Circuit Mathematics – Chapter 2: Power and RMS

RMS – a shortcut for power calculation in \textit{linear} circuits

\textbf{Power} is $V \times I$ – for time-varying circuits, this is known as \textit{“Instantaneous Power”}\n
The \textbf{average power} $<P>$ over interval $T$ is

$$<P> = \frac{1}{T} \int_0^T V I \, dt = R/T \int_0^T I^2 \, dt \quad \text{(if $V=IR$).}$$

Define \textbf{RMS current $I_{\text{RMS}}$} as the \textit{equivalent current which causes the same average heating effect as the actual, time-varying current} in a linear circuit,

then $R \, I_{\text{RMS}}^2 = R/T \int_0^T I^2 \, dt \quad \rightarrow$

$$I_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T I^2 \, dt} \quad \text{.........................(1)}$$

Remember:  
1/ RMS strictly applies (to allow power calculations) only to \textit{linear} components, but allows \textbf{arbitrary} waveforms.  
2/ For sinusoids only, the $1/\sqrt{2}$ relationship holds. $V_{\text{RMS}} = (1/\sqrt{2})V_{\text{Ampl}}$  
3/ \textbf{Otherwise}, use the \textbf{integral} form (or RSS see below).
RSS – summing RMS for uncorrelated signals

RSS: shortcut for power calculation in linear circuits when RMS values are known

\[ I_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T I^2 \, dt} \] \hspace{1cm} (1)

**Root Sum Squares:**

Extend RMS concept to harmonics, or any set of uncorrelated sinusoids (more strictly “orthogonal”):

\[ I_{\text{RMS}} = \sqrt{\sum_{k} I_{\text{RMS}}^2(k)} \] \hspace{1cm} (2)

This works because, in the integral (Equn 1), for cross terms \( V_I \) if \( I \) and \( V \) have different frequencies, the products of \( I \) and \( V \) have zero average over time – only \( I \) and \( V \) with matching frequencies cause real power dissipation or flow.

**Example:** The spectrum of the current in a heater element, resistance of 24Ω, is

- 50Hz: 10A RMS
- 100Hz: 0.1A RMS
- 150Hz: 1A RMS

What is the RMS Current?

**Answer:** These are harmonics, so they are orthogonal – can use RSS of individual RMSs

\[ I_{\text{RMS}} = \sqrt{10^2 + 0.1^2 + 1^2} \]

\[ I_{\text{RMS}} = 10.05 \text{ Amps} \]

What is the average power dissipation?

\[ P = R \times I_{\text{RMS}}^2 = 2,424 \text{ Watts} \]

(Actually pretty close to 2400 Watts – goes to show that the harmonics don’t provide much heat (but not always the case – e.g. transformers, motors).

Mathematics for AC Circuit Analysis: Blackwell, Walshe

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**Fourier Series**

Mohan: 3.2.4.1 Fourier Analysis of Repetitive Waveforms

In general, a nonsinusoidal waveform \( f(t) \) repeating with an angular frequency \( \omega \) can be expressed as

\[ f(t) = F_0 + \sum_{h=1}^{\infty} f_h(t) = \frac{1}{2}a_0 + \sum_{h=1}^{\infty} \left( a_h \cos(h\omega t) + b_h \sin(h\omega t) \right) \] \hspace{1cm} (3-21)

where \( F_0 = \frac{1}{2}a_0 \) is the average value. In Eq. 3-21.

\[ a_h = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(h\omega t) \, d(\omega t) \hspace{1cm} h = 0, \cdots, \infty \] \hspace{1cm} (3-22)

\[ b_h = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(h\omega t) \, d(\omega t) \hspace{1cm} h = 1, \cdots, \infty \] \hspace{1cm} (3-23)

From Eqs. 3-21 and 3-22, the average value (noting that \( \omega = 2\pi T \))

\[ F_0 = \frac{1}{2\pi} \int_0^{2\pi} f(t) \, d(\omega t) = \frac{1}{T} \int_0^T f(t) \, dt \] \hspace{1cm} (3-24)

