. Again, the stability limits shown here are beyond the pressures attainable in H-1, so within the limitations of this model we can rule these out of having any contribution to the observed MHD activity.

5.3.5 Discussion of the Interchange Mode Analysis

We now discuss several limitations to the present interchange mode analysis. Firstly, we have restricted ourselves to cylindrical coordinates, a crude approximation to the heliac geometry. However, our qualitative understanding of the spectra provided by the cylindrical model is sufficient to rule out resonant interchange modes as a candidate for the observed MHD activity, simply because the resonant mode spectra are not present on the high-$\kappa_h$ side of the ‘whale-tail’ resonance structures. We expect variation in the spectrum upon consideration of full 3D geometry; while we do not attempt that here, in section 5.6 we consider a similar physical model for the case of Alfvén eigenmodes and compare the CAS3D results for H-1 to cylindrical coordinate results.

Pressure profiles parabolic in $\langle r \rangle$ have been used throughout this section; while we do not have accurate temperature profiles for this dataset we can use the ELSI electron density profiles to better approximate the pressure profiles. Our interest here lies in the possibility of $\beta$ thresholds being reduced on application of more
realistic pressure profiles. From equation 5.10, we see that for the resonant inter-
change, a larger pressure gradient will reduce the stability threshold. A typical
ELSI profile with a steep edge, as described in appendix E, is shown in figure
5.13, and we see that at the outer third of the radius it matches the normalised
parabolic pressure profile. If we assume identical temperature and density profiles
then we have $|p'| \sim 1$ in this region, around half that of the assumed parabolic
profile. A much larger gradient would be required to substantially affect the
stability threshold, we therefore conclude that the assumed parabolic profile is
likely to decrease the $\beta$ threshold when compared to the pressure profiles esti-
mated from ELSI data, i.e.: use of a more realistic pressure profile will not reduce
the critical $\beta$ to a value attainable in H-1.

As discussed in section 2.2.2, we expect growth rates of resonant modes to
be altered when resistivity is included in the model. The transverse Spitzer
resistivity $\eta_\perp$ is

$$\eta_\perp = 1.03 \times 10^{-2} Z \ln(\Lambda) T^{-3/2} \Omega \text{ cm}, \quad \text{(5.11)}$$

where $Z$ is the ion charge, $\ln(\Lambda)$ is the Coulomb logarithm (the factor by which
small angle collisions between charged particles are more effective than large angle
collisions), and the temperature $T$ is measured in electron-volts. For the presently
studied H-1 conditions we have $Z = 1, T_e \approx 20 \text{ eV}$ and $\ln(\Lambda) \approx 24 - \ln(n_e^{1/2} T_e^{-1}) \approx 13$ for electron-ion collisions, giving $\eta_\perp \approx 1.5 \times 10^{-3} \Omega \text{ cm}$. The plasma resistivity
is often expressed as the dimensionless magnetic Reynolds number, or Lundquist
number $S$, which is the ratio of the fluid flow and magnetic diffusion velocities:

$$S = \frac{\mu_0 a V_A}{\eta}, \quad \text{(5.12)}$$

where the minor radius $a$ and Alfvén velocity $V_A$ represent the typical length
and velocity scales of the system. For $V_A \approx 5 \times 10^6 \text{ m s}^{-1}$ we get $S \approx 8 \times 10^4$. 

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This is a small value in the context of fusion devices and describes a resistive, low-temperature and high density plasma; for high-temperature plasmas $S \sim 10^7$ and for a reacting plasma in a fusion reactor $S \sim 10^9$.

The pursuit of an accurate model of the resistive MHD instabilities in H-1 is beyond the scope of this thesis. While finite resistivity allows interchange modes to overcome shear stabilisation, it does not destroy magnetic well stabilisation. We have seen that the limits in $\kappa_h$-space to where resonant interchange modes can be observed are determined by the relative locations of the rational surface and the edge magnetic hill; while the growth rates will vary with resistivity, their location in $\kappa_h$ configuration space will not. The addition of resistivity to the model affects the “skin depth” region on either side of the resonant surfaces. However, as the observed modes span configurations in $\kappa_h$ both above the $\iota = n/m$ resonance (absence of rational surface) and below it (rational surface within the plasma volume) it is clear that the mode itself is not due to the presence of the rational surface in the plasma volume, and we deduce that the addition of resistivity to the model would not even account qualitatively for the general “whale-tail” $\kappa_h$-scan spectral characteristics.
(a) $(4,3)$ mode at $\kappa_h = 0.5$, $Im(\gamma) = 0.1644$.

(b) $(4,3)$ mode at $\kappa_h = 0.8$, $Im(\gamma) = 0.0535$.

(c) $(4,3)$ continuum modes at various frequencies at $\kappa_h = 0.8$. These modes lie in the blue region in figure 5.8(a).

(d) $(6,5)$ at $\kappa_h = 0.35$, $Im(\gamma) = 0.1758$ and $(5,4)$ at $\kappa_h = 0.5$, $Im(\gamma) = 0.1148$.

Figure 5.9: Non-resonant eigenmodes (a,b,d) and continuum modes (c) for the cylindrical H-1 model.
Figure 5.10: Location of resonant surfaces lying in the magnetic hill region.

Figure 5.11: Suydam stability thresholds $\beta_S$ for resonant modes.
(a) Numerical calculation of $\beta_0 = 3 \times 10^{-2}$ for threshold of the (4, 3) non-resonant eigenmode at $\kappa_h = 0.75$

(b) Comparison of $\beta$ thresholds of resonant (solid lines) and non-resonant (o) modes, (4, 3) and (5, 4).

Figure 5.12: $\beta$ thresholds of non-resonant modes.

Figure 5.13: Comparison of pressure profiles: H-1 and parabolic.
5.4 Global Alfvén Eigenmodes

The importance of the global Alfvén eigenmode (GAE) to the confinement of toroidal plasmas has been discussed in section 2.2.3.2; in this section we investigate the extent to which the GAE model can be used to explain the fluctuations observed in the configuration scan. The GAE frequency lies just below the Alfvén continuum which, in a cylindrical model, gives
\[ f_{\text{GAE}} \lesssim \frac{1}{2\pi} k_\| v_A, \]
where \( k_\| = m|\ell - n/m|/R \), \( v_A = B/\sqrt{4\pi\rho} \) is the Alfvén velocity, \( \rho \) is the mass density, and \( R \) is the major radius. We are led to investigate the GAE as a candidate model by the \( \ell = n/m \) resonance in the dispersion relation; firstly, because near resonance the mode frequency is much lower, and closer to our observations, and secondly, the observed frequencies have a local minimum near resonances. This would reproduce the observed features if \( \ell \) were constant across the plasma. The radial variation of \( \ell \) creates complications which require further consideration.

