Introduction to Plasma Physics
C17 Lecture Notes

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Chapter 1

BASIC PLASMA PHENOMENA

1.1 What is a plasma?

Plasma is the fourth state of matter. As we increase the heat added to a solid, it will eventually make phase transitions to the liquid state, become gaseous and then finally the bonds binding electrons and ions are broken and the gas becomes an electrically conducting plasma. As a loose definition we may regard plasma as matter whose behaviour is dominated by electric and magnetic forces.

Important examples of plasma are interstellar gas, stars, stellar atmospheres, and on earth, lightning, arcs, aurorae, fluorescent tubes, the ionisphere, material processing plasmas and fusion plasmas such as that studied at the H-1NF National Plasma Fusion Research Facility at ANU.

A more quantitative definition relies on concepts to be developed below. However, in general, a plasma must satisfy the following criteria:

1. Quasi-neutrality. A plasma maintains almost perfect charge balance:

   $$-q_\mathrm{e} n_\mathrm{e} = q_\mathrm{i} n_\mathrm{i} \pm \Delta$$

   where Δ is tiny.

2. Interactions between individual charged particles are insignificant compared to collective effects. This introduces the concept of *Debye screening* (more below). This condition requires that the number of particles in the *Debye sphere* $\Lambda \gg 1$. Λ is also called the *plasma parameter*.

3. The electron-neutral collision cross-section is much smaller than the electron-ion cross section $\sigma_\mathrm{en} \ll \sigma_\mathrm{ei}$. Thus a weakly ionized ($\sim 1\%$) gas can behave like a plasma because the long range Coulomb forces have an effect much

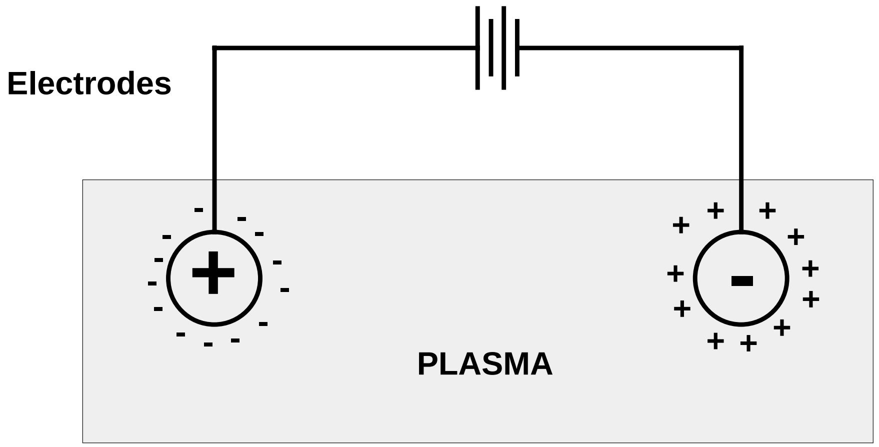


Figure 1.1: The electric field applied between electrodes inserted into a plasma is screened by free charge carriers in the plasma.

more significant than that associated with collisions with neutrals (1 atomic radius). For the plasma state to exist, collisions with neutrals must not be so frequent ($\nu_{\rm en} = 1/\tau_{\rm en}$) that charged particle dynamics are governed by hydrodynamic forces rather than electromagnetic.

1.1.1 Basic dimensionless parameters

The yardsticks for our dimensionless analysis are time t and length ℓ. Let us consider three physically meaningful parameters related to our yardsticks:

$$\begin{aligned} p_1 &= n = \text{particle number denisty} \sim \ell^{-3} \\ p_2 &= v_{\rm th} = \text{particle thermal velocity} \sim \ell t^{-1} \\ p_3 &= \frac{e^2}{m\varepsilon_0} = \text{interaction strength} \sim \ell^3 t^{-2} \end{aligned} \tag{1.1}$$

Observe that the final quantity, which contains only powers of ℓ and t, measures the strength of the Coulomb interaction between free plasma charged particles. We now combine the three physical quantities in an attempt to obtain a dimensionless parameter that characterises the plasma:

$$\begin{aligned} \mathcal{P} &= p_1^\alpha p_2^\beta p_3^\gamma \\ &\sim \ell^{3(\gamma-\alpha)+\beta} t^{(-2\gamma-\beta)}. \end{aligned} \tag{1.2}$$

We now set the combined indices to zero to obtain $\beta = -2\gamma$ and hence $\gamma = 3\alpha$. Arbitrarily taking $\alpha = 1$ then gives the dimensionless quantity

$$\mathcal{P}_0 \sim \frac{p_1 p_3^3}{p_2^6} \equiv \frac{1}{\Lambda^2} \tag{1.3}$$

where

$$\Lambda = n\left[\frac{v_{\rm th}}{\left(\frac{ne^2}{m\varepsilon_0}\right)^{1/2}}\right]^3 \tag{1.4}$$

and $m = m_{\rm i}$ or $m = m_{\rm e}$. Any other choice for α would give a power of $\mathcal{P}_0$ that would nonetheless remain dimensionless.

Λ is known as the *plasma parameter*. It is the only dimensionless parameter that characterises unmagnetized plasma systems. We idenitify two limits for Λ – the strongly coupled case $\Lambda \ll 1$ in which the potential energy of the interacting particles is more significant than thier kinetic motions and the weakly coupled case $\Lambda \gg 1$ where the particle thermal motions are more important. This is the case almost always encountered for naturally occurring and man-made plasmas.

Given that $n \sim \ell^{-3}$ we can obtain a "characteristic" plasma length scale using Eq. (1.4):

$$\lambda_{\rm D} = \frac{v_{\rm th}}{\left(\frac{ne^2}{m\varepsilon_0}\right)^{1/2}} \tag{1.5}$$

known as the Debye length. This length is a measure of the distance in a plasma over which the electric field of a charged particle is "screened out" by the random thermal motions of the other charged plasma particles. We encounter this scale again below. In terms of the Debye length, the plasma parameter becomes

$$\Lambda = n\lambda_{\rm D}^3 \tag{1.6}$$

which is a measure of the number of plasma particles in a *Debye sphere* (more correctly a cube) surrounding our isolated test charge.

Asscoiated with this length scale is the inverse time scale

$$\omega_p \sim v_{\rm th}/\lambda_{\rm D} = \left(\frac{ne^2}{m\varepsilon_0}\right)^{1/2} \tag{1.7}$$

known as the "plasma frequency". Below we examine the meaning of this natural plasma timescale.

Important macroscopic scaling parameters are the system size L and the observation time τ. We can normalize our characteristic plasma length and time scales to obtain new dimensionless parameters

$$\lambda_{\rm D}/L \ll 1 \tag{1.8}$$

and

$$\omega_p\tau \gg 1 \tag{1.9}$$

which indicate respectively that particles must stay in the system long enough for it to be regarded as a plasma and that we must be able to see plasma oscillations. These are fairly basic and weak constraints. The observation time τ can be loosely identified with the time taken for a particle to traverse the plasma system, or the inverse *transit frequency* ω_T:

$$\tau = L/v_{\mathrm{th}} \sim 1/\omega_T \tag{1.10}$$

so that the small Debye length ordering $\lambda_{\mathrm{D}}/L \ll 1$ can also be written as $\omega_T/\omega_p \ll 1$.

1.1.2 Coulomb force and the plasma frequency

The plasma frequency arises as a basic consequence of the restoring Coulomb interaction between oppositely charged particles. It is the plasma "ringing" response to a charge perturbation. First, consider the electrc field due to a 1-D line of charge (see Fig. 1.2). Applying Gauss's theorem to the pillbox shown, we find

$$\int \boldsymbol{E}.\mathrm{d}\boldsymbol{s} = 2AE = \rho A\mathrm{d}x/\varepsilon_0 \tag{1.11}$$

where ρ is the charge per unit length of the line. We now allow the line of charge to extend from $x = -\infty$ to the origin. The associated field at the test charge is

$$E_{\mathrm{tot}} = \int_{-\infty}^{0} \frac{\rho \mathrm{d}x}{2\varepsilon_0} = \frac{\rho}{2\varepsilon_0}\left[x\right]_{-\infty}^{0} \rightarrow \infty \tag{1.12}$$

and the force can be arbitrarily large! Thus, in a plasma, the electrons and ions cannot be separated on average. This is the meaning of property 1 above.