In Eq. 3-21, each frequency component \( [f_h(t) = a_h \cos(h\omega t) + b_h \sin(h\omega t)] \) can be represented as a phasor in terms of its RMS value,

\[ F_h = F_{h0} e^{j\phi_h} \] \hspace{1cm} and phase \( \phi_h \) is given by

\[ \tan(\phi_h) = \frac{-b_h}{a_h} \]

where the RMS magnitude

\[ F = \left( F_0^2 + \sum_{h=1}^{\infty} F_h^2 \right)^{1/2} \]

Mathematics for AC Circuit Analysis: Blackwell, Walshe
Power Factor

Power, Apparent Power, Power Factor

Power (the rate of doing work) is defined as V.I instantaneously. However, because
- AC varies in time and
- energy can flow back and forth between inductors and capacitors (and motors),

the actual work done must obtained by integrating in time over those flows (normally one cycle is enough), and so we can defined average power as

\[
<P> = \frac{1}{T} \int_0^T VI \, dt
\]

For sinusoidal waveforms, and linear loads, we can use the RMS value and the phase difference between V and I to avoid integration.

Average Power is NOT simply \(V_{RMS} \times I_{RMS}\), except for resistors.

We define Apparent Power as \(V_{RMS} \times I_{RMS}\)

Power Factor is the ratio of Real Power to Apparent Power

\[
\text{Power Factor} \quad PF = \cos(\alpha)
\]

Example: If \(V = 240 \angle 0\) and \(I = 2 \angle -65^\circ\) then phase lag \(\alpha = +65^\circ\) and \(PF = \cos(\alpha) = 0.423\). Also \(Z = V/I = 120 \Omega \angle 65^\circ\) has \(\text{arg} = +65^\circ\), a positive imaginary part. We can deduce that the load is a lossy inductor – an ideal inductive load would have a phase lag of 90°.

Power Factor, Phasors

Power Factor is the ratio of Real Power to Apparent Power (Apparent Power: \(V_{RMS} \times I_{RMS}\))

\[
\text{Power Factor} \quad PF = \cos(\alpha)
\]

Example: If \(V = 240 \angle 0\) and \(I = 2 \angle -65^\circ\) then phase lag \(\alpha = +65^\circ\) and \(PF = \cos(\alpha) = 0.423\). Also \(Z = V/I = 120 \Omega \angle 65^\circ\) has \(\text{arg} = +65^\circ\), a positive imaginary part. We can deduce that the load is a lossy inductor – an ideal inductive load would have a phase lag of 90°.

So

\[
\text{Power} = V \times I \times \text{Power factor} = V I \cos(\alpha)
\]

\textbf{RMS phasor notation} is conventional unless time or \(e^{\omega t}\) appear explicitly.

So \(240 \angle -15^\circ\) would look as illustrated, and refers to a waveform of RMS voltage 240V, phase of \(-15^\circ\) or a phase lag of \(+15^\circ\). So the voltage would be written as a function of time as

\[
V(t) = 339.4 \cos(\omega t + 15^\circ\pi/180), \text{ where } \omega = 2\pi f \sim 314.6 \text{ rad/sec}
\]

or

\[
339.4 \cos((\omega t - 15^\circ\pi/180)) = 339.4 \cos(\omega t - 15^\circ\pi/180) + j \sin(\omega t - 15^\circ\pi/180)
\]

Note the \(\sqrt{2}\) difference between the phasor notation and the explicit time variation.

Phasors rotate \textbf{anti-clockwise} in time.

Fig 1. is a vector representation of \(V_a\) at three consecutive time intervals.

A waveform delayed in time (see \(V_b\), dashed, red) is said to have a phase \text{lag}.

\(\rightarrow\) negative imaginary part, and the magnitude phase representation has a negative angle (\(240 \angle -15\) in the figure). But …….

Mathematics for AC Circuit Analysis: Blackwell, Walshe
Power Factor, Phasors (ii)

Phasors rotate anti-clockwise in time. Fig 1. is a vector representation of $V_a$ at three consecutive time intervals. A waveform delayed in time (see $V_b$, dashed, red) is said to have a phase lag, $\rightarrow$ negative imaginary part, and the magnitude phase representation has a negative angle ($240^\circ - 15^\circ$ in the figure). But……

Consider the current in an ideal inductor lags the voltage by $90^\circ$, $\rightarrow$ current phasor has a negative imaginary part, and a negative angle ($\angle -90$).