For a given configuration we find a collection of modes corresponding to the various low order \((n, m)\) resonances. The Alfvén continua for a cylindrical model for a range of \( \kappa_h \) configurations are shown in figure 5.14. If a configuration has a rational surface within the plasma volume then equation 5.13 contains a root such that \( \min(f_{\text{GAE}}) = 0 \) (for example, see the \((9, 8), (8, 7), (7, 6), \) etc... modes in figure 5.14(a)). In H-1, this occurs on the low \( \kappa_h \) side of the resonance, so the left arm of the “whale-tail” shape would be lowered to the axis \( (f = 0) \). Therefore, the \( \kappa_h \)-scan GAE spectrum for \( \min(f_{\text{GAE}}) \) will appear as shown in figure 5.15(a), which is not in agreement with our observations. If we instead assume for these configurations a localisation of the mode away from the rational, near the station-
Figure 5.14: Lower limit of $(n, m)$ Alfvén continua in a cylinder for a parabolic pressure profile in various $\kappa_h$ configurations with $1 \leq n, m \leq 10$. Central density of $n = 1 \times 10^{18} \text{m}^{-3}$ is used for a hydrogen/helium plasma ($\mu = 2.5$).

For any point (the local maximum in $f_{\text{GAE}}$, e.g.: (4, 3) in figure 5.14(d) at normalised radius 0.6) we retain the ‘whale-tail’ resonance spectra in the configuration scan, as shown in figure 5.15(b).

We now examine the (4, 3) whale-tail mode with respect to our simple cylindrical GAE model. Figure 5.16 shows the observed Mirnov frequencies from cluster EM:4.10:5 scaled by $n_e^{1/2}$ compared to $n_e^{1/2} \times f_{\text{GAE}}$ scaled by a factor $\lambda = 0.27$, determined by minimising the root mean square discrepancy between $f_M$ and $\lambda f_{\text{GAE}}$ (with assumed mass $\mu = 2.5$). Here, the mode is plotted against $\iota$ rather than $\kappa_h$, where the $\iota$ used for each datapoint is evaluated at the radius of min($f_{\text{GAE}}$) or local max($f_{\text{GAE}}$) (cf. figure 5.15(b)), and a small correction $\Delta \iota = -7.17 \times 10^{-3}$.
is applied to the HM3 transform values to minimise the RMS difference between the data and the fit. We have taken the \( n_e \) value at the same radial location as the transform value, using ELSI inversion data. In the lower panel we see the \( n_e \) dependence of the same model and data. With the scale factor \( \lambda \) included, there is good agreement between observed frequencies and the simple cylindrical GAE model for \( \iota \) and density scaling.

The (5, 4) mode does not conform as closely to the simple \( f = \lambda f_{\text{GAE}} \) interpretation as the (4, 3) mode. We see from the \( f_M \times \sqrt{n_e} \) scaling in the lower panel of figure 5.2 that the mode spectrum has a small, but clear, discontinuity at \( \kappa_h \sim 0.4 \) which cannot be explained with a rotational transform that is a continuous, slowly varying function of \( \kappa_h \). One may consider that, rather than lying below \( \lambda f_{\text{GAE}} \), the modes might have a dispersion relation where the relevant rotational transform value is \( \iota_{zs} \). Away from the plasma edge, where \( v_A \rightarrow \infty \)
Figure 5.16: Comparison of $f_M$ and $f_{GAE}$ for the (4,3) mode. Here, $A = \frac{m}{2\pi R} \nu_A$ where $\nu_A = n_1^{1/2} v_A$, and $\lambda = 0.27$. The top panel shows the selection of this value of $\lambda$ as that which minimises error between $f_{GAE}$ and the observed Mirnov fluctuation frequency. A transform offset of $\Delta \iota = -0.00717$ has been used.

As $\rho \to 0$, the radial location of $\text{min}(f_{GAE})$ is largely determined by the location of $\iota_{zs}$ for non-monotonic $\iota$ profiles, which means that such $\iota_{zs}$-modes would show a similar $\kappa_h$ dependence to the GAEs and would also be unable to explain the discontinuities in the modes at $\kappa_h \gtrsim 0.4$.

However, such discontinuities can be explained if we assume the modes to have a fixed radial location when their associated resonant surfaces are not present in the plasma volume. Taking the rotational transform value at either the fixed radius ($\iota = n/m$ not in plasma volume) or the zero-shear radius (when $\iota = n/m$ is within the plasma volume) we find the parallel wavenumber $k_\parallel = m|\iota - n/m|/R$ to have good agreement with the spectra, as shown in figure 5.17.

We see in figure 5.17 that the zero-shear / fixed radius interpretation of $k_\parallel$
agrees with modes other than the (5, 4) mode. For the (4, 3) mode, the radial zero-shear location is close to the fixed radius used here \( \langle r \rangle = 0.15 \text{ m} \) such that there is only a slight discontinuity between the two sides of the ‘whale-tail’ resonance and so, to a good approximation, figure 5.16 remains valid for this interpretation. The (6, 5) mode is also in agreement, with a discontinuity at \( \kappa_h \sim 0.24 \) which is larger than that of the (5, 4) mode due to the increasing shear at \( \langle r \rangle = 0.15 \text{ m} \) for lower \( \kappa_h \). The (7, 6) mode, shown in part for \( 0.16 \leq \kappa_h \leq 0.23 \), somewhat matches the \( k || \) for this model; it follows the same gradient as \( k || \) but has small offset which could be due to slightly different position of the fixed radius for the mode.

For the model to match the (7, 6) mode frequency we require a smaller value
of \( \iota \) than shown in figure 5.17, i.e.: the radial location at which we evaluate \( \iota \) needs to be closer to the axis than \( \langle r \rangle = 0.15 \text{ m} \) (normalised average minor radius = 0.75). From figure 5.7 we see that the magnetic well/hill boundary moves towards the axis by \( \sim 10 - 15\% \) compared to the slowly varying boundary location at normalised radius \( \sim 0.8 \). A quantitative study of the correlation between the magnetic well boundary and the mode frequency is subject to the accuracy of the HELIAC vacuum field model (where the well boundary location is more sensitive to error fields than the rotational transform), and will be addressed in future work.

5.4.1 The Scale Factor \( \lambda \)

While the observed modes show some qualitative agreement with \( k_{||} \), quantitative agreement with \( f_{\text{GAE}} \) is found only with the inclusion of a correction factor \( \lambda \) applied to the frequency, such that \( f_{M} = \lambda f_{\text{GAE}} \). In figure 5.16 we have applied a correction factor of \( \lambda = 0.27 \) to the cylindrical GAE model frequency spectra, whereas in figure 5.17 the difference between \( f_{M} \sqrt{n_{e}} \) and \( k_{||} \) give a frequency scaling factor of \( \lambda = 0.2 \). Note that the \( \Delta \iota \) correction in figure 5.16 accounts for most of the difference between these two scaling factors.