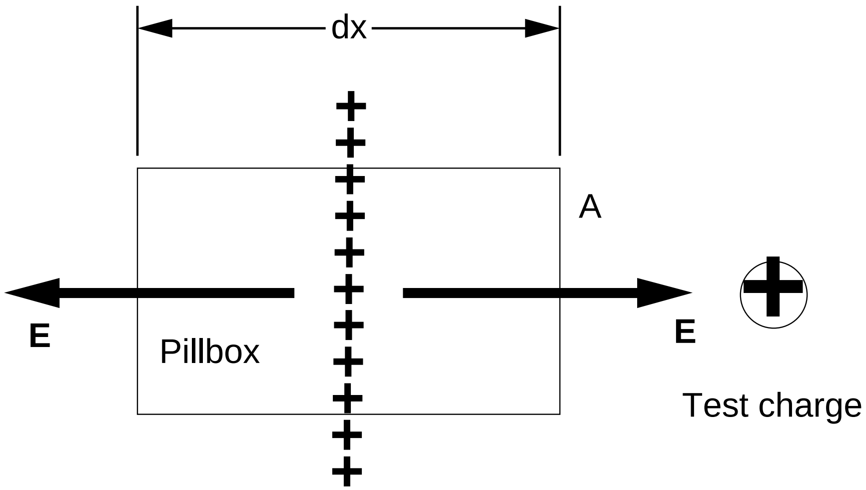


Figure 1.2: Electric field generated by a line of charge

We now consider the dynamic response of the plasma to an imposed charge separation. To study this, we displace electrons and ions (as shown in Fig. 1.3) and calculate the transient response. Since the electrons ($q_\mathrm{e} = -e$) are much lighter than the ions, we assume the ions are immobile. Then

$$\begin{aligned} F &= qE \\ m_\mathrm{e}\frac{\mathrm{d}^2x}{\mathrm{d}t^2} &= -\frac{e\rho x}{\varepsilon_0} \\ \frac{\mathrm{d}^2x}{\mathrm{d}t^2} &= -\omega_\mathrm{pe}^2 x \end{aligned} \tag{1.13}$$

which is the equation of a simple harmonic oscillator with *electron plasma frequency*:

$$\omega_\mathrm{pe} = \left(\frac{e\rho}{m_\mathrm{e}\varepsilon_0}\right)^{1/2} = \left(\frac{ne^2}{m_\mathrm{e}\varepsilon_0}\right)^{1/2} \tag{1.14}$$

Note that the solution contains no wavenumber dependence ($kx = 0$), that is, $x = x_0 \exp(\mathrm{i}\omega_\mathrm{ce}t)$ and the oscillation is not a propagating wave.

The ions oscillate much more slowly (by factor $\sqrt{m_\mathrm{e}/m_\mathrm{i}}$) about the centre of mass. At these frequencies, one can regard the ions as an irregular lattice of charge.

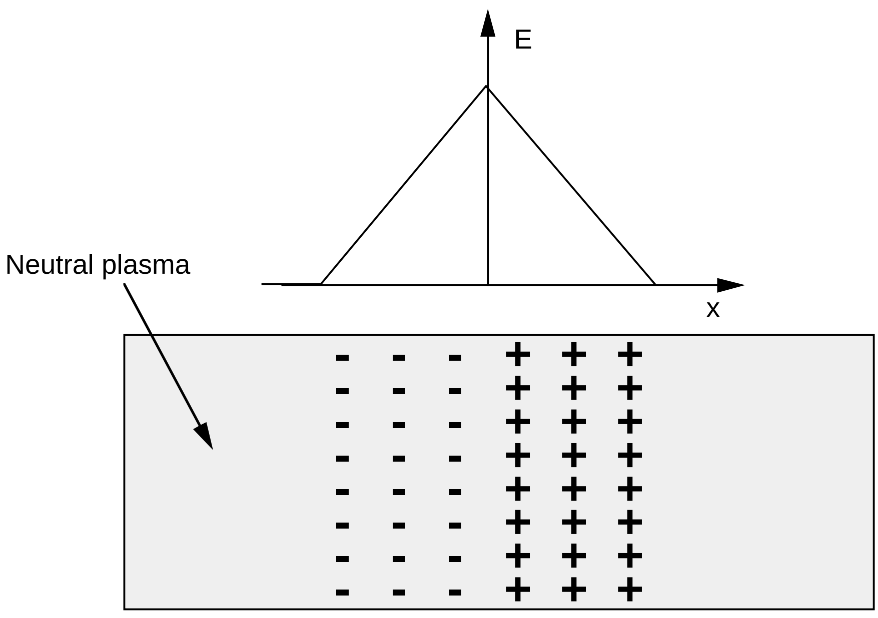


Figure 1.3: The figure shows regions of unbalanced charge and the resulting electric field profile that results.

As a quick estimate, one can use

$$f_\mathrm{pe} = 9\sqrt{n_\mathrm{e}}\ \mathrm{Hz}$$

For example, for $n_e = 1 \times 10^{18}\mathrm{m}^{-3}$ (H-1NF) we have $f_{pe} = 9$ GHz - well into the microwave region of the spectrum. For $n_e = 1 \times 10^{14}\mathrm{m}^{-3}$ (a flame) we have $f_{pe} = 90$ MHz (FM radio)

This is the most fundamental of the "natural" plasma frequencies. It appears in the plasma refractive index N and so can be measured using laser interferometers that sense the phase velocity c/N of light compared with vacuum. Interferometry is an unperturbing way to measure the electron density in the plasma (see Fig 1.4).

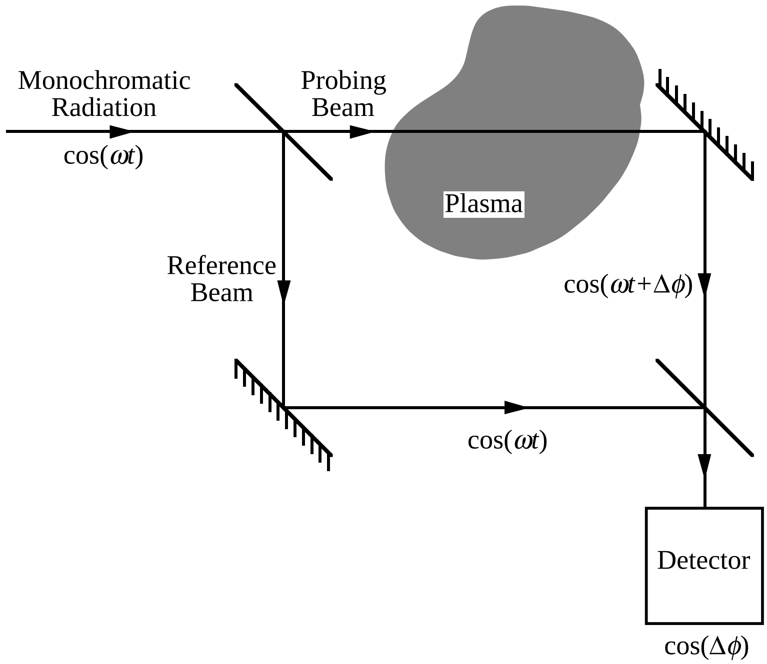


Figure 1.4: Principle of the Mach Zehnder interferometer

1.1.3 Debye shielding

We will later examine more rigorously the nature of *Debye shielding* using the results of kinetic theory. For now, however, we take a more intuitive appraoch. We have established that plasmas are neutral on timescales of order ω_{pe}^{-1}. An electron travelling at thermal speed v_{the} moves a distance $\Delta x = v_{the}/\omega_{pe}$ in this time. This gives the distance scale over which any charge imbalance in the plasma is neutralized or shielded. The rigorous result is that free charges in the plasma are shielded out in a *Debye length* $\lambda_D = v_{the}/(\sqrt{2}\omega_{pe})$:

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{n_e e^2}\right)^{1/2} \tag{1.15}$$

where k_{B} is the Boltzmann constant and we have anticipated a result [Eq. (2.26)] relating v_{the} and the electron temperature T_{e} for a plasma in thermal equlibrium. The Coulomb force between particles in a plasma is thus shielded by the mobility of free charges, and so is reduced in range from ∞ to $\sim \lambda_{\mathrm{D}}$. The hotter the particles, the more mobile they are and the greater is the range. When the density n_{e} of electrons is high, the Debye length shrinks. The Debye shielding picture is valid provided there are enough particles in the charge cloud:

$$\Lambda = \frac{4}{3}\pi\lambda_{\mathrm{D}}^3 n_{\mathrm{e}} \sim T_{\mathrm{e}}^{3/2} n_{\mathrm{e}}^{-1/2} \gg 1. \tag{1.16}$$

As we mentioned earlier, this is the situation for weakly-coupled plasmas.