By the same token, an inductor has a lagging power factor, but its power factor is given with a positive angle and positive complex power $S$.

Be careful!

Summary:
A lagging power factor is given a positive angle (corresponds to the complex power and impedance), but the current (which lags the voltage) has a negative angle and imaginary part.

$$I = I_0 e^{j(\omega t + \phi)}$$

if $I = I_0 e^{j(\omega t + \phi)}$, then $\phi$ is $-ve$ for lag.

Complex (and Imaginary or Reactive) Power

$Q$ (or “Reactive Power”)

$$Q = VLN ILine \sin(\alpha)$$. Per phase……………………………………..(7)

$V,\cos$ does no work against $I,\sin$: Energy oscillates in and out of load, between the inductance and capacitance of the system.

Reactive power” is inaccurate (it is more like the imaginary part of the power).

The complex power (Volt-amperes) is defined as

$S$ (or “Complex Power”)

$$S = V^T \times I^{conj}$$

NB :-for matrix use both $V$ & $I$ are single column vectors (e.g. $1 \times 3$ for three phase) so the transpose of $V$ ($V^T$) must be taken.

In single phase systems this reduces to

$$S = P + jQ \quad \text{where} \quad P = \text{power (kW)} \quad \text{and} \quad Q = \text{VARs} \quad \text{as defined previously}$$

This can be rearranged as $|V||I| \angle \delta \quad \text{where} \quad \delta$ is the phase difference ($\angle V - \angle I$)

NB :-the definition of the sign of VAR is important: “positive VAR are consumed by an inductive load and generated by a capacitive load”. For example, in Equn 8 we use $\delta = (\angle V - \angle I)$. From this definition it follows that:

1. a capacitive load consumes negative VAR and generates positive VARs
2. $S$ is calculated as voltage times the conjugate of current
3. $S^2 = P^2 + Q^2$
Complex Power (ii)

The complex power (Volt-amperes) is defined as

\[ S = V \times I^\text{conj} \]

NB: For matrix use both V & I are single column vectors (e.g. 1x3 for three phase) so the transpose of V (V^T) must be taken.

In single phase systems this reduces to

\[ S = P + jQ \]

where \( P = \text{power (kW)} \) and \( Q = \text{VARs as defined previously} \ldots \ldots \) (8)

This can be rearranged as \( |V| |I| \angle \delta \) where \( \delta \) is the phase difference \( (\angle V - \angle I) \)

Why is complex power \( S = V \times I^* \)?

We need to use a formalism which depends only on phase difference between V and I;

\( \rightarrow \) the answer should not be changed if the phases of V and I are both changed by the same amount.

In three phase, \( V_B \) is displaced in phase \( \angle 120^\circ \), but the average power is still the same.

Example: Calculate the power when \( Z = 1 \Omega \) (i.e. real), and \( V = 2+j \). (as \( |V| = \sqrt{5} \), we know the power is \( V^2/Z = 5 \)).

\[ S = V \times I^* = (2+j)(2-j) = 4 - 1 + j(2-2) = 5 \text{ Watts (Real only – reactive power = 0)} \]

This is correct as V and I are in phase, so we expect P=V^2/Z, and it is. If we had (incorrectly) used \( V \times I \), (omitting the *) we would have calculated 3 Watts + 4[VAR] (4 units of reactive power).

Note also that Power = Real(S), and \( \text{VARs} = \text{Imag}(S) \).

Transformers and Per-Unit Impedance Notation

This is introduced in Chapter 2 at this point, but we won’t use until Ch 3 and 4 (Transformers and Motors)

Details of Transformers and motors are often described more simply using impedances, voltages, currents and power in per-unit notation, where the quantity is expressed as a fraction of a base unit, usually the nominal or “identification plate” power of the device.

e.g. if a transformer designed for 110kVA power transmission at a nominal voltage of 11kV, the primary current would be 100A, and the ratio \( V/I \) is 100Ω This is the Base Impedance

If the equivalent series impedance (due to losses and stray inductance) was 5Ω, then it would be described as

0.05 (or 5%) per unit, referred to the nominal ratings.