From equation 5.13 we find 3 quantities where the scale factor could possibly be absorbed: \( n_{e} \sim \lambda^{-2} \), \( \mu \sim \lambda^{-2} \), and \( k_{||} \sim \lambda \). We have confidence in the measurement of \( n_{e} \) through the agreement of the separate ELSI and 2 mm interferometers and so rule out the possibility that inaccuracies in \( n_{e} \) measurements make any significant contribution to \( \lambda \). Also, while small populations of impurities (O, C, Ar) are measured in H-1, it is implausible that a modified value of \( \mu \) is responsible, as that would imply an ion mass of \( \mu' = \lambda^{-2} \mu \sim 35 \) for the bulk plasma, much greater than could be attained by an impurity-induced increase
in effective mass. A correction to $k_{||}$ is clearly required due to the difference between cylindrical and H-1 geometry and can be expected to contribute to $\lambda$; the extent to which it does is discussed in section 5.6 with reference to present CAS3D modelling of modes in H-1.

### 5.4.2 Other Alfvén Eigenmodes

Aside from the GAE, other Alfvén eigenmodes caused by poloidal/toroidal couplings exist and are described by equation 2.7 and table 2.1. We now consider the possibility that the observed fluctuations that are not part of the main whale-tail resonance structures (e.g.: cluster EM:4.10:51) may be of such type. Shown in figure 5.18 are eigenmodes with $|\delta_{n,m}| \leq 5$, calculated with the same parameters (i.e.: parabolic profile) as previous examples. The only modes existing in the frequency range $f < 200\, \text{kHz}$ are of the helical Alfvén eigenmode (HAE) type, with $|\delta_n|, |\delta_m| > 1$. Toroidal Alfvén eigenmodes (TAE) commonly observed in other devices are generally not seen in the heliac geometry. This is because the very low shear prohibits $(n, m)$ and $(n, m + 1)$ rational surfaces from coexisting in the plasma volume (for low values of $n$ and $m$). We generally expect the strongest mode couplings to be those which correspond to the largest Fourier components of the H-1 geometry; namely $(n, m) = (\delta_n N_{fp}, \delta_m) = (0, 1)$ and $(3, 1)$, with smaller $(3, 0)$ and $(3, 2)$ smaller again at 10% to 20% of the larger components. Of these, only the $(3, 2)$ HAE modes are found in figure 5.18.

Of the modes in figure 5.18, the $\{(n, m), (\delta_n, \delta_m)\} = \{(7, 5), (1, 2)\}$ and $\{(10, 7), (1, 2)\}$ near the lower limit of their frequency range resemble somewhat the observed modes in cluster EM:4.10:51. For a quantitative comparison between theory and experiment we require the actual, rather than parabolic, density profiles. Unfortunately, the available density profiles reconstructed from ELSI data
become less precise as \( \kappa_h \) is increased due to inaccuracies in the implementation of the Gourdon code \cite{71} used in the reconstructions, and can be unreliable for configurations at \( \kappa_h \gtrsim 0.6 \) \cite{72}. In the absence of time resolved profile information, we choose an ELSI profile which is typical of the configuration scan and scale it with the time-resolved 2 mm interferometer density measurements. Shown in figure 5.19 is the ELSI reconstruction for the profile, as well as a least squares fit parametrisation which we use. The equation for the parametrisation is given in appendix D. We can justify using a single density profile shape because the modes of most interest lie in a restricted region of configuration space \( 0.8 \lesssim \kappa_h \lesssim 1.1 \), where we expect variation in the profile to be minor. The main determinant for profile shape is the presence of low order rational surfaces; for these configurations, the dominant \( (3, 2) \), \( (4, 3) \) and \( (5, 4) \) are absent. There may be a minor perturbation to the profile shape due to the \( (7, 5) \) rational, however we expect such an effect to be smaller than that due to the use of simple cylindrical geometry in our model.

A comparison between \( f_M \) and the \( (n, m)(\delta_n, \delta_m) = (7, 5)(1, 2) \) cylindrical model HAE for three H-1 shots is shown in figure 5.20. Here, the typical ELSI profile of figure 5.19 has been used with ion mass \( \mu = 2.5 \). We see in each case that \( \lambda_{HAE} f_{HAE} \) matches the magnetic fluctuations quite well, and that the time evolution of \( f_M \) follows the Alfvénic scaling. It is not surprising that the cylindrical model does give the exact frequency of the observations; here the scale factor \( \lambda_{HAE} = 0.8 \) has been used. In section 5.6 we see that similar modes modelled by CAS3D show a comparable reduction in frequency when a cylindrical model is replaced with full three-dimensional geometry which suggests that our 20% offset between the cylindrical HAE model and observed frequencies is in the range one should expect.
Figure 5.18: Other Alfvén eigenmodes. In this frequency range, only HAEs are present. Labels represent pairs of coupled \((n, m)\) modes.

Figure 5.19: Parametrisation of a typical ELSI profile, shot 58082 at \(\kappa_h = 0.92\), see appendix D for details.
Figure 5.20: Timeseries comparison of $f_M$ and $\lambda_{HAE} f_{HAE}$ for shots at $1.03 \leq \kappa_h \leq 1.05$. The density profile in figure 5.19 has been used with $\mu = 2.5$, and the scale factor is $\lambda_{HAE} = 0.8$. The top panel shows fluctuation structures as used in chapter 4 with colour and size of $f_M$ proportional to normalised energy. The lower panel shows combined power spectra of the four largest singular values for each 1 ms time segment, with size and colour proportional to signal power.
5.4.3 Alfvén Mode Instability Drive

The Alfvén eigenmodes described above are normal modes of a toroidal plasma which require a driving force to become unstable. The free energy source most commonly associated with the destabilisation of Alfvén eigenmodes is that of a fast particle pressure gradient, where the fast particles with velocity $v_p$ are resonant with the Alfvén velocity $v_A$.

Particles in a stellarator geometry are generally classified as either trapped or passing, where trapped particles are those localised in the torus due to ripples in the magnetic field. The pitch angle between the particle trajectory and the field defines a loss-cone in velocity space, particles with $v_\perp^2/|v|^2 < B/B_{\text{max}}$ can pass through the strong field region $B_{\text{max}}$ untrapped. Particles trapped by the toroidal ripple are called blocked particles, while those trapped by the helical ripple are known as localised particles and have a shorter orbit. For passing particles, excitation of Alfvén eigenmodes has been observed using the sideband resonance $v_p/v_A = 1/3$ as well as the fundamental resonance $v_p/v_A = 1$ [43, 41].