A major consequence of this result is that plasma theory now becomes tractable—we can treat the plasma as a collection of independent fluid elements described by a distribution function which evolves under the influence of local forces and collisions. We develop this treatment in the next chapter.

If the plasma dimension L is much larger than λ_{D} then charge perturbations or potentials are shielded out within the plasma and it remains quasi-neutral. However, because of the statistical nature of thermal motion, it is possible for small charge imbalances with associated potentials $\phi \sim k_{\mathrm{B}}T_{\mathrm{e}}/e$ to arise spontaneously.

Some Debye shielding implications for plasma physics:

- Limits the range of Coulomb collisions.
- Dominates plasma wall interactions - more about "sheaths" later.
- For same reason, determines behaviour of material probes in a plasma.
- Electron plasma waves (Chapter 2) are strongly damped for wavelengths close to λ_{D}.
- Important in laser light scattering from plasmas where $\boldsymbol{k}_{\mathrm{Scatt}} = \boldsymbol{k}_{\mathrm{Inc}} - \boldsymbol{K}_{\mathrm{Plasma}}$. For $|\, K_{\mathrm{Plasma}}\lambda_{\mathrm{D}}\,| \ll 1$ the laser senses plasma "collective" or fluid behaviour. For $|\, K_{\mathrm{Plasma}}\lambda_{\mathrm{D}}\,| \gg 1$ the electron thermal motion is observed.

Table 1.1 lists typical numerical values for some of the key parameters characteristing important man-made and naturally occurring plasmas [1].

	n (m^{-3})	T (°K)	ω_{pe} sec^{-1}	λ_{D} (m)	Λ
glow discharge	10^{19}	3×10^3	2×10^{11}	10^{-6}	3×10^2
chromosphere	10^{18}	6×10^3	6×10^{10}	5×10^{-6}	2×10^3
interstellar medium	2×10^4	10^4	10^4	50	4×10^4
magnetic fusion	10^{20}	10^8	6×10^{11}	7×10^{-5}	5×10^8

Table 1.1 Some plasmas and their key parameters. In all these examples the plasma parameter is large and the plasma is characterised by Coulomb interaction potentials that are weak compared with particle kinetic energies.

1.1.4 Saha equilibrium

The amount of ioinzation to be expected in a gas in thermal equilibrium is given by the *Saha equation*:

$$\frac{n_{\mathrm{i}}}{n_{\mathrm{n}}} = 2.410^{21}\frac{T^{3/2}}{n_{\mathrm{i}}}\exp\left(-U_i/k_{\mathrm{B}}T\right) \tag{1.17}$$

where n_{n} is the neutral density, n_{i} is the ion density, T is the temperature in degrees Kelvin and U_i is the ionization energy of the gas. $n_{\mathrm{i}}/n_{\mathrm{n}}$ is a balance between the rate of ionization (T dependent) and the rate or recombination (density dependent). As shown in Fig. 1.5, the fractional ionization is a very sensitive function of temperature and density. These considerations bear on the regimes for which plasma criterion 3 noted above is valid.

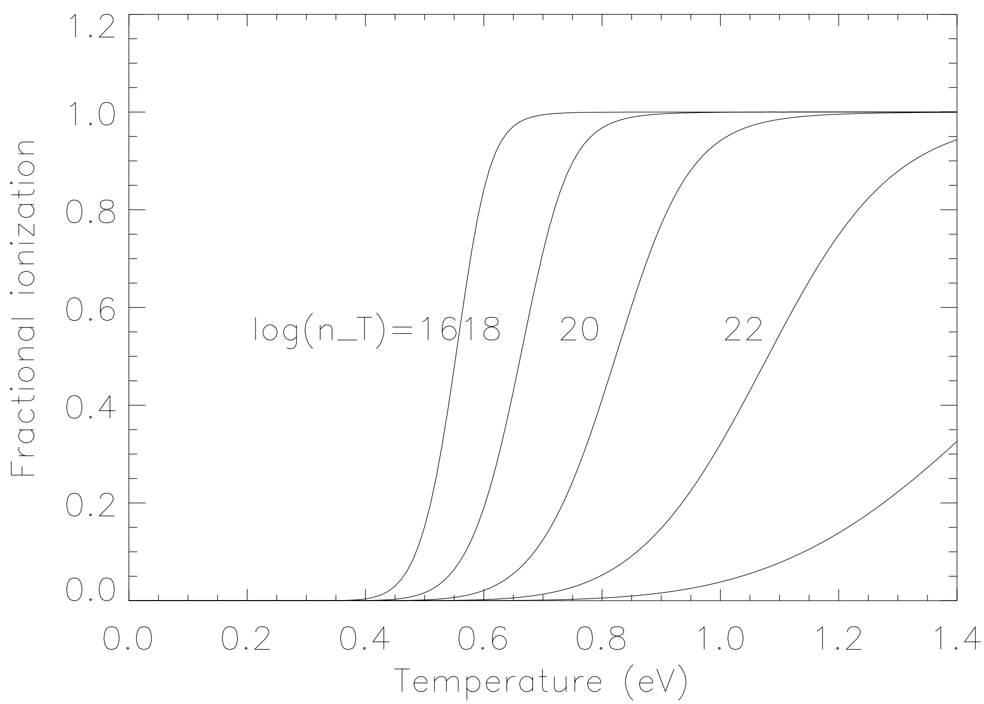


Figure 1.5: Curve showing the dependence of the fractional ionization of a hydrogen (U_i=13.6 eV) as a function of temperature and number density.

1.1.5 Collisions

Because of the long range Coulomb force, collisions between charged particles (in the sense that the particle momentum is significantly altered) are more likely the result of a large number of glancing encounters each of which slightly perturbs the particle trajectory. The plasma "collisionality" often refers to a dimensionless measure such as ν/ω_T where ν is the actual collision frequency and ω_T is the system transit frequency. An alternative and more intuitive measure is the ratio

$$\lambda_{\mathrm{mfp}}/L \sim \omega_T/\nu \tag{1.18}$$

where

$$\lambda_{\mathrm{mfp}} \equiv v_{\mathrm{th}}/\nu \tag{1.19}$$

defines the mean free path between collisions. A "collisionless" plasma satisfies the condition $\lambda_{\mathrm{mfp}} >> L$. This condition can arise in hot plasmas for reason that the Coulomb collision frequency varies as

$$\nu \sim \frac{\log \Lambda}{\Lambda}\omega_p \tag{1.20}$$

whereupon

$$\nu \sim nT^{-3/2}. \tag{1.21}$$

This important result will be derived in Chap. 2. Collision frequencies for the representative plasmas of Table 1.1 are presented in Table 1.2.