The advantage of this is that it gives a quick idea of the worst voltage loss under load, which is about 5% of the input voltage.

It is not the exact voltage loss or drop unless the voltage drop is in phase with the voltage across the transformer. Usually it is less (and could be 0 or negative). The phasor diagram shows the relationship btw. Vin and Vload.
Why AC?

- **Transformers:** change voltage/current level to suit mechanical design considerations
  - for long-distance transmission, high voltage low current reduces I^2R loss, c.f.
  - low voltage for safe distribution inside buildings.
- **Transformers:** isolate for safety – no electrical path between primary and secondary circuits
- **Fuses, Circuit Breakers** interrupt AC much more readily than DC – regular current zeros.
- **Sliding contacts are much simpler (or not required at all)** in AC motors
  - higher currents and powers are possible.
  DC motors must have a multi-segment commutator
  AC motors can use “slip rings” - simple sliding contacts, or induction, (no contact!).

Why three phase?

- More efficient transmission (less copper)
  - regard as three separate circuits, \(V_o, V_o \angle -120^\circ, V_o \angle -240^\circ\), return currents cancel
  - no return conductor required in theory
  \(\Rightarrow\) half the conductor compared to single phase!
- **Natural source of rotation for machinery** (in principle a two phase, quadrature \(V_o, V_o \angle -90^\circ\), system would work too, but neutral is not balanced, so still need three wires)
- **Interfaces better with rectifiers** – load current has much less ripple, line current drawn is closer to a sinusoid than single or two phase.
- Naturally extends to 6 phase by simple inversion using a 3\(\phi\) transformer. (invert v)

### Three Phase Power: the Wye and Delta circuits

The complex power (Volt-amperes) is defined as \(S = V^T \times I^{conj}\)

For one phase of a three phase circuit, we need to be careful so “double-counting” is avoided.

The voltage and currents in the loads (Y’s) are different by a factor of \(\sqrt{3}\) in an ideal circuit (if balanced \(V_{CN} = \sqrt{3} V_{CN}\)), so it is important to be consistent

- if we measure \(V_{CN}\), then we should use the corresponding current into terminal C (Ic) when calculating power. \(V_{CN}\) is known as \(V_{LINE-NEUTRAL}\), \(V_{LL}\) or \(V_{PHASE}\). \(I_c(I_{BC}, I_{CA})\) are known as \(I_{LINE}\).

Note that the short names, \(V_{PHASE}, V_{LINE}\), although used commonly in practice, can be confusing! Try to use the long name (\(V_{LINE-NEUTRAL}\)). Similarly \(V_{CA}\) is known as \(V_{LINE-LINE}\) or \(V_{LINE}\) and \(I_{CA}\) is known as \(I_{LINE-LINE}\) or \(I_{PHASE}\). Once again, use the long form to avoid confusion. Also note that \(I_c = I_{CA} + I_{CB}\) or \(I_{CA} - I_{BC}\).

\[
\text{Power} = V_{PHASE} \times I_{LINE} \times \text{Power factor} = V_{LN} I_{Line} \cos(\alpha) \quad \text{(per phase)} \ldots \ldots (5)
\]

For balanced three phase circuits, \(\text{Power(3)} = 3 V_{PHASE} I_{LINE} \cos(\alpha) \quad \ldots \ldots (6)\)

In “mixed” quantities this is equivalent to \(\sqrt{3} V_{LL} I_{Line} \cos(\alpha) = \sqrt{3} V_{LINE} I_{LINE} \cos(\alpha)\)

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AC/Three phase Calculations

neutral currents, un/balanced loads, and three wire systems.