For the H-1 conditions, $B_0 = 0.46\,T$, $\beta_0 = 10^{-4}$ and $\mu = m_i/m_p = 2.5$ we have an Alfvén velocity $v_A \simeq 5 \times 10^6\,\text{ms}^{-1}$. A hydrogen ion at this velocity has energy of approximately 130 keV, or 43 keV at the $v_A/3$ sideband. As this is several orders of magnitude greater than $T_i$ we can clearly discount any contribution from thermal ions to this resonance. If such fast ions are present in H-1 they are likely to be produced by the ICRF heating, as has been observed in other devices with hydrogen-minority ICRF heating [38, 41], and since ICRF supplies energy perpendicular to the field the fast ions will mostly be trapped ($v_\perp \gg v_\parallel$). In H-1, these ions, with $v_T \sim v_A$, have a gyroradius which is a large fraction of the minor radius and, as seen in figure 5.21, are very poorly confined. At the present time, H-1 does not have the capability to detect fast ions.
Figure 5.21: Trajectories of 100 keV $H^+$ ions with pitch angle $\Theta_p = 80^\circ$ at the $\kappa_h = 0.5$ configuration. The red trajectory follows an ion launched at $\phi = 0$ near the axis at $R = 1.25$ m and remains in the plasma volume for only a few gyro-orbits. The blue trajectory, launched at $R = 1.4$ m, does not complete a single gyro-orbit.

Trapped particles, as well as passing particles, are capable of resonantly destabilising Alfvén eigenmodes. One mechanism by which trapped particles can drive Alfvén modes is through the matching of the precessional drift velocity of the particle orbit with the Alfvén velocity [43]. This condition implies that the trapped particles are highly energetic, with $v_T \gg v_d \sim v_A$, where $v_d$ is the precessional drift velocity. We have seen that fast ions with $v_T \sim v_A$ are not confined in H-1, therefore these more energetic ions will not be confined either. Another possibility is for the bounce frequency $f_b$ of the trapped particle to match the Alfvén eigenmode frequency [73].

We now explore the properties of trapped ions with $f_b \sim f_{\text{MHD}}$. Shown in
Figure 5.22: Trapped orbits of 100 eV H$^+$ ions with pitch angle $\Theta_p = 80^\circ$ at the $\kappa_h = 0.5$ configuration. The red trajectory follows an ion launched at $\phi = 0$ near the axis at $R = 1.25$ m, with the blue launched at $R = 1.4$ m. The outer orbit slightly modulated by the ripple between TF coils.

Figure 5.22 are the trajectories of two 100 eV hydrogen ions with pitch angle $\Theta_p = 80^\circ$, launched at $R = 1.25$ m and 1.4 m at $\phi = 0$ for the $\kappa_h = 0.5$ configuration. Both orbits are trapped by an additional ripple component appearing in low aspect ratio helical axis configurations, the $1/R$ ripple due to the helical axis excursion ($\delta_{n,m} = \delta_{3,0}$, occasionally referred to as “bumpiness or corner ripple”), with the outer orbit slightly modulated by the ripple between TF coils. We see from the oscillation that the bounce frequency is $f_b \sim 10$ kHz.

For a wider range of parameters, we see that the 100 eV trapped particles generally have $f_b \sim 5 - 20$ kHz. Shown in figures 5.23(a) and 5.23(b) are the dependences of the $\kappa_h = 0.5$ bounce frequency on the launch radius and pitch.
angle respectively. We see the lower bounce frequency ($\lesssim 10\,\text{kHz}$) is relatively insensitive to launch radius and pitch angle, while the higher bounce frequencies ($f \gtrsim 60\,\text{kHz}$) are prominent at larger radii where the field ripple due to the individual TF coils is larger. We see from figure 5.24 that there is an increase in the bounce frequency of 100 eV ions from $f_b \lesssim 18\,\text{kHz}$ at $\kappa_h = 0$ to $f_b \lesssim 30\,\text{kHz}$ at $\kappa_h = 1$; with 200 eV ions extending up to $f_b \simeq 40\,\text{kHz}$ at $\kappa_h = 1$. In the present experiment, we expect 100 – 200 eV ions to be present in the high energy tail of the thermal distribution. However, to determine whether such particles are capable of driving the observed modes we require detailed kinetic modelling which is beyond the scope of this thesis.

Figure 5.23: Bounce frequency of 100 eV $H^+$ ions for the $\kappa_h = 0.5$ configuration.
5.5 Other Fluctuation Models

In light of the differences between theory and experiment for the GAE and interchange models, notably the correctional scaling factor $\lambda$, we now consider several other models.

5.5.1 The Drift Wave

As the experimental data shows fluctuations below the typical MHD frequency $k v_A$, we consider processes which involve longer timescales, such as drift waves. The classical drift wave [74] describes an electrostatic perturbation $\mathbf{E} = -\nabla \Phi$ with frequency satisfying $k || v_{ti} \ll \omega \ll k || v_{te}$ in a plasma with $T_i = 0$, where $v_{ts}$ is the thermal velocity of species $s$. In the absence of collisions, the wave propagates due to an adiabatic response to the electrostatic potential $n_1/n_0 = e\Phi/T_e$, where $n_1$ and $n_0$ are the perturbed and background density respectively. The electron

Figure 5.24: Dependence of bounce frequency on $\kappa h$ for 100 eV (o) and 200 eV (x) $H^+$ ions with $\Theta_p = 80^\circ$ at various launch radii.
drift frequency $\omega_{ne}$ is defined by density gradient:

$$\omega_{ne} = \frac{k_\perp T_e}{eB} \frac{d \log n_0}{dx}.$$  \hfill (5.14)

Using the H-1 relevant values $k_\perp = 2\pi (2\pi a/m)^{-1} = 15$ m$^{-1}$ for $m = 3$, $T_e = 20$ eV and $B = 0.46$ T giving $T_e/eB \approx 44$ m$^2$s$^{-1}$. From the ELSI profiles (i.e.: figure 5.19), we estimate $\frac{d \log n_0}{dx} \approx -10$ which gives an electron drift frequency $\omega_{ne} \approx 6600$ rad/s.