To summarize this section, the three criteria for the existence of the plasma state (noted at the ouset) can be quatified as

1. $\Lambda \gg 1$ (plasma parameter large)
2. $\lambda_{\mathrm{D}} \ll L$ (small Debye length)
3. $\omega_{\mathrm{pe}}\tau_{\mathrm{en}} > 1$ (low neutral collisionality)

1.2 Plasma Generation

Terrestrial plasmas are generated by depositing large amounts of energy either resonantly or by some other means into a small amount of material. Examples of such techniques are

- Thermal: Q-machine, shock-waves, Stellar atmosphere
- Electrical: Discharge tubes, tokamak, stellarator
- EM waves: ICRH, ECRH, laser fusion
- Particles: high energy neutral beam injection

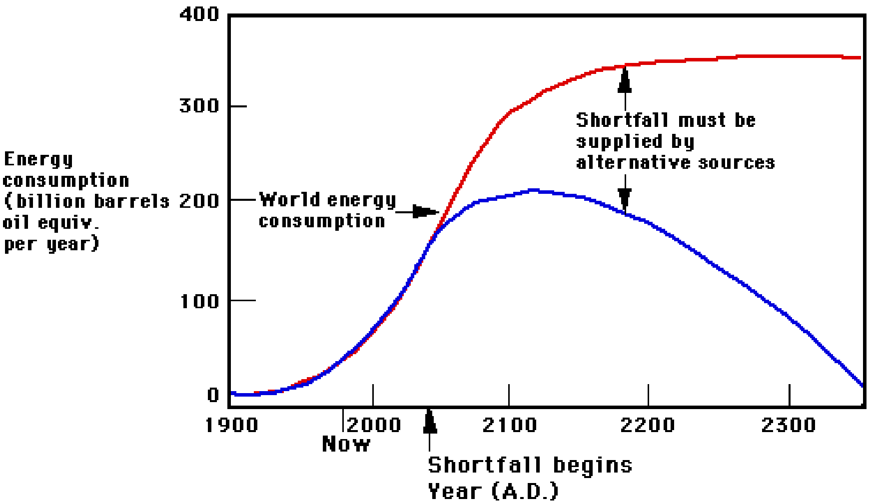


Figure 1.6: Unless alternatives can be found, a serious shortfall in expendable energy reserves will be apparent by the middle of the next century. (reproduced from `http://FusEdWeb.pppl.gov/`)

1.3 Nuclear fusion

Based on reasonable projections of the world's likely energy consumption and estimates of the available energy reserves, it is clear that a major shortfall will become apparent sometime in the next century (see Fig. 1.6). The potential for conflict and major disaster is great.

The ultimate goal of fusion is the controlled release of clean and effectively inexhaustible nuclear energy for mankind's use. This is achieved through the fusion of nuclear isotopes of hydrogen (deuterium and tritium) under conditions of extreme temperature and pressure as shown in Fig. 1.7.

A 1 GW fusion power reactor requires little fuel and produces little waste compared with a coal-fired plant (see Fig. 1.8).

You will find the web site `http://wwwofe.er.doe.gov/Education.html` an excellent resource for educational material on fusion plasma physics.

1.4 Magnetic confinement

Because of the extreme conditions, high temperature plasmas must be confined away from material walls. For long duration terrestrial plasma, this requires the use of magnetic fields. Because plasma particles are charged, they are constrained

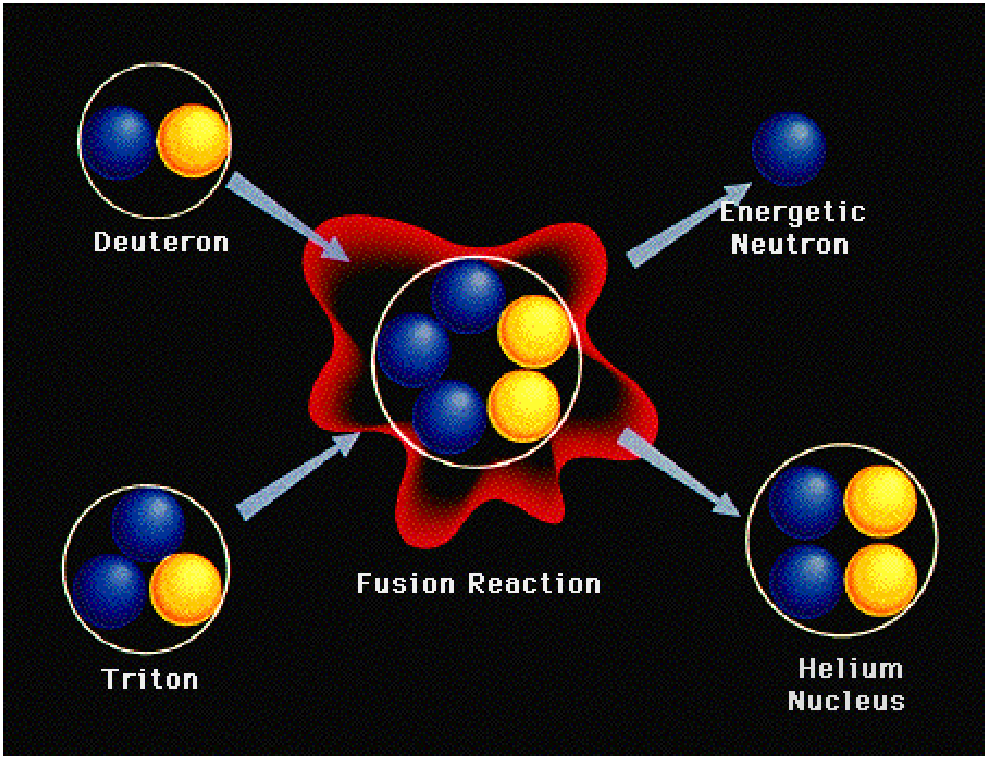


Figure 1.7: The terrestrial fusion reaction is based on the fusion of deuterium and tritium with the release of a fast neutron and an alpha particle. (reproduced from `http://FusEdWeb.pppl.gov/`)

to move in helical orbits about the magnetic lines of force (see Fig. 1.9) by the *Lorentz force*:

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}). \tag{1.22}$$

The particles orbit at the *cyclotron frequency*

$$\omega_{\mathrm{c}} = \frac{eB}{m} \tag{1.23}$$

where B is the magnetic field strength and m is the particle mass. Positive particles rotate in a left hand sense about the magnetic field (align your left thumb in the direction of the field - your hand clasp indicates the rotation direction); negative particles - electrons - rotate in a right hand sense. Injection of electromagnetic waves at ω_{c} or its harmonics can be used to resonantly heat electrons or ions (for example, ion cyclotron resonance heating - ICRH).

Two new dimensionless parameters can be constructed when a magnetic field is present. The first is the ratio of the Larmor radius r_{L} of the circulating particle to the plasma dimension $\delta = r_L/L$. For strongly magnetized plasmas, $\delta \ll 1$

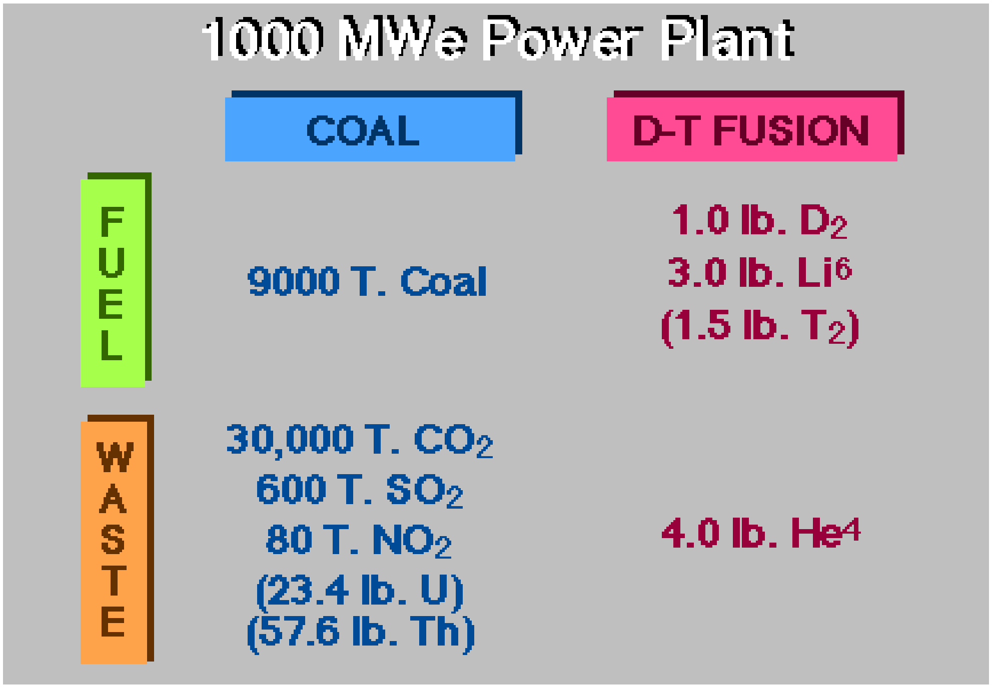


Figure 1.8: Comaprison of fuel needs and waste by-products for 1GW coal fired and fusion power plants. (reproduced from `http://FusEdWeb.pppl.gov/`)