• Consider a case where a solidly grounded star connected load comprises impedances of
  
  \[ \text{Z}_a = 10 \Omega \angle 15^\circ, \text{Z}_b = 12 \Omega \angle 7^\circ & \text{Z}_c = 4 \Omega \angle 32^\circ \]

\[ h := \frac{-1}{2} + \frac{\sqrt{3}}{2}j \]

Unbalanced example

<table>
<thead>
<tr>
<th>Phase</th>
<th>Impedance</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;a&quot;</td>
<td>Za = 10 \Omega \angle 15^\circ</td>
<td>Va = 240</td>
</tr>
<tr>
<td>&quot;b&quot;</td>
<td>Zb = 12 \Omega \angle 7^\circ</td>
<td>Vb = Va \times b^2</td>
</tr>
<tr>
<td>&quot;c&quot;</td>
<td>Zc = 4 \Omega \angle 32^\circ</td>
<td>Vc = Va</td>
</tr>
</tbody>
</table>

If there is no influence from the supply or cables, and if the neutral is solidly grounded at the transformer

<table>
<thead>
<tr>
<th>Phase</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;a&quot;</td>
<td>V_a</td>
</tr>
<tr>
<td>&quot;b&quot;</td>
<td>V_b</td>
</tr>
<tr>
<td>&quot;c&quot;</td>
<td>V_c</td>
</tr>
<tr>
<td>Neutral</td>
<td>V_n</td>
</tr>
</tbody>
</table>

\[ I_a := \frac{V_a}{Z_a}, \quad I_b := \frac{V_b}{Z_b}, \quad I_c := \frac{V_c}{Z_c}, \quad I_n := -(I_a + I_b + I_c) \]

\[ \text{Phase"a"} \quad I_a = 23.182 - 6.212j, \quad |I_a| = 24 \]
\[ \text{Phase"b"} \quad I_b = -12.036 - 15.973j, \quad |I_b| = 20 \]
\[ \text{Phase"c"} \quad I_c = 2.094 + 59.963j, \quad |I_c| = 60 \]
\[ \text{Neutral} \quad I_n = -13.24 - 37.779j, \quad |I_n| = 40.032 \]

: Obtaining Z, given power and power factor

Lagging PF \( \rightarrow \) positive reactive power (consumption), care with conj: \( S = VI^* \). Lag corresponds to a negative phase angle (e.g. \( \angle -60^\circ \)) for waveforms, but a positive impedance phase angle (as \( Z = V/I \): current lags in an inductor, phase is negative, but division makes phase or imaginary part of \( Z \) positive).

Assume we have a largely inductive (i.e. lagging) load. \( Z \) has a positive imaginary part, and as

\[ Z = V/I, \quad V = ZI, \quad \text{so } S = VI^* = Z(I^*) \], so \( S = Z|I|^2 \).

That is, \( S \) has the same signed imaginary part as \( Z \). Similarly (and more usefully in practice),

\[ Z = |V|^2/S^* \quad \text{i.e. } Y = S^*/|V|^2 \]

- very handy for circuit calculations (but be careful of the \( 1/S^* \), the sign of the imaginary part flips twice). Also note that Mohan uses bold \( S \) for the complex power and italic \( S \) for the absolute value of bold \( S \). We will normally use the convention that \( S \) is always complex unless specifically noted.

Example: You are given that a load is a 5kW cooling fan, connected to a single phase of a 415V line-line system with a power factor of 0.71, lagging. This means that the load is partially inductive, and that the current lags the voltage. (The units “kW” show that 5kW refers to the Real power \( P \).)

Now \( S = P + jQ \), and \( \cos(\phi) = 0.71 \), so \( Q/P = \tan(\phi) = 1 \) (see note below) , so \( S = 5e3 (1 + j) \).

Using the above info, the line to neutral voltage is \( 415/\sqrt{3} = 240V \) (all voltages in RMS), \n\[ Z = |V|^2/S^* = 240^2/(5e3-j5e3) = 5.76\Omega + j5.76\Omega. \]
Nodal Admittance Matrix Method

Nodal Admittance Matrix ("Node Voltage Method" in Schaum/Edminster)

Power networks → large number of nodes whose voltage is referenced to a common node.

To a good approximation we know the voltages, and we need the current

→ Admittance makes sense: \[ \text{[I]} = \text{[[Y]]}[\text{V}] \]

No. of equations needed = number of nodes minus one (which is the ref. node).

In contrast, if a mesh impedance solution is attempted, the number of independent equations depends on the number of nodes and branches i.e. it depends on the precise topology.