The drift wave dispersion relation in the frequency regime described above is

$$\omega \approx \omega_{ne} + \frac{k_\parallel^2 c_s^2}{\omega_{ne}}.$$  \hfill (5.15)

The ion sound speed is $c_s = \sqrt{T_e/m_i} \approx 2.8 \times 10^4$ ms$^{-1}$; assuming $k_\parallel = (\nu m - n)/R$, then for $\kappa_h = 0.8$ and $(n, m) = (4, 3)$ (where $f_{Mirnov} \approx 20$ kHz) we have $k_\parallel \approx 0.05$ m$^{-1}$. We therefore obtain a drift wave frequency of $\omega \approx 6.9 \times 10^3$ rad/s, substantially below the observed Mirnov frequency. Hence, we rule out the drift wave as a candidate for the observed Mirnov spectra. Initial results from the 2 mm electron density interferometer show some additional modes in the drift frequency range, and are examined in [68].

### 5.5.2 The Acoustic Wave

Because the sound spectrum lies below the Alfvén spectra, we take into consideration the possible involvement of sound waves in the observed fluctuations. Previous results from CAS3D modelling of W7-AS show a high degree of similarity between CAS3D eigenmodes and analytical sound spectra in the straight cylindrical approximation [59]; this result was recently shown to be true in H-1 also [75]. The relation between sound $\omega_s$ and Alfvén $\omega_A$ frequencies in the analytic cylindrical case is:

$$\omega_s^2 = \frac{\gamma_s \beta}{2 + \gamma_s \beta} \omega_A^2.$$  \hfill (5.16)
Figure 5.25: Ratio of sound to Alfvén eigenmode frequencies, \( \omega_s / \omega_A = \sqrt{\gamma_s / (2 + \gamma_s \beta)} \). The ratio of H-1 ‘whale-tail’ mode frequency to \( f_{GAE} \), \( \lambda = 0.27 \), is also shown.

Note that the \( s \) subscript denotes sound in the case of \( \omega_s \) and the ratio of specific heats for \( \gamma_s \). The scale factor \( [\gamma_s / (2 + \gamma_s \beta)]^{1/2} \) is shown in figure 5.25 as a function of \( \beta \). We can compare this scale factor to the scale factor \( \lambda \) required to match \( f_M \) and \( f_{GAE} \). In figure 5.25 we see that for H-1 conditions (\( \beta = 10^{-4} \)) the sound spectrum is entirely too low to account for our observations.

### 5.5.3 Alfvén Cascades and the Beta-Induced Alfvén-Acoustic Eigenmode

Alfvén cascades (AC), also known as reversed shear Alfvén eigenmodes (RSAE), have been observed in toroidal plasmas with non-monotonic \( \iota \)-profiles [76, 77]. The AC is generally characterised by a collection of GAE-like modes, with dispersion relation \( \omega_{AC}(t) \approx |m\iota - n|v_A/R \), observed as the value of \( \iota \) is swept during a shot. Deviations from the expected GAE behaviour were seen at low frequencies where, rather than reaching \( f = 0 \), the modes were seen to either flatten out
at finite frequency common to all modes, or terminate above this frequency.

The AC dispersion relation, modified to account for the low frequency component, was found to be [77]:

\[
\omega_{AC}(t) = \left[ (m_t - n)^2 \frac{v_A^2}{R_0^2} + \frac{2T_e}{M_t R_0^2} \left( 1 + \frac{7}{4} \frac{T_i}{T_e} \right) \right]^{1/2}
\]

(5.17)

where the temperature dependent term accounts for the finite pressure and geodesic curvature which prohibit the Alfvén perturbations from being strictly incompressible. This effect is closely related to the geodesic acoustic mode (GAM) [78]. Because both terms in equation 5.17 are positive definite, such that \( \omega_{AC} > \omega_{GAE} \) holds, we will not find an interpretation for the reduced frequency of H-1 modes here (as \( \omega_{H-1} < \omega_{GAE} \)). However, we note that the modes responsible for the whale-tail structures are also modified near \( \omega = 0 \) as shown by \((n, m) \rightarrow (0, 0)\).

A recent analysis of the low frequency Alfvén–acoustic regime has found new global eigenmodes existing in gaps in the Alfvén–acoustic continuum [79]. These global modes have been called beta–induced Alfvén–acoustic eigenmodes (BAAE), and can appear in relatively low and high beta plasmas. By including sideband coupling between \( m \) and \( m \pm 1 \) modes, a generalised dispersion relation with three roots was found [79]. Two roots, an acoustic sideband and a modified shear Alfvén wave, form a gap in which the BAAE resides; the other root is related to the BAE and becomes the GAM in the high–\( \beta \) limit, and the usual shear Alfvén wave in the low–\( \beta \) limit. For high aspect-ratio tokamak geometry with H-1 parameters the BAAE gap is around \( 10 - 20 \) kHz with a width \( \Delta f \simeq 1 \) kHz, however as the gap frequency scales as \( f_{\text{max}} \sim n/m \) rather than \( |\iota - n/m| \) the BAAE is unable to account for the resonant structure of the H-1 observations.
5.6 Preliminary Results from CAS3D Modelling of H-1 Plasmas and the Beta-induced Alfvén Eigenmode

At the time of writing, the research collaboration working on CAS3D (see section 2.2.5) modelling of global eigenmodes in H-1 has produced some preliminary results. In figure 5.26 the CAS3D global modes (sub-figure b) are compared to modes from a cylindrical model (sub-figure a) for the $\kappa_h = 0.37$ configuration. Aside from the fully three-dimensional geometry, the conditions here are different to those of the models in this chapter. The central number density is $n_0(\text{CAS3D}) = 10^{19}$ m$^{-3}$ for a parabolic profile (linear in flux, $s \sim r^2$) and $\mu_{\text{CAS3D}} = 1$, with $\beta_{\text{CAS3D}} = 2 \times 10^{-4}$, and compressibility is included with a ratio of specific heats of $\gamma_s = 5/3$. Compared to the values $\mu = 2.5$, $n_0 = 1.5 \times 10^{18}$ m$^{-3}$ used elsewhere in this chapter, we expect the frequencies modelled here to be scaled by a factor $[\mu n_0/(\mu n_0)_{\text{CAS3D}}]^{1/2} \approx 0.39$. Note also that the CAS3D model does not extend to the plasma edge, with normalised flux $s = 1$ equivalent to $\langle r \rangle = 0.14$ m.

On comparison of the two models in figure 5.26 we note that the (5, 4) mode (black) and its harmonic (10, 8) (red) are shown to have non-zero global minimum ($f \approx 5$ kHz) due to the inclusion of compressibility. As described in section 2.2.4, beta-induced Alfvén eigenmodes (BAEs) can exist in the gap and have both acoustic and Alfvénic characteristics. In previous CAS3D models BAEs have been seen to also exist at higher frequency, within the Alfvén continuum, depending on the density and rotational transform profile [75]. This frequency shift appears to occur in the H-1 CAS3D model, where two modes at $f = 16$ kHz and $f = 23$ kHz shown in figure 5.27, both with $(n, m) = (10, 8)$, are probably associated with the BAE. The $f = 23$ kHz eigenmode (figure 5.27(a)) is peaked near the axis at $s \sim 0.1$, and the 16 kHz mode (figure 5.27(b)) has its peak at mid radius. We see from the sharp peaks in the radial structure that these modes
exist in a continuum (cf. figure 5.9(c)). We would require a more detailed analysis to find the damping rate of the modes, and to determine if such modes can exist within the continuum. Such an analysis may be the subject of future work.