and the particle orbits closely approximate ideal helical trajectories. This is discussed further in Chap. 4. The second dimensionless parameter that can be derived is the ratio β of the plasma kinetic pressure $p = nk_{\mathrm{B}}T$ to the magnetic pressure: $\beta = p/(B^2/2\mu_0)$. This parameter is related to the fact that a plasma is diamagnetic - We will see more of this later.

Table 1.2 includes important additional parameters useful for characterising the plasmas of Table 1.1 in the case where a magnetic field is present. Note also the wide variation in collision frequencies.

	$\nu_i\,(\mathrm{s}^{-1})$	$\nu_e\,(\mathrm{s}^{-1})$	B (Tesla)	$r_{Li}\,(\mathrm{m})$	$r_{Le}\,(\mathrm{m})$	β
glow discharge	5×10^{14}	10^{10}	5×10^{-2}	5×10^{-4}	4×10^{-5}	0.1%
chromosphere	10^{6}	6×10^{7}	10^{-2}	10^{-2}	2×10^{-4}	0.4%
interstellar medium	2×10^{-8}	10^{-6}	5×10^{-10}	3×10^{-5}	7×10^{3}	6%
magnetic fusion	10^{2}	6×10^{3}	5	3×10^{-3}	7×10^{-5}	3%

Table 1.2 Additional parameters for the representative set of plasmas of Table 1.1. In the case of a glow discharge, the collision frequency for ions is higher than for electrons due to the higher electron temperatures and the larger ionic charge.

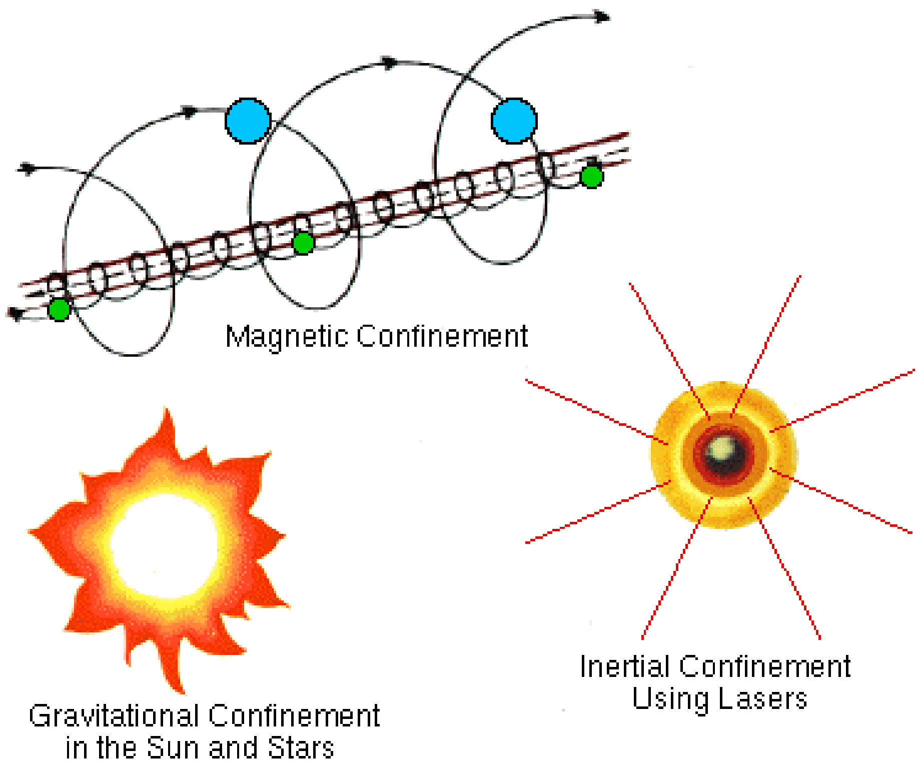


Figure 1.9: The principal means for confining hot plasma are gravity (the sun), inertia (laser fusion) and using electric and magnetic fields (magnetic fusion). (reproduced from `http://FusEdWeb.pppl.gov/`

The magnetic lines of force, produced using magnetic solenoid coils, can be closed into a doughnut shape to prevent plasma end losses. However, the lines of force also require a twist (helicity), in order to stably hold the plasma. In a *tokamak*, this twist is produced by inducing a toroidal current to flow. This current also serves to produce and heat the plasma. Figure 1.12 shows the world's largest tokamak device, the Joint European Torus, located at Culham in England.

Stellarators, another type of magnetic confinement device, produce the entire twisted helical and toroidal magnetic bottle using magnetic coils external to the plasma. The world's largest stellarators are the superconducting device LHD in Japan (see Fig. 1.13) and the W7X device in Germany (see Fig. 1.14).

The *H-1 heliac* is a particular type of stellarator device, a "heliac", in which the magnetic axis executes a three period helix about a central planar ring conductor (see Fig. 1.10). It is usual for the magnetic coils to surround a vacuum chamber into which is "puffed" the working gas (usually hydrogen, but sometimes helium or another noble gas). This gas is then ionized to produce an electrically conducting plasma. An alternative approach is to immerse the coils in the vacuum tank itself. This is the preferred construction for the H-1NF heliac. More pictures and information about the H-1NF program can be found at `http://rsphysse.anu.edu.au/prl/PRL.html` and `http://rsphysse.anu.edu.au/prl/H-1NF.html` as well as in Sec. 1.5 below.