In the column vector "I" non-zeros only occur at nodes where a generator is connected.

Entries in “Y” can be written down by inspection. Consider a node (1) with a generator connected, a shunt admittance \( Y_{10} \) to the common node “0” and connections to several other nodes. Apply Ohm’s Law (for admittance) to obtain \( I_1 \) via Kirchoff’s Current Law:

\[
I_1 = (Y_{10} + Y_{12} + Y_{13} + \ldots + Y_{1n})V_1 + (-Y_{12}V_2 - Y_{13}V_3 - \ldots - Y_{1n}V_n)
\]

Re-arranging these terms gives

\[
I_1 = (Y_{10} + Y_{12} + Y_{13} + \ldots + Y_{1n})V_1 + Y_{10}V_1 - Y_{12}V_1 - Y_{13}V_1 - \ldots - Y_{1n}V_1
\]

Thus the nodal admittance matrix can be written down in an automated manner without any reference to network topology.

### Current into node | Current to ref node | Current flow to other nodes
|\( I_1 \) | \( Y_{10} * V_1 \) | \( Y_{12} * (V_1 - V_2) \)
|\( I_1 \) | \( Y_{13} * (V_1 - V_3) \)
|In general| \( Y_{1n} * (V_1 - V_n) \)

This leads to the general rule for the “automatic” formulation of the Nodal Admittance matrix:

- The diagonal term is the sum of all admittances connected to the node,
- The off-diagonal term is the negative of the admittance between the respective nodes, and
- The matrix is symmetrical

Nodal Admittance Matrix Method

\[
I_1 = (Y_{10} + Y_{12} + Y_{13} + \ldots + Y_{1n})V_1 + (Y_{12}V_2 - Y_{13}V_3 - \ldots - Y_{1n}V_n)
\]

- The diagonal term is the sum of all admittances connected to the node,
- The off-diagonal term is the negative of the admittance between the respective nodes, and

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} =
\begin{bmatrix}
Y_{CA} + Y_{AB} & -Y_{AB} & -Y_{CA} \\
-Y_{AB} & Y_{AB} + Y_{BC} & -Y_{BC} \\
-Y_{CA} & -Y_{BC} & Y_{BC} + Y_{CA}
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix}
\]
Nodal Admittance Matrix Method:
Partitioning to eliminate internal nodes

To solve the case of the broken neutral we had to eliminate a node by using $I_N=0$.

Provided an equation for $V_N = (Y_AV_A + Y_BV_B + Y.CV_C)/(Y_A + Y_B + Y_B)$

The technique can be generalised for the removal of any or several internal nodes.

If $I = YV$ …………………………………………………(1)

and $I$, $V$ can be reordered into source nodes and internal nodes $I = [I_S, I_i]$

\[
\begin{bmatrix}
I_S \\
I_i
\end{bmatrix} = 
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
Y_S \\
Y_i
\end{bmatrix}
\] ……. (2) where $A, B, C, D$ are different size sub-matrices

(e.g. prob. p2-11) $I_A = [Y_AV_A + 0V_B + 0V_C] + [-Y_AV_N]$ i.e. $A = \begin{bmatrix} Y_A & 0 & 0 \\ 0 & Y_B & 0 \\ 0 & 0 & Y_C \end{bmatrix}$, $B = \begin{bmatrix} -Y_A \\ -Y_B \\ -Y_C \end{bmatrix}$

$C = [-Y_A \ -Y_B \ -Y_C]$ and $D = [Y_A + Y_B + Y_C]$

Challenge: Parts of the LHS AND RHS are unknown – (see circles – we know $I_i=0$, need $I_S$, elim. $V_i$)

Solution: $\begin{align*}
I_S &= AV_S + BV_i \quad \text{(expanding (2))} \\
I_i &= CV_S + DV_i \quad \text{(4)}
\end{align*}$

$I_i = 0$ in (4) $\Rightarrow V_i = -D^{-1}CV_S$, and sub this in 3 to eliminate $V_i$ $\Rightarrow$

\[
I_S = AV_S + B(-D^{-1}CV_S) \Rightarrow I_S = (A - BD^{-1}C)V_S
\]