The effects of helical coupling between continuum branches can also be seen in figure 5.26. The helical coupling of the \((9, 7)\) and \((6, 5)\) and also of the \((4, 3)\) and \((1, 1)\) branches, at \(f \approx 130\ kHz, s \approx 0.75\), in the cylindrical model, induces a continuum gap. Global modes (HAEs) are present within the gap and are marked in the figure by black dots at \(f = 78, 80, 98\) and \(128\ kHz\) for the \((9, 7)\), \((6, 5)\) HAE, and at \(f = 83\ kHz\) for the \((4, 3), (1, 1)\) HAE. Thus we see that the general trend is for HAEs to have reduced frequency in three-dimensional heliac geometry compared to the cylindrical model, an observation which is consistent with the use of the scale factor \(\lambda_{\text{HAE}} = 0.8\) in section 5.4.2 to match the cylindrical HAE frequency with \(f_M\). Work is presently underway to explore the full range of \(0 \leq \kappa_h \leq 1.1\) configurations with CAS3D modelling, which should clarify whether or not the modes in figure 5.20 are of HAE type.

### 5.7 Discussion

In this chapter we have compared the observed ‘whale-tail’ resonance features to several physical models which might account for their \(\kappa_h\), or \(\iota\), dependence. The resonant interchange mode fails, as does any magnetic island model, on account of not being capable of describing the high-\(\kappa_h\) side of the resonance. The non-resonant mode (section 5.3.4) and the global Alfvén eigenmode (section 5.4) qualitatively show the correct \(\kappa_h\), and to a degree the correct \(n_e\), behaviour; however a scale factor \(\lambda \approx 0.27\) is required for quantitative agreement. Initial results from fully three-dimensional modelling of H-1 with CAS3D do not suggest that 3D effects can account for the \(\lambda\) scale factor in the case of GAEs, however
Figure 5.26: Preliminary CAS3D results for the $\kappa_h = 0.37$ configuration (sub-figure b), and corresponding cylindrical model (sub-figure a). Note that the horizontal axis here is flux rather than the avg. minor radius used elsewhere in this thesis. Mode numbers are shown as $(m, n)$, see text for details. Figure provided by C. Nührenberg.

(a) (10, 8) BAE-like mode at 23 kHz  (b) (10, 8) BAE-like mode at 16 kHz

Figure 5.27: Radial structure of the BAE-like modes with continuum features found in preliminary CAS3D results. Figures provided by C. Nührenberg.
$\beta$-induced Alfvén eigenmodes (BAE) appear a possible candidate. In section 5.2 we noted that the $B$ scaling of the whale-tail modes does not appear to be consistent with the seemingly Alfvénic $n_e$ dependence, however as BAEs do not necessarily scale linearly with $B$ [41, 49] this does not rule out the possibility of the observed modes in H-1 being of BAE type. One subject which should be addressed in future work is whether or not the BAEs with increased frequency observed in CAS3D results (figure 5.27) can avoid continuum damping through the inclusion of additional physics, such as finite Larmor radius effects.

Regarding the excitation of the modes, we found the bounce frequency of trapped thermal ions to be in the same range as the observed MHD activity, meaning that coupling with the trapped ions may be a source of energy for the modes. Some fluctuations apart from the main ‘whale-tail’ modes appear consistent with helical Alfvén eigenmodes (HAE). The cylindrical model for the $(n,m) = (7,6)$ and $(10,7)$ HAE coupling fits well with the Mirnov spectra at $\kappa_h \sim 1.05$ when a scaling factor $\lambda_{\text{HAE}} = 0.8$ is applied. From the preliminary CAS3D results we expect the HAE frequency to be lower in the fully three-dimensional H-1 geometry than in the cylindrical approximation, as the observed Mirnov frequencies suggest; however we require further results from the CAS3D model in the relevant configurations in order to conclusively identify the modes.
CHAPTER 6

Summary and Conclusions

A critical issue for plasma fusion reactors is the potential for fast ions such as fusion born alpha particles to drive magnetohydrodynamic (MHD) activity, in particular Alfvén eigenmodes, which could degrade confinement. The primary aim of this project is to investigate MHD activity in the Alfvén range of frequencies in the (much smaller) H-1 heliac. The H-1 heliac is unique in its capability to access, with very fine resolution, a wide range of vacuum magnetic field configurations having different rotational transform $\iota$ and magnetic well. It follows that scanning through magnetic field configuration parameters is a useful method for investigating MHD behaviour. A practical parametrisation of the variation in $\iota$ is $\kappa_h$, which is proportional to the ratio of helical winding current to toroidal- and poloidal-field coil currents. For a given flux surface $a$, the transform $\iota(a)$ scales approximately linearly with $\kappa_h$ over the range of $\kappa_h$ used in this study.

Basic plasma diagnostics pre-existing on H-1 were utilised in initial investigations; these included a 2 mm interferometer measuring electron density and a linear array of 5 Mirnov coils to record magnetic fluctuations. A more detailed examination of MHD mode structure was made possible by a pair of toroidally separated poloidal coil arrays, each with 20 Mirnov coils, mounted outside the last closed flux surface, which was designed and installed specifically for this work. During the course of this project, colleagues have assisted with progress through their research and introduction of additional diagnostics to H-1. The most im-
portant of these are the electronically scanned interferometer (ELSI) installed by Dave Oliver and the accurate mapping of error fields in H-1 by Santhosh Kumar.

Results from several experimental campaigns showed the $\kappa_h$ dependence of confinement and fluctuation to be very reproducible. However, for such a large dataset it was thought that a novel data mining approach may prove fruitful. The algorithm developed in chapter 3 requires minimal $a$ priori information or manual interaction, and clusters together fluctuations with similar phase structure. A cluster tree mapping has been used both as a visualisation tool and as a method for determining which clusters are well defined. This method has simplified the process of isolating a single mode out of a large configuration scan dataset in order to study its properties. In developing the data mining algorithm the process has been generalised sufficiently for it to be applicable to situations other than the analysis of Mirnov signals, i.e.: any set of geometrically ordered timeseries data.