1.5 The H-1NF National Facility

1.5.1 The coil set

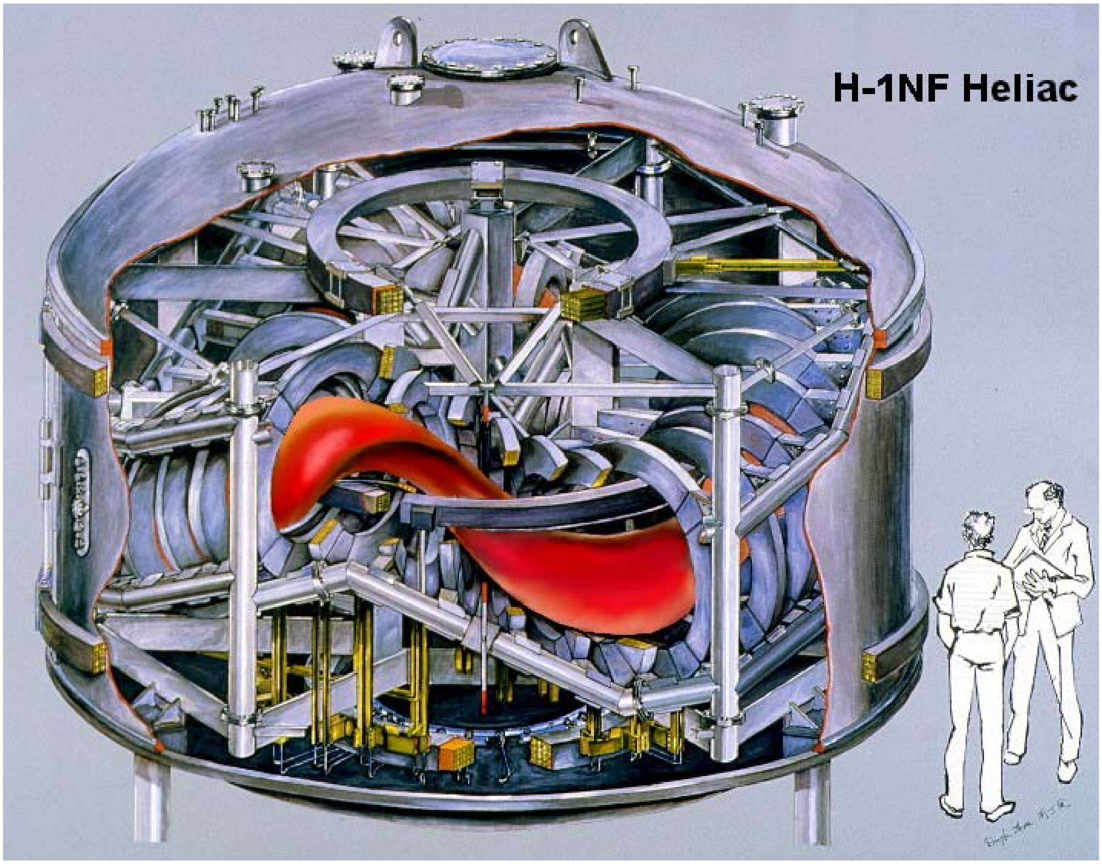
H-1NF consists of a set of 36 toroidal field coils (TFCs) having centres lying on a toroidal three-period helix of major radius $R_0 = 1$ m and helical radius 0.22m. The TFCs are linked by a planar central ring conductor (poloidal field coil PFC) of radius 1m. A helical control winding (HCW) wraps helically about the PFC three times, and in phase with the TFCs, with a mean helical radius of .095 m. Finally, two pairs of vertical field coils, one pair inside and one outside the vacuum tank. All coils are fed in series with various taps and shunts provided to vary the current ratios as required. Some pictures of the H-1NF device and the magnetic coil support structure are shown in Fig. 1.10.

The minor, or "poloidal" cross-section of the plasma manetic bottle produced by the magnetic coil set is shown in Fig. 1.11. The dots correspond to the "punctures" in the poloidal plane produced by a helical magnetic line of force as it makes many circuits toroidally around the machine. The helicity causes the magnetic field line to be angularly displaced in the poloidal direction for each toroidal turn. The surface mapped out by the line, is called a *magnetic flux surface*. The outermost magnetic flux surface in H-1NF has mean radius of ~ 0.2m. Because charged particles are free to move along a magnetic field line, a magnetic surface must also be a plasma pressure iso-surface. Thus, the shape of the magnetic surfaces defines the shape of the plasma pressure (density and/or temperature) surfaces.

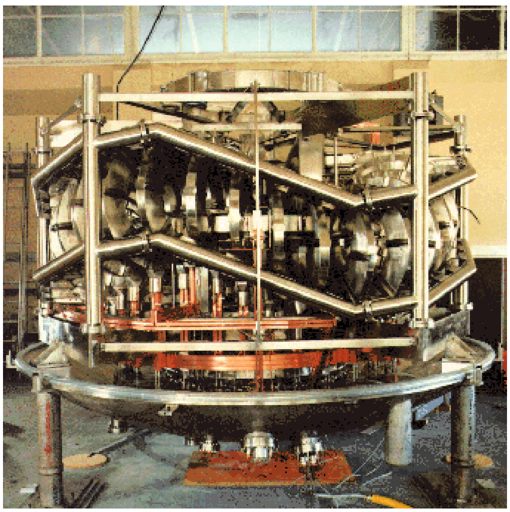
The surface shape can be measured by launching electrons in the direction of the magnetic field using a simple heated filament inserted into the magnetic volume, and measuring their point of intersection with a fluorescent screen inserted between two toroidal field coils in a poloidal cross-section.

1.5.2 Operations

In its present arrangement, the H-1NF water-cooled magnetic coil set is operated cw to produce a magnetic field strength on axis of the machine that can be varied between 600G and 10000G (1. Tesla). The coils are powered by a computer controlled transformer-rectifier system directly from the mains. A maximum of 14 kA current is provided in series to all of the coils with a maximum pulse duration of 1s limited by ohmic heating of the copper conductors.



(a)



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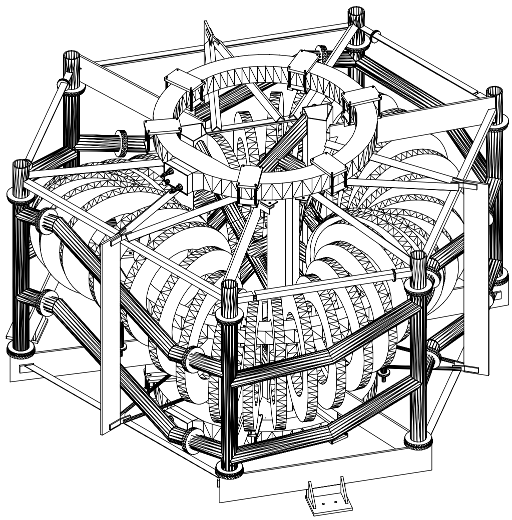


(c)

Figure 1.10: H-1NF. (a) Artist's impression (the plasma is shown in red), (b) during construction and (c) coil support structure.

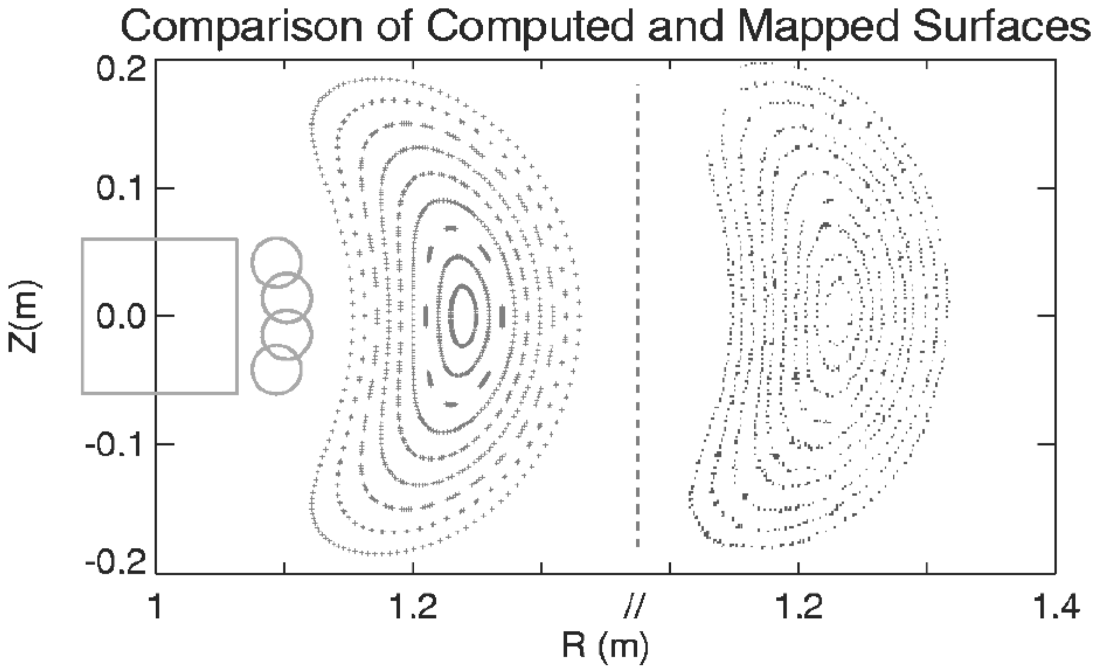


Figure 1.11: Computed magnetic surfaces (left) and surfaces measured using electron gun and fluorescent screen (right).

The operating gases are typically argon, neon, helium or hydrogen. Plasma is produced by filling the vacuum tank (base pressure $\sim 10^{-7}$ Torr) to a fill pressure in the range $\sim 10^{-5} \rightarrow\sim 10^{-4}$ Torr) and igniting the discharge using a high power (up to 500kW) pulse (typically 50-500 ms duration) of radio-frequency radiation in the range 4-28 MHz. The rf radiation is transmitted to the plasma via antennas that are matched in impedance to the plasma for optimum power transfer. The rf antennas conform in shape with the plasma edge and are effective at launching a type of plasma wave known as a "helicon" that damps effectively on plasma electrons. There also appears to be strong heating of plasma ions that is believed to be related to the high rf oscillating electric fields in the vicinity of the antenna.

An alternative heating scheme uses high frequency microwaves (28 GHz, 200kW) to resonantly heat electrons (ECRH). This system is being installed and will be operational in 1999.

By changing the current in the helical control winding that wraps around the central planar ring conductor, H-1NF can access a wide variety of magnetic configurations. Part of the experimental program is to study the properties of the plasma produced in these various configurations. Such properties include the effectiveness at confining heat and particles, the resilience of the plasma to various instabilities that can be excited at high plasma pressure, and the effectiveness of various heating schemes for the electrons and/or ions.

1.5.3 Control system

The H-1 heliac is a 1 Tesla, 1 metre radius plasma confinement machine, powered by a 13kA 8MW phase-controlled rectifier. Many of the subsystems (magnet supply, rf heating, water cooling, vacuum) are monitored and some are controlled by a computer control and display system. The interface is designed along 'fail-safe' lines to minimize the reliance on the computer for protection of equipment, while allowing the computer system to detect subtle faults and notify the operator or take some action automatically.

The control computer has a colour graphics display and touch-screen and track-ball input, and connects through a simple CAMAC interface to chain of 7 control racks distributed through the laboratory. These communicate on a 32 wire multiconductor ring bus with individual optical isolation at each station. Typical control modules plug into a "eurocard" bus and include 16 bit input and output registers (optical and relay), ADCs and thermocouple preamplifiers.

The control program is written in C++ using object-oriented programming, driven by a data file that describes the connections between the "sensors" and "displays" that make up the system. A reverse polish macro language serves for improvising new functions until they are incorporated into the main C++ code.

1.5.4 Data acquisition

The signals from the various H-1 diagnostics are cabled through rigid copper ducting that minimizes rf pickup and electrical noise, to an electrostatically shielded room that houses the high speed data acquisition hardware. The information from the various systems is acquired using either synchronously or independently clocked analog-to-digital (ADC) converters.

The hardware is controlled over an ethernet link from a DEC alpha work station using a commercially available, menu-driven software package for configuring and controlling the various digitizers. The acquired data are stored on disk in a database file that facilitates easy access. Two software packages based on the IDL programming language, and using a user-friendly graphical user interface have been developed for interacting with the H-1NF database.

Problems

Problem 1.1 For the following plasmas, calculate the Debye length (λ_D), plasma frequency (f_p Hz), plasma parameter (Λ) and Coulomb collision frequency $\nu_{\rm ei}$.

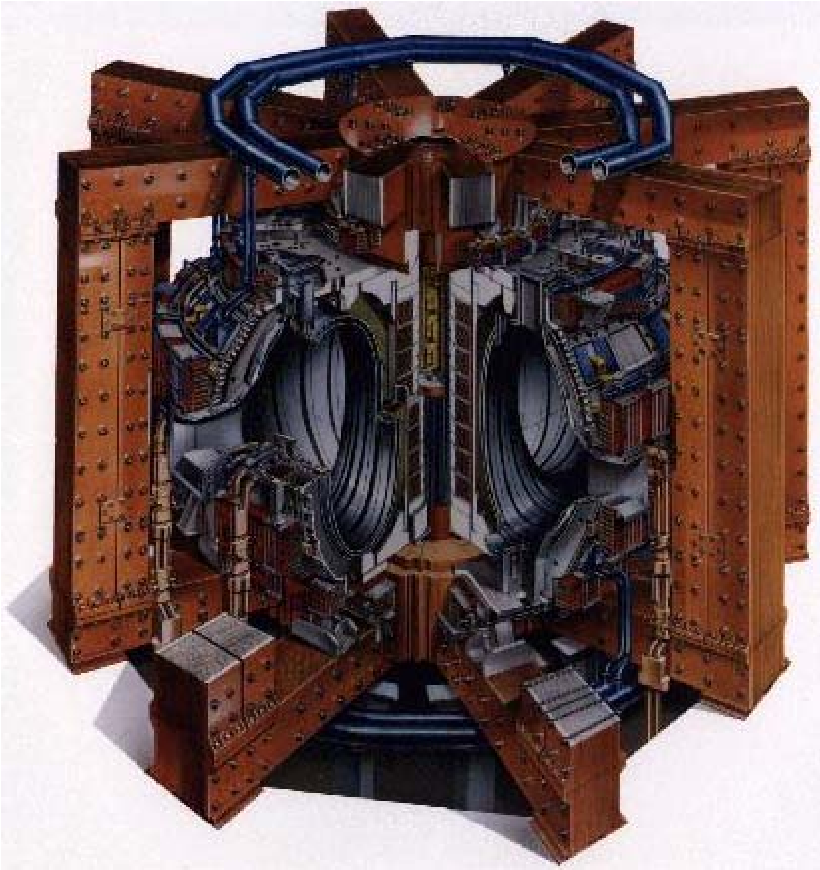
a. Processing plasma (argon) $n = 10^{15}$ m^{-3}, k$\mathrm{T}_e = $ k$\mathrm{T}_i = 1$ eV

b. Fusion plasma (hydrogen) $n = 10^{20}$ m^{-3}, k$\mathrm{T}_e = $ k$\mathrm{T}_i = 10$ keV

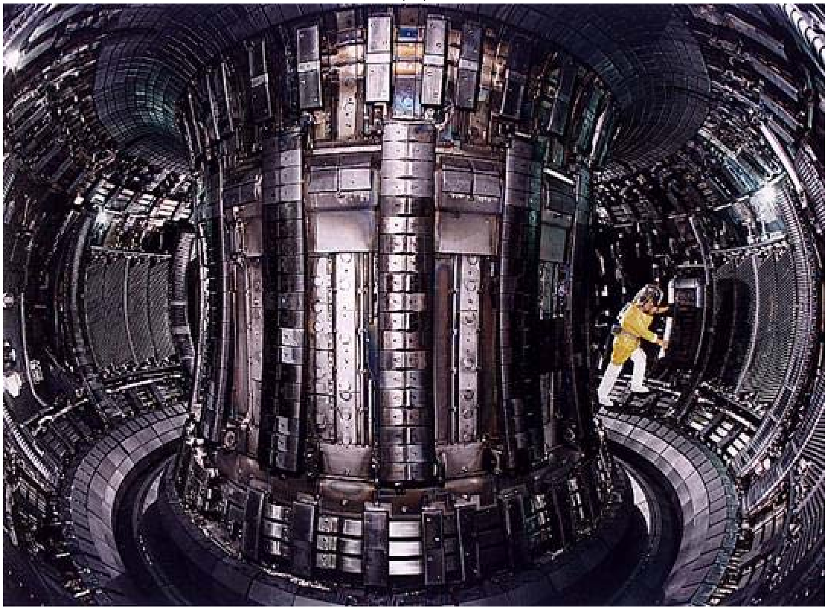
c. Fusion plasma at edge (hydrogen) $n = 10^{18}\ \mathrm{m}^{-3}$, k$\mathrm{T}_e$ = kT_i = 10 eV

Assume parameters of c. Calculate the energy flux $\Gamma_E = \bar{U}\Gamma_n = 3/2kT\, n\bar{v}$ in W/sq m. What is the temperature rise of the wall if the plasma lasts one second, the wall thickness is 1 cm and the specific heat is 4 J/cc/K.