The data mining process was applied to Mirnov fluctuation data from an experimental campaign in which the $\kappa_h$ parameter was scanned. The observed $B$ and $n_e$ fluctuation spectra were found to have a strong relationship to the low order rational values of rotational transform, $\iota = n/m$, and the clustering results from the data mining process clearly distinguish modes related to separate $(m, n)$ resonances. To check that the clustering results were not biased by the constraint that clusters are defined by Gaussian distributions in phase-space (a condition imposed by the expectation maximisation clustering algorithm), the results were compared with those found using a different clustering process (agglomerative hierarchical). It was found that there was a high degree of similarity between the two cluster sets. However, the extent to which the choice of clustering metric influences the results was not further investigated. The analysis of alternative metrics in $\Delta \psi$-space is an area in which further research may prove fruitful.
The structures of the dominant modes were analysed using standard Fourier techniques. The complicated geometry of the magnetic field, as well as uncertainty in the radial location of the mode, make the task of resolving the poloidal mode structure difficult. The modes about $\iota = 4/3$ and $5/4$ were found to have dominant $m = 3$ and $4$ components respectively, though the structure of higher $m$ modes is less clear. While toroidal mode numbers $n$ could only be resolved to an accuracy of $n \pm lN_{fp}$, for integer $l$ and $N_{fp} = 3$ is the number of field periods in H-1, the observed values of $n$ are consistent with $(n, m)$ expected for the $\iota = 4/3$ and $5/4$ modes.

There is some evidence that the observed fluctuations show Alfvénic characteristics. This comes from the observation of scaling of the mode frequency $f_M$ as both $f_M \propto |\iota - n/m|n_e^{1/2}$ for a wide range of configurations near $\iota_{zs} = n/m$, and $f_M \propto n_e^{-1/2}$ for a radially localised electron density $n_e$ within a shot. There has been no clear measurement of how the fluctuation frequency scales with $B$ in the RF-heated plasmas because it is not possible to decouple $B$ from the resonant heating mechanism. However the general trend appears to be that the mode frequencies decrease with increasing field, which is inconsistent with Alfvénic behaviour ($f_M \propto B$).

The modes near rational $\iota$ are consistent with a cylindrical model of global Alfvén eigenmodes (GAE) up to a frequency scaling factor $\lambda \simeq 0.27$ and a modification to $k_\parallel$. The modified $k_\parallel$ assumes the mode to be localised at the zero-shear radius when the corresponding $\iota = n/m$ rational surface is within the plasma volume, and at a fixed radius $\langle r \rangle \simeq 0.15$ m when the rational surface is absent from the plasma. The scale factor $\lambda$ cannot be adequately explained by any uncertainty in plasma parameters, and the frequency shift is too large to be accounted for by geometrical effects, which points to conclusion that the modes
may not be GAEs. Interchange models with Alfvénic scaling similar to the GAE were also considered; resonant interchange modes cannot explain the frequency spectra about the rational \( \tau \), while non-resonant modes are subject to the same \( \lambda \) frequency correction as the GAEs.

In seeking a physical explanation of the fluctuations several other candidate models have been considered. The most plausible model appears to be the beta-induced Alfvén eigenmode (BAE). In the literature, there remains some uncertainty about the true nature of the BAE, which is complicated by the entwining of Alfvén and acoustic continua. Initial results from CAS3D models of H-1 show BAEs in the frequency range of the observed fluctuations in H-1, even though \( \beta \) is low (\( \simeq 10^{-4} \)). Further modelling is required for a comprehensive comparison. It is interesting that the BAE frequency does not necessarily follow the Alfvénic (linear) \( B \) scaling, though clearly not much can be inferred from this given the restricted \( B \)-scan capability.

Higher frequency modes were also observed at \( \kappa_h \simeq 1 \) configurations. These modes appear consistent with the helical Alfvén eigenmode (HAE) model, with \( f_M \) lying below the cylindrical model \( f_{HAE} \) by an amount similar to the degree to which the full three-dimensional model lies below the cylindrical model. If such modes are Alfvénic, a plausible driving mechanism must be found. It is probable that ICRH induced fast ions exist in H-1, although there has not been, to date, any experiments to directly measure them. Confinement of ions whose velocity matches \( v_A \) (40 – 120 keV) is marginal, especially for perpendicular ions, so straightforward fast ion drive is unlikely. Calculations of the orbits of trapped particles in H-1 configurations show that the bounce frequency of toroidally trapped ions is in the range of \( f_b \sim 5 – 40 \text{ kHz} \) for ions in the thermal tail, suggesting that resonance with trapped thermal ions may be a source of energy for excitation of
the MHD modes.

Regarding the nature of the observed fluctuations, the work carried out for this thesis appears to have produced more questions that it has answered. At present, the investigation of the resonance modes is expanding, with near-term projects aiming to resolve the radial structure of the modes, and further work is progressing on the accurate CAS3D modelling over a range of magnetic configurations. An important goal of this further work is to accurately identify the modes seen in H-1 with results from larger devices. Such an understanding would allow reactor-relevant experiments to be undertaken in the accessible and easily diagnosed H-1 plasmas. Although the H-1 magnetic field, density and temperature are one or two orders of magnitude smaller than those of ITER, the countering effects of field and density in the Alfvén velocity allow comparison of Alfvén activity in H-1 to the edge region environments of ITER and future reactor devices.
APPENDIX A

Mirnov Coil Calibration

Shown in figure A.1 is the coil response, calibrated before the Mirnov array was installed into H-1. Each point plotted represents the mean value across all coils, with the variation between measurements barely noticeable on this scale with the exception of the 1 MHz measurement where standard deviation is 6.5 V/T.

Figure A.1: Mirnov coil response, including stainless steel shielding.
APPENDIX B

Alternative Cluster-tree Representation

In figure B.1 is an alternative cluster tree representation of the $\kappa_h$ scan data. The figure in the bottom left corner contains the whole dataset; its upper panel shows the fluctuation structures mapped to $f$ and $\kappa_h$, the numbers 1 (2000) at the top right are the tree level, $N_{Cl}$, and cluster population respectively, EM:XX labels correspond to cluster labels in figure 4.10. For clarity, only a subset of clusters within the tree have their contents displayed and EM:4 has been displaced to prevent overlap. Vertical parent-child distance is proportional to the distance between cluster means, while line thickness is inversely proportional to the Gaussian width of the cluster. Several AH clusters are also shown, these are essentially equivalent to the $N_{Cl} = 10$ level EM clusters, see table 4.2 for comparison.
Figure B.1: An alternative cluster tree representation of the $\kappa_h$ scan data
APPENDIX C