Problem 1.2 Caluclate the cyclotron frequencies for electrons and ions for the various conditions given in Table 1.2. Defining a collisionality parameter ν/ω_p for each species, in each case, comment on whether or not the electrons and ions are effectively magnetized.



(a)



(b)

Figure 1.12: Joint European Torus (JET) tokamak. (a) CAD view and (b) inside the vacuum vessel. From JET promotional material.

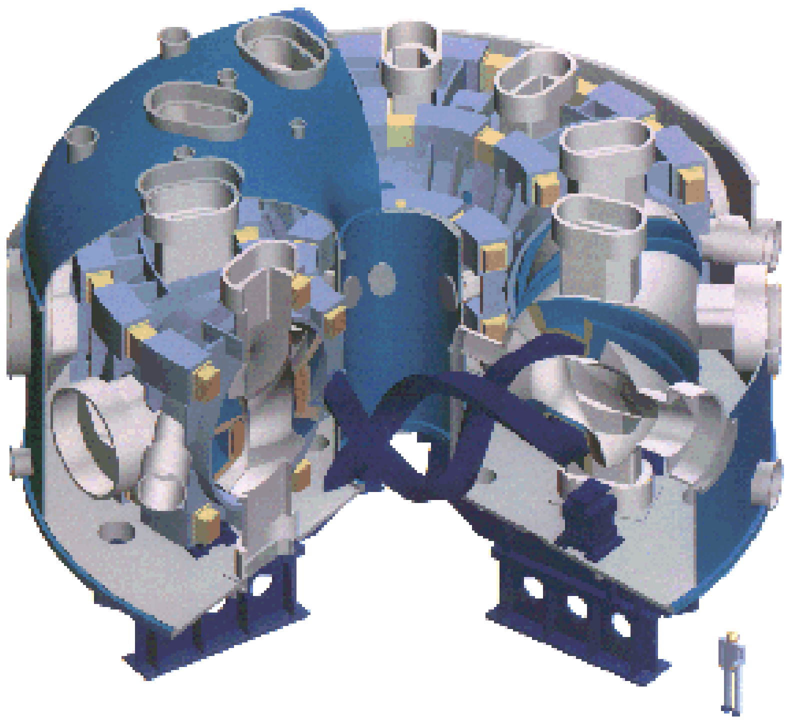


Figure 1.13: Large Helical Device (LHD) heliotron.

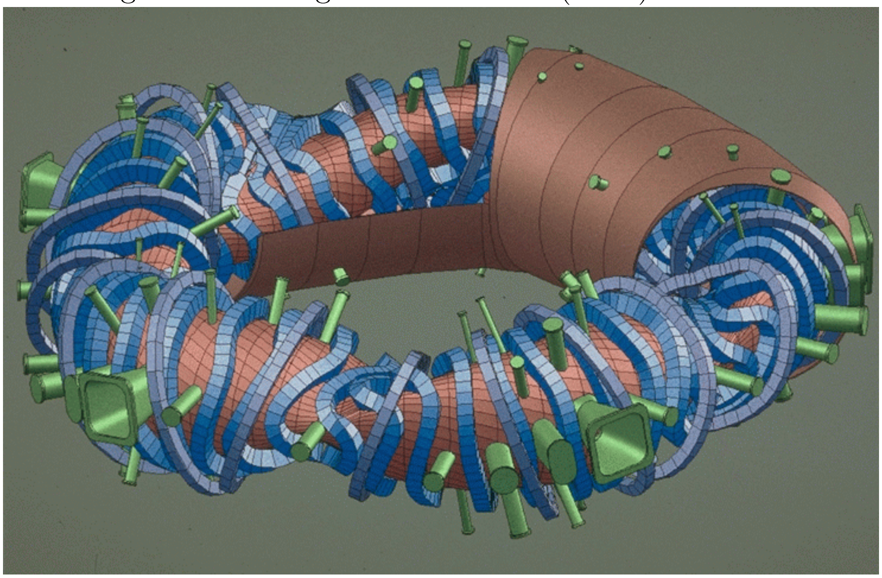


Figure 1.14: Coil system and vacuum vessel for the Wendelstein 7-X (W 7-X) modular helias.

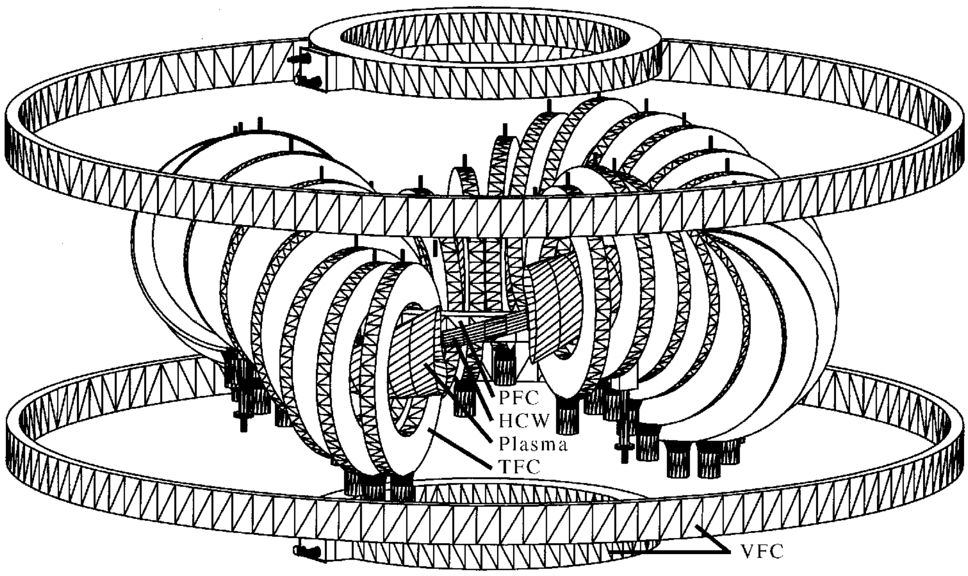


Figure 1.15: Coil system and plasma of the H-1NF flexible heliac. Key: PFC: poloidal field coil, HCW: helical control winding, TFC: toroidal field coil, VFC: vertical field coil.

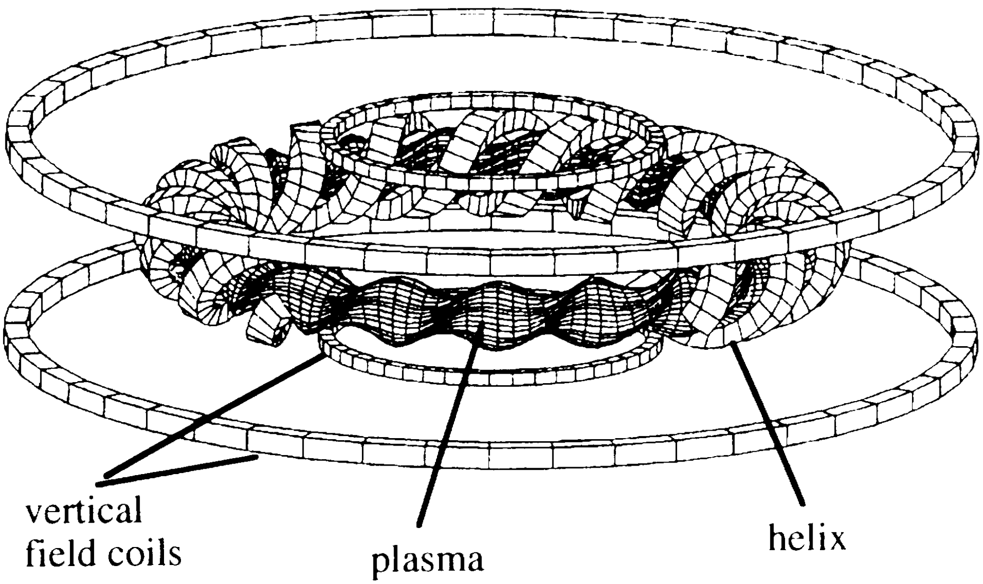


Figure 1.16: Plasma and coil system for the Heliotron-E torsatron.