Normalised MHD Variables

In section 5.3 we follow the variable normalisation of reference [30]:

\[ \psi \rightarrow aB_0 \psi \]  
\[ u \rightarrow \frac{aR_0}{\tau_A} u \]  
\[ t \rightarrow \tau_A t \]  
\[ p \rightarrow p_0(r = 0)p \]  
\[ r \rightarrow ar \]  
\[ J_z \rightarrow \frac{B_0}{\mu_0 a} J_z \]  
\[ \nabla^2 u \rightarrow \frac{R_0}{a\tau_A} \nabla^2 \nabla^2 u \]  
\[ A_z \rightarrow aB_0 A_z \]

where \( \tau_A = R_0 \sqrt{\mu_0 \rho}/B_0 \) is the Alfvén transit time.
APPENDIX D

Parameterisation of a Typical ELSI Profile

In section 5.4.2 we introduce a parameterisation of a typical ELSI profile for the analysis of HAEs. The profile used here is from shot 58082, at kh 0.92, ELSI sweep #35 at $t = 50$ ms. A least squares fit for a 6th order polynomial was used, giving the result:

$$n_e(r) = \sum_{j=0}^{6} a_j r^j$$ \hspace{1cm} (A-1)

with

$$(a_0, a_1, \ldots, a_6) = (2.148, -0.240, 5.993, -34.937, 39.692, -6.491, -6.151).$$ \hspace{1cm} (A-2)$$
APPENDIX E

SVD Analysis of a Typical ELSI Profile

To find a density profile which is typical for a shot, we consider the spatial singular vectors of the ELSI data for $t > 10$ ms for a given shot. In figure E.1 we see the singular values of the ELSI inversion data for shot 58061 at $\kappa_h = 0.5$, with a single dominant component, 4 higher order components which are 1 to 2 orders of magnitude lower than the dominant component, and 6 which are around 4 orders of magnitude smaller again and contribute only to the noise. The topos and chronos of the 5 largest singular values are shown in figure E.2 where we can see the separate components of the temporal evolution of the density profile. The largest, SV0, is essentially constant for $t > 10$ ms shown here and has a roughly linear radial profile, while the perturbations described by SV1 and SV2 are easily understood. For example; the SV1 topo shows, for early time, a flattening of the density profile for $\rho \lesssim 0.4$ and $\rho \gtrsim 0.8$ with an increased gradient elsewhere; the magnitude of the effect decreases linearly with time until $t \sim 35$ ms where the sign changes, inverting the perturbation.

In section 5.3.5 we are interested in a steep pressure gradient near the plasma edge. For the region of interest, $0.8 \lesssim \rho \lesssim 1.0$, we see the greatest density gradient is towards the end of the shot as $t \to 60$ ms. We therefore take the ELSI sweep at $t \sim 58$ ms as a typical density profile for this case.
Figure E.1: Singular values of typical electron density profile. Shot 58061, $\kappa_h = 0.50$

Figure E.2: Corresponding topos and chronos for singular values of figure E.1 showing the main components of a typical H-1 density profile.
APPENDIX F

Publications During PhD Candidature


REFERENCES


[72] D. Oliver. personal communication.


Glossary

α fluctuation structure, 52

$B$ magnetic field, 7

$\beta$ magnetic field normalised plasma pressure, 22

$C_s$ Collision operator, 11

c Speed of light, 2

$D_M$ Mercier criterion, 26

$D$ Deuterium, 3

$\delta$ expansion parameter, 14

$\delta(\alpha_a, \alpha_b)$ Distance between $\alpha_a$ and $\alpha_b$, 59

$v_d$ drift velocity, 8

$dB$ decibel, 49

$E$ Electric field, 11

$\epsilon_0$ permittivity of free space, 6

$\eta$ resistivity, 15

$e$ electron charge, 6

$eV$ Electron volt, 3

$f_M$ Mirnov frequency, 104
\( \gamma_s \) ratio of specific heats, 15

\( \gamma_{a,b} \) normalised average cross-power spectrum, 52

\( H \) normalised entropy, 47

\( \iota \) Rotational transform, 9

\( \iota' \) Magnetic shear, 10

\( t_{0(a)} \) \( t \) at magnetic axis (plasma edge), 68

\( \kappa \) curvature, 23

\( \text{keV} \) kiloelectronvolt = \( 10^3 \text{eV} \), 7

\( \kappa_h \) helical current ratio, 35

\( L \) characteristic length scale, 6

\( \lambda \) scaling factor: \( f_M/f_{\text{GAE}} \), 124

\( \lambda \) wavelength, 29

\( \lambda_D \) Debye length, 6

\( m \) poloidal mode number, 58

\( m_e \) electron mass, 6

\( \mu_g \) geometric mean, 48

\( \mu_i \) mean value of cluster \( i \), 60

\( n \) Particle density, 2
$n$ toroidal mode number, 94

$n_{e}$ electron density, 6

$N_{a}$ number of singular values, 47

$N_{c}$ number of channels, 46

$N_{Cl}$ number of clusters, 59

$N_{fp}$ number of field periods, 31

$N_{m}$ number of modes, 50

$N_{s}$ number of samples, 46

$\omega_{c}$ cyclotron angular frequency, 8

$\omega_{p}$ Plasma frequency, 6

$\psi$ magnetic flux, 23

$\psi_{p}$ Poloidal magnetic flux, 9

$\psi_{t}$ Toroidal magnetic flux, 9

$\pi$ anisotropic stress tensor, 15

$p$ normalised energy of fluctuation structure, 54

$p_{k}$ normalised energy of singular value $k$, 47

$q$ electric charge, 7

$\rho$ fluid mass density, 15
\( \textbf{r}_c \) cyclotron radius, 8

\( \langle r_a \rangle \) average minor radius, 34

\( S \) Magnetic Reynolds number, 28

\( S \) short time segment matrix, submatrix of \( S \), 46

\( \mathcal{S} \) matrix of timeseries channels, 46

\( \langle \sigma v \rangle_f \) fusion reaction rate, 2

\( \sigma \) Cross-section of interaction, 2

\( \sigma_i \) standard deviation of cluster \( i \), 60

\( T_e \) electron temperature, 6

\( T_i \) Ion temperature, 3

\( \textbf{T} \) Tritium, 3

\( \tau \) characteristic time scale, 6

\( \tau \) sample time, 46

\( \tau_E \) Energy confinement time, 7

\( \tau_A \) Alfvén transit time, 104

\( \textbf{T W} \) Terawatt = \( 10^{12} \) Watts, 4

\( \textbf{V}_d \) diamagnetic drift velocity, 18

\( v \) particle velocity, 7
$v_A$ Alfvén velocity, 28

$v_s$ sound speed, 29

$\xi$ fluid displacement vector (perturbation), 25

$z_0$ resistive damping distance, 29